



## International Journal of Control Theory and Applications

ISSN : 0974-5572

© International Science Press

Volume 10 • Number 35 • 2017

### An Efficient Adaptive Decision Feedback Equalizer based on Variable Step Size Strategy for Inter Symbol Interference Cancellation

K. Murali Krishna<sup>a</sup>, Md. Zia Ur Rahman<sup>b</sup>, G. Bhargavi<sup>c</sup>, A. Deepthi<sup>d</sup>, B. Hyndavi<sup>e</sup> and D. Lakshmi<sup>f</sup>

<sup>a-f</sup>Department of E.C.E., KKR & KSR Institute of Technology & Sciences, Vinjanampadu, Guntur, A.P, India. Email: <sup>a</sup>muralikondaveeti1@gmail.com; <sup>b</sup>mdzr55@gmail.com

**Abstract:** One of the most challenging problems encountered in wireless data communication is Multipath broadcasting. It introduces signal fading, delay spread, and Doppler spread, and can significantly impair the performance of a data communication system. For low-error-rate, high-speed wireless data communications Multi-path mitigation techniques such as adaptive decision-feedback equalizers (ADFE) are thus required. This paper examines these methods for wireless data communications. It is the efficient way for solving the signal fading problems. But, wireless radio channels exhibit frequency-selective fading which causes inter-symbol interference (ISI), as a result the receiver alone cannot provide satisfactory performance, and more effective signal processing techniques are often required. Adaptive equalization is known to be an effective measure against ISI. In this paper two versions of Least Mean Square (LMS) algorithm i.e., Normalized LMS and Variable step size LMS algorithms are proposed for adaptive decision feedback equalizer, which is suitable for cancellation of ISI and high data rate wireless. In the transmission of digital signals physical channels can be rarely represented by a nondistorting channel model with additive noise. The performance analysis of various adaptive algorithms is done in terms of bit error rate (BER) and signal to noise ratio (SNR). The variable step size ADFE shows better performance compared to normalized LMS.

**Keywords:** Adaptive, Bit error rate, Equalizer, ISI, Variable Step size.

#### 1. INTRODUCTION

In a wireless communication channel, the desired signal from the transmitter antenna generally propagates to the receiver antenna through several different paths. This phenomenon is called multipath propagation. Multipath propagation is caused due to the multiple reflections originated by reflectors and scatterers in the environment. Possible reflectors and scatterers may contain hills, mountains and trees in rural areas, buildings and vehicles in urban environments, or floors and walls in indoor environments. Therefore the receiver antenna will receive several copies of the transmitted signal. The output of the receiver antenna is the sum of the multiple transmitted signal copies weighted by the antenna gain pattern. Delay spread is caused due to the multipath propagation.

In a digital communication system, narrow pulse sequence is used to indicate the stream of desired signal bits. Due to delay spread, each narrow pulse is widens in time when transmitted through a wireless channel. As a result, succeeding pulses interfere with each other, causing inter-symbol interference (ISI). This problem can be overcome by a special form of discrete time filter which is known as equalizer.

Equalizer is a filter that compensates the scattering effect of the transmission channel. In fact the signal is suffering from the ISI while transmitting through the channel and at the receiver front end white Gaussian noise is added. To overcome the effect of the channel, different equalization techniques are used at the receiver end. Decision feedback equalizer [4] uses previously detected decisions to eliminate ISI from the received symbol. The distortion on the current symbol that is caused by the previous symbol is subtracted. An adaptive decision feedback equalizer is designed by using least mean square algorithm. In [1-2] the problem of control algorithms for the ISI cancellation is proposed.

The fading and multipath acoustic channel has always been a great impediment to building reliable communication systems. Many complex physical phenomena cause the propagating acoustic wave intensity and phase to vary temporally and spatially. Thus, one advantage of spatial diversity equalization is its ability to improve the limited signal-to-noise ratio at the receiver through coherent combining. The multi-path spread in these channels is largely caused by reflections from ocean boundaries and refraction due to sound speed variations as a function of depth. Since the degradation in the signal is caused by multipath inter symbol interference (ISI), simply increasing the signal-to-noise-ratio will not alleviate the problem. It is well known that coherent equalization techniques improve the bandwidth efficiency of the communication system, thus increasing the data rate [6]. Equalization techniques are well understood in radio communications [5-7]. A variety of equalization techniques are available, such as zero-forcing equalization, minimum mean square error (MMSE) equalization [3]. Decision Feedback Equalization (DFE) can be considered an effective technique because it can help to eliminate causal ISI in addition to compensating for the channel. When the communication channel is unknown to the designer, adaptive equalization techniques can be used to first extract the channel response from a training sequence and then compensate the channel distortion in the incoming data symbols [8]. The basic adaptive algorithm is least mean square algorithm [9]-[10]. In this paper we proposed two techniques for adaptation process in the equalizer. One is normalized least mean square (NLMS) algorithm and another one is Variable step size LMS (VSSLMS) for updating the filter coefficients in the equalizer. The performance of these algorithms are taken into consideration in terms of bit error rate and signal to noise ratio.

