I J C T A, 9(34) 2016, pp. 765-778

Design and Control of Ball on Plate System

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ABSTRACT

This paper presents the design and implementation of ball on plate system. The system consists of plate, touch screen and servo motor. Touch screen is placed over the plate and the objective is to balance a freely rolling ball in a specific position or to move it in a trajectory on the plate with the least possible error and smallest settling time achieved for the dynamics of the real time system. Linear mathematical model of the system is derived to find the relationship between input and output. MATLAB is used to evaluate the closed loop system response and to determine the PID parameters. ATMega 2560 is used as the controller in which the PID control algorithm is implemented.

Keywords: Ball on plate, PID algorithm, 4 wire resistive touch screen, servo motor.

1. INTRODUCTION

Balancing systems are one of the most challenging systems in control field. Inverted pendulum, ball on beam is some of the classic examples of balancing systems. Ball and plate system is an enhanced version of ball and beam system. In this system, ball is balanced in a specific position of the plate. The plate is manipulated in two perpendicular directions (x and y). Resistive touch screen is used as sensor to find the position of ball in the plate. Servomotor is used as the actuator and Atmega2560 is used as controller. When disturbance is given to the ball, the touch screen produces a change in resistance depending on the position of ball and controller gives the signal to the servomotor to set the ball in desired position.

The paper is organized as follows. In Section 2 the construction of ball and plate system is described. The mathematical modelling of the system is presented in Section 3. The mechanical and electronic design of the system is given in Section 4&5. Section 6 describes about controller design and Section 7 concludes the paper.

2. SYSTEM DESCRIPTION

The system consists of a top plate and a bottom support. The ball rolls freely over the top plate on which a touch screen is placed to determine the co-ordinates of the ball. The top plate is connected to the bottom plate by means of ball and socket joint in order to provide free motion in x and y directions.

Horizontal position of the plate in both the axis were found using the spirit-level measurement and the corresponding servo angle were measured using the internal control loop in the digital servo using the 'read' command. This was fixed to be the angle of the servo where the plate angle was zero with respect to the horizontal.. The measured angles for the system were found out to be 87^o for x direction and 89^o for y direction of the plate. From [1], [2], [3] the resistive touch screen was used as image processing required more computing power when compared to the analog sensor. The touch screen provides a signal for every change in the position of the ball. The PID control algorithm was implemented in the Atmega2560 controller and the sensor data was also retrieved by the controller. The set-point given was compared with the input from touch screen for the specific direction and corresponding PID controller output was given to the servo

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motor of the specified direction. From the transfer function derived from the generalized model the PID parameters were derived by the PID application in MATLAB and implemented in the real time model which provided satisfactory response compared to the generalized model.

3. MATHEMATICAL MODELING

The mathematical modelling is done taking some assumptions to simplify the analysis. It is simplified into a particle system consisting of two rigid bodies. The plate has three geometrical limitations in the translation along the x-axis, y-axis and z-axis. It also has a geometrical limitation in its rotation about the z-axis. The ball has a geometrical limit in the translation along the z-axis; therefore it has 4 degrees of freedom (DOF) in movement along the x-axis and y-axis.The change in angle of the plate is limited to 15o in each direction in order to limit the velocity of the ball so as to prevent it from going out out of the plate. The friction component between the plate and the ball is neglected for ease of mathematical analysis. From [8] the rolling and slip of the ball are neglected to provide a linear equivalent. Initially we determined the transfer function for one axis system (Ball and Beam system). Ball and Beam system is shown in Figure [1]

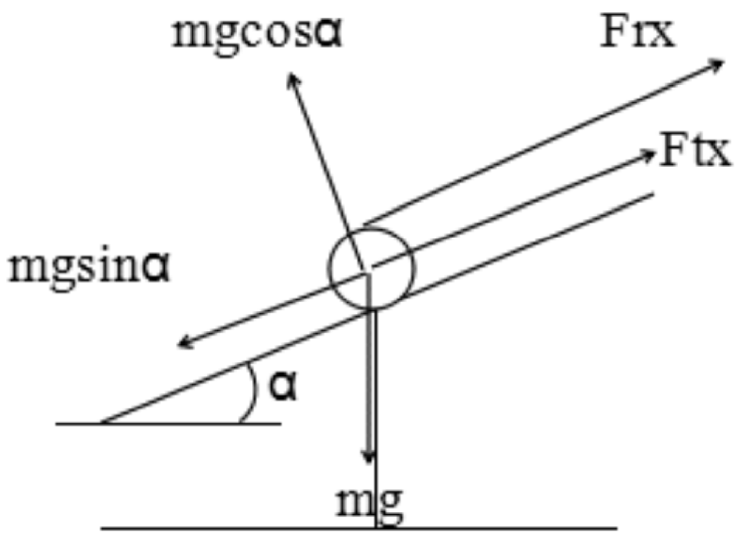

Figure 1: Ball and Beam system

The inclination is considered with respect to x-coordinate where

m = Mass of the ball (grams)

g = gravitational constant (9.81 meter/sec^2)