## **2. ADAP2TIVE DECISION FEEDBACK EQUALIZATION**

### **A. Linear Equalization (LE)**

The most common implementation of the linear equalizer is as a transversal filter as shown in Figure 1. The input to the filter is the sequence  $u_n$  of sampled values and the output is the estimate of the current data symbol. Each of the  $2n + 1$  samples are weighted by  $b_i$  and summed to form the estimate. Formally, the estimate for the  $n^{\text{th}}$  data symbol is

$$\hat{J}_n = \sum_{i=-n}^n b_i u_{n-i} \quad (1)$$

Typically the symbol estimate  $\hat{J}_n$  will not exactly represent a data symbol and a decision must be made as to which is the nearest data symbol. If the decision results in  $\hat{J}_n$  that is not identical to the transmitted data symbol  $J_n$  then a symbol error has occurred. For a simple (non-adaptive) linear equalizer this error will not affect

further data symbols. To optimize the filter coefficients some criterion must be used to measure the equalizer performance. The optimization of the filter coefficients is also complex. Substantially more tractable criteria include the peak distortion criterion and the mean square error criterion, described in the next section.

The structure of a decision feedback equalizer is shown in Figure 2. The equalizer consists of a feed-forward and feed back sections, which is identical to the linear equalizer discussed in [14-17]. The feed-forward filter can either be symbol spaced or fractionally spaced. The feedback section contains a symbol-by- symbol detector and a symbol spaced feedback filter. The feedback section removes ISI due to previously detected symbols.

The equalizer output can be expressed as

$$\hat{J}_n = \sum_{i=-n_1}^0 b_i u_{n-i} + \sum_{i=1}^{n_2} b_i \tilde{J}_{n-i} \tag{2}$$

where,  $\tilde{J}_n$  the detected symbol. The equalizer is composed of  $(n_1 + 1)$  feed-forward taps and  $n_2$  feedback taps. In contrast to the linear equalizer the decision feedback equalizer is non-linear due to the decision device in the feedback path.

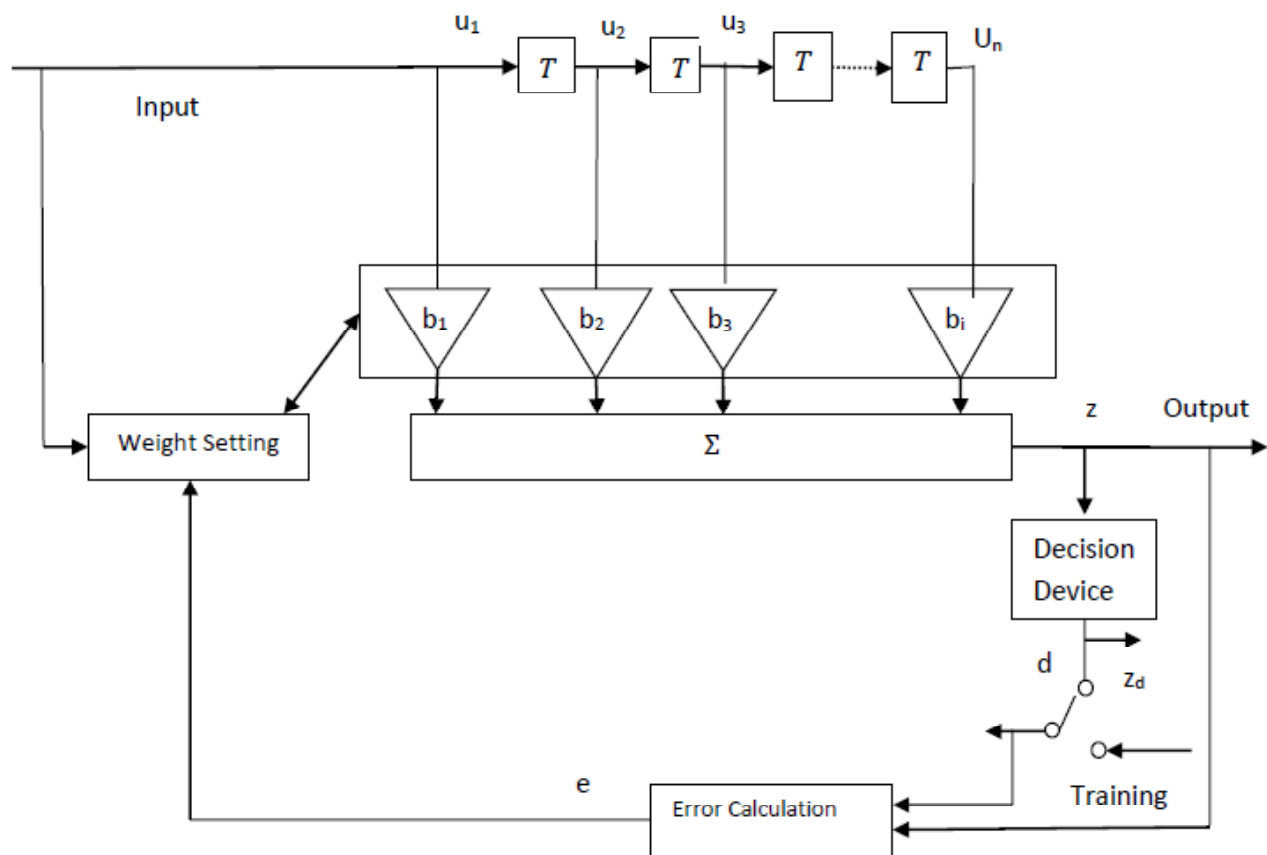


Figure 1: Linear equalizer structure

### Minimum Mean Square Error Optimization Criterion

The optimization of the filter coefficients (both feed-forward and feedback) can be done with either the peak distortion criterion or the mean square error [11-13] criterion. In practice the mean square error criterion is almost universally used and thus only the mean square error criterion is discussed here.

The minimization leads to the following set of equations for the feed-forward coefficients

$$\sum_{n=-n_1}^0 \varphi_{mi} b_i = h_{-m}^* \quad m = -n_1, \dots, -1, 0 \quad (3)$$

where,

$$\varphi_{mi} = \sum_{l=0}^{-m} h_l^* h_{l+m-i} + K_0 \rho_{mi}, \quad m, i = -n_1, \dots, -1, 0 \quad (4)$$

The coefficients of the feedback filter can be found from the feed-forward coefficients as follows

$$b_n = - \sum_{i=-n_1}^0 b_i h_{n-i} \quad n = 1, 2, \dots, n_2 \quad (5)$$

Provided that the previous decisions are correct and that  $n_2 \geq M$  (i.e. the feedback filter length is greater than or equal to the channel impulse response length) then the ISI from previously detected symbols can be completely removed. In [5] the superiority of the decision feedback equalizer over the linear equalizer is demonstrated for a number of different channels.

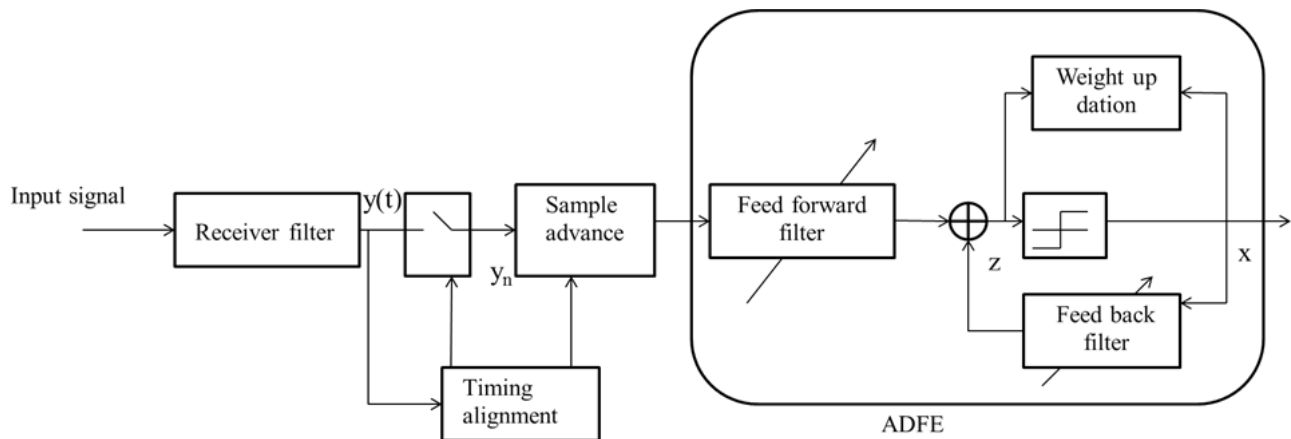


Figure 2: Adaptive Decision Feedback Equalizer structure

### A. Filter Coefficient Update Methods

The optimization of equalizer coefficients was derived. During the derivations it was implicitly assumed that the channel characteristics were known. In practice this is rarely the case and often the channel may be time varying. In such situations the equalizer must be able to be initially adjustable for the input and, if necessary, adapt to time variations. This is called Adaptive decision feedback equalizers shown in Figure 2. Two algorithms to achieve this are introduced.

#### NLMS

NLMS algorithm is another class of adaptive algorithm [18-21] used to train the coefficients of the adaptive filter. One of the problems in design and implementation of the LMS adaptive filter is the selection of the step size. For the stationary process the LMS algorithm converges in the mean if  $0 < S < \frac{2}{\lambda_{\max}}$  and converges in

the mean square if  $0 < S < \frac{2}{tr(R_x)}$ , however, since the  $R_x$  is generally unknown then either,  $\lambda_{max}$  or  $R_x$ , must be estimated in order to use these bounds.

The bound on the step size for mean-square convergence:

$$0 < S < \frac{2}{u_n^T u_n}$$

more over the upper bound is given as

$$S_n = \frac{S}{u_n^T u_n} = \frac{S}{\|u_n\|^2} \tag{6}$$

In overcoming the gradient noise amplification problem associated with the conventional LMS filter, the normalized LMS filter [15] introduces a problem of its own, namely the tap input vector  $u_n$  is small, numerical difficulties may arise because then we have to divide by a small value for the squared norm. To overcome this problem, we modify the above recursion by adding a small positive constant  $\alpha$ . The parameter  $\epsilon$  is set to avoid denominator being too small and step size parameter is too big.

Now the step size parameter is written as,

$$S_n = \frac{S}{\alpha + \|u_n\|^2} \tag{7}$$

where,  $S_n$  is a normalized step size with  $0 < S < 2$ . Replacing  $\mu$  in the LMS weight vector update equation with  $S_n$  leads to the NLMS, which is given as:

$$w_{n+1} = w_n + \frac{S}{\|u_n\|^2} e_n u_n \tag{8}$$

In the LMS algorithm, the correction that is applied  $w_n$  is proportional to the input vector  $u_n$  is large, the LMS algorithm experiences a problem with gradient noise amplification. With the normalization of the LMS step size by  $\|u_n\|^2$  in the NLMS algorithm, however, this noise amplification problem is diminished. Although the NLMS algorithm bypasses the problem of noise amplification, we are now faced with a similar problem that occur when  $\|u_n\|$  becomes too small. An alternative is to use the following modification to the NLMS algorithm:

$$w_{n+1} = w_n + \frac{S}{\alpha + \|u_n\|^2} e_n u_n \tag{9}$$

The update equation of NLMS is a scaled version of that of LMS algorithm. The size of the change to weight vector  $w_n$  is therefore be in inversely proportional to the norm of data vector  $u_n$ . The data vector  $u_n$  with a large norm will generally lead to a small change to  $w_n$  than a vector with a smaller norm. This normalization results smaller step size values than conventional LMS. The normalized algorithm usually converges faster than the LMS algorithm, since it utilizes a variable convergence factor aiming at the minimization of the instantaneous output error.

Hence NLMS is applicable to very high speed application such as biotelemetry. The step size parameter  $S$  of this algorithm is independent of the input signal power. But this algorithm required more computation to evaluate the normalization term  $\|u_n\|^2$ . At the same time NLMS requires less a priori information than LMS. Thus, the resulting mean-square error of NLMS is larger than that of LMS.

### **VSSLMS**

A popular approach for the improvement of fixed step-size algorithms is to implement the step-size that is time-varying in the steepest descent manner. The weight update equation only changes in the sense that now the learning parameter  $\mu$  becomes time varying.

So it becomes

$$W_{n+1} = W_n + S_n \cdot u_n e_n \quad (10)$$

The update equation of the step-size is given as

$$S_{n+1} = \delta S_n + \gamma e_n^2 \quad (11)$$

where,  $0 < \delta < 1$  and  $\gamma > 0$ .

where,  $S_n$  is the time-varying step-size. The variable step-size LMS (VSSLMS) [17–20] provides improved performance while maintaining the inherent simplicity and robustness of the conventional fixed step-size algorithm.

Generally, the approach with VSSLMS is to devise step-size rules that give large steps when the estimated error is large and small steps when the error is small, thereby avoiding the trade-off between convergence rate and steady-state error for fixed step-size LMS. This mechanism is mostly data-dependent and is implemented in an iterative manner (steepest descent). For this reason, there has been a lot of research in the VSSLMS field and their stability. The two main aims are to either improve the convergence rate of the algorithm for a given Excess Mean-Square Error (EMSE) or to improve the EMSE for a given convergence rate. In terms of the convergence rate [21–22], the desired performance enhancement is the speed with which an algorithm attains the steady-state. With high data rates, it is a desirable feature of any algorithm to achieve the steady-state in the minimum number of iterations. Conversely, in terms of the excess mean-square error, the desired performance is to attain lower steady-states at a certain convergence rate. Lower excess mean-square error means the filter attains a steady-state error that is closer to the minimum achievable steady-state error, that is, the minimum mean-square error. With the advancement of modern digital communication systems, both performance aspects are of great importance.

### **3. RESULTS AND DISCUSSION**

Figure 3 compares the BER performance of a linear and a ADFE equalizer. The linear equalizer was contained a 33 tap  $\frac{1}{4}$  symbol spaced feed-forward equalizer. The ADFE contained a 33 tap  $\frac{1}{4}$  symbol spaced feed-forward and a 32 tap feedback filter. In both cases the VSSLMS algorithm is used to update the filter coefficients. It can be observed that for very low SNR neither equalizer is able to correctly estimate the data symbols. As the SNR increases the linear equalizer is able to correctly estimate some of the data symbols. The ADFE continues to have poor performance as the large number of incorrect decision feedback and disrupt the equalizer. As the SNR improves further the number of incorrect decisions rapidly decreases and the ADFE is able to obtain superior performance.

Figure 4 compares a ADFE using the NLMS and VSSLMS update algorithms. It is clear that the DFE using VSSLMS is able to obtain significantly better performance than the ADFE using NLMS.

### **4. CONCLUSION**

To remove inter symbol interference in transmitted symbols Adaptive decision feed back equalizers are used. Adaptive decision feedback equalizers have less complexity. ADFEs show best results. ADFEs do not allow any residual intersymbol interference (ISI). The proposed equalizer is robust and is stable even under channel

step changes but it has a higher mean square error. This robustness avoids the overhead of additional training symbols, improving the efficiency of the communications and the reducing the time.

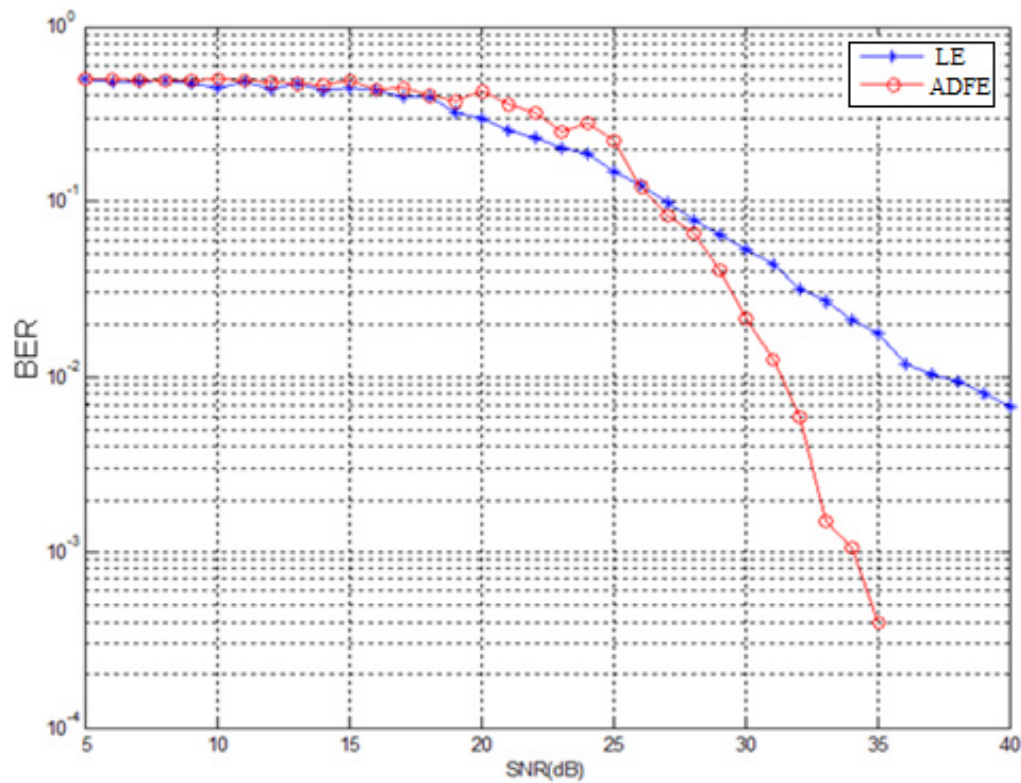


Figure 3: BER performances of a linear and ADFE equalizer

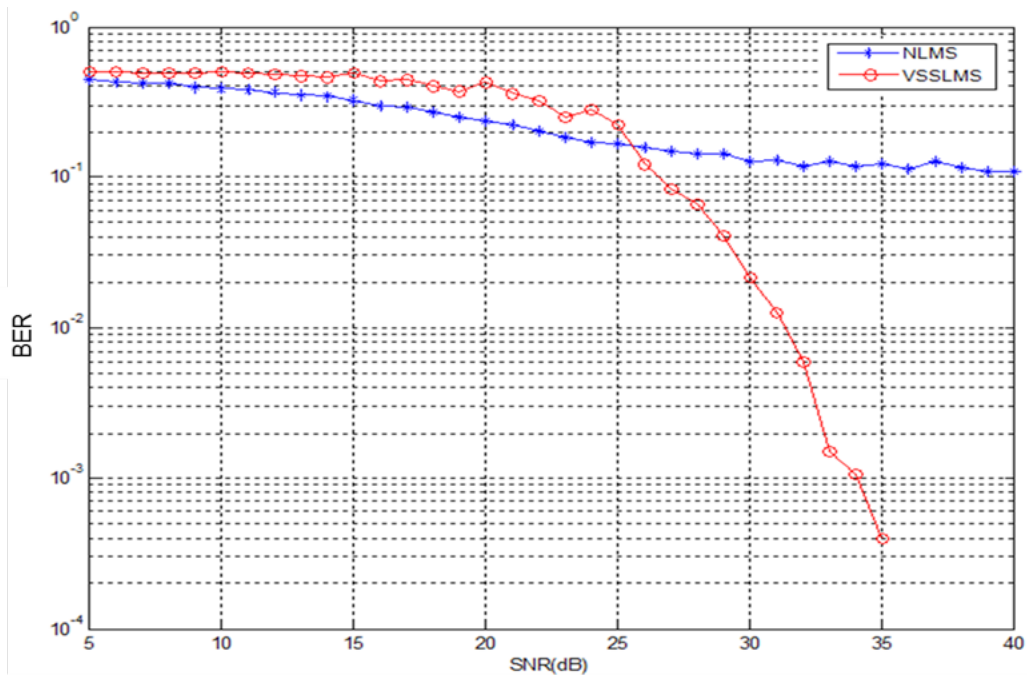


Figure 4: Comparing a ADFEs using the NLMS and VSSLMS update algorithms

## 5. REFERENCES

- [1] Prlja A, and Rusek F “Optimal hannel shortening for MIMO and ISI channels,” IEEE Trans. Wireless Commun.,vol.11, no.2,pp.810–818, 2012.
- [2] De Lamare RCand Sampaio-Neto R “Adaptive MBER decision feedback multiuser receivers in frequency selective fading channels,” IEEE Commun. Lett.,vol.7, no.2,pp.73–75, 2003.
- [3] Collings I B, McKay M R, and Tulino AM (2010) “Achievable sum rate of MIMO MMSE receivers: a general analytic framework,” IEEE Trans .Inf. Theory, vol.56,no.1,pp.396–410.
- [4] Haiquan Zhao, Xiangping Zeng, Jiashu Zhang, and Tianrui Li, Nonlinear Adaptive Equalizer Using a Pipelined Decision Feedback Recurrent Neural Network in Communication Systems, IEEE Transactions on Communications, vol. 58, no. 8, pp-2193-2198, Aug. 2010.
- [5] Alban Goupil and Jacques Palicot, An Efficient Blind Decision Feedback Equalizer, IEEE Communications Letters, vol. 14, no. 5, pp-462-464, May 2010.
- [6] J.G. Proakis, *Digital Communications*, 4<sup>th</sup> ed., McGraw-Hill, New York, USA 2001.
- [7] J. G. Proakis , “Adaptive equallzation techniques for acoustic telemetry channels,” *IEEE J. Oceanic Eng.*, vol. 16, no. 1, pp. 21-31, 1991.
- [8] Yu Gong and Colin F. N. Cowan, A Self-Structured Adaptive Decision Feedback Equalizer, IEEE Signal Processing Letters, vol. 13, no. 3, pp-169-172, Mar. 2006.
- [9] B. Widrow, “Adaptive Filters, I: Fundamentals,” Stanford Electronics Laboratory, Stanford University, Stanford, CA, Tech Report No. 6764-6, 1996.
- [10] B. Widrow, S. Stearns, *Adaptive Signal Processing*, Prentice Hall Inc., United States of America (1985).
- [11] O. Macchi, *Adaptive Processing – The Least Mean Squares Approach with Applications in Transmission*, John Wiley & Sons, Chichester, England (1995).
- [12] Rainfield Yutian Yen, “ Stochastic Unbiased Minimum Mean Error Rate Algorithm for Decision Feedback Equalizers”, IEEE Transactions on Signal Processing, vol. 55, no. 10, pp-4758-4766, october 2007.
- [13] O.Macchi, Adaptive processing, the least mean squares approach with applications in transmission. West Sussex. England: John Wiley and Sons,1995.
- [14] X. I. Reed, Error control coding for data networks. Boston: Kluwer academic publishers, 1999.
- [15] Ching-An Lai “NLMS algorithm with decreasing step size for adaptive IIR filters”, *Signal Processing*, vol. 82, pp. 1305-1316, 2002.
- [16] F C Souza, O J Tobias, R Seara and D R. Morgan, “A PNLMS Algorithm With Individual Activation Factors”, *IEEE Transactions on Signal Processing*, vol. 58, no. 4, pp. 2036-2047, April, 2010.
- [17] Christian Huemmer, Roland Maas, and Walter Kellermann, “The NLMS Algorithm with Time-Variant Optimum Stepsize Derived from a Bayesian Network Perspective”, IEEE Signal Processing Letters, Vol. 22, no. 11,pp. 1874-1878, Nov. 2015.
- [18] Long Shi and Haiquan Zhao, “Variable step-size distributed incremental normalised LMS algorithm”, Electronics Letters, Vol. 52, no. 7, pp. 519–521, April 2016
- [19] Y. Zhang, Ning Li, Jonathon A. Chambers and A. H. Sayed, “Steady-State Performance Analysis of a Variable Tap-Length LMS Algorithm”, *IEEE Transaction on Signal Processing*, vol. 56, no. 2, pp. 839-845, Feb., 2008.
- [20] J Ni and F Li, “A Variable Step-Size Matrix Normalized Subband Adaptive Filter”, *IEEE Transactions on Audio, Speech and Language Processing*, vol. 18, no. 6, pp. 1290-1299, August, 2010.
- [21] Z Ramadan, A Poularikas “Performance Analysis of a New Variable Step-Size LMS Algorithm with Error Nonlinearities”, 0-7803-8281-1/04, *IEEE*. pp. 384-388, 2004.



- [22] Tyseer. A and K. Mayyas, "A Robust Variable Step-Size LMS-Type Algorithm: Analysis and Simulations", *IEEE Transactions on Signal Processing.*, vol. 45, no. 3, pp. 631- 639, March, 1997.
- [23] S. Zhang and J. Zhang, "New Steady-State Analysis Results of Variable Step-Size LMS Algorithm With Different Noise Distributions", *IEEE Signal Processing Letters*, Vol. 21, no. 6, pp. 653-657, June, 2014.