F_{tx} = Translational force (Newton)

F_{rx} = Rotational force (Newton)

x = displacement of the ball from the centre (meter)

R = Radius of the ball (meter)

α = Deviation of the plate from the horizontal (degrees)

The acceleration of ball is given by

$$dx^2 / dt^2 = \ddot{x} \tag{1}$$

The force due to translational motion is given by

$$F_{tx} = m\ddot{x} \tag{2}$$

The torque developed through ball rotation is determined by the force at the edge of the ball multiplied by the radius. It is given by

$$T_r = F_{rx}R = J\frac{d\omega_b}{dt} \tag{3}$$

$$J\frac{d\omega_b}{dt} = J\frac{d(V_b/R)}{dt} \tag{4}$$

$$J\frac{d(V_b/R)}{dt} = J\frac{\partial^2(x/R)}{\partial t^2} \tag{5}$$

$$J\frac{\partial^2(x/R)}{\partial t^2} = \frac{Jx''}{R} \tag{6}$$

Where

J = moment of inertia

$$J = \frac{2}{5}mR^2$$

Here for a solid sphere

W_b = angular velocity of the ball (degrees/sec)

V_b = speed of the ball along x axis (meter/sec)

The equation is arranged such that the final result is expressed solely in terms of position or its derivatives as well as variables associated with the ball.

The rotational force by dividing torque of the ball by its radius

$$F_{\tau x} = \frac{T_{rx}}{R} = \frac{Jx''}{R^2} \tag{7}$$

substituting the moment of inertia into the equation we get

$$F_{rx} + F_{tx} = mg\sin\alpha \tag{8}$$

$$\frac{2mx''}{5} + mx'' = mg\sin\alpha \tag{9}$$

$$\frac{2x''}{5} + x'' = g\sin\alpha \tag{10}$$

Rearranging gives

$$\frac{5g\sin\alpha}{7} = x'' \tag{11}$$

we utilize the approximation $\sin\alpha = \alpha$, since the angle of the beam will not exceed 10-15 degree -inclination. This means that in radians, sine of the angle is approximately the angle itself, so the equation is further approximated as

$$\frac{5g\alpha}{7} = x'' \tag{12}$$

Taking Laplace transform of position with respect to angle gives

$$H_x(S) = \frac{X(S)}{\Theta(S)} = \frac{5g}{7s^2} \tag{13}$$

The constant in the numerator is a theoretical constant that neglects surface imperfections and friction. The measured constant is approximately 0.91 therefore

$$\frac{X(S)}{\Theta(S)} = \frac{0.91}{s^2} \tag{14}$$

For Ball on Plate system with 2DOF

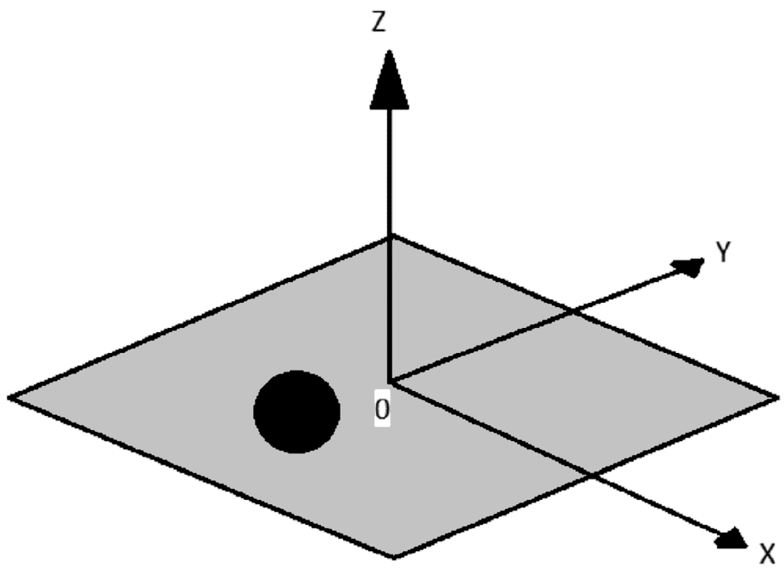

Figure 2: Ball on Plate system

Mass addition due to Moment of Inertia

$$m = \frac{I_b}{r_b^2} \tag{15}$$

In X axis

$$\left[M_b + \left(\frac{I_b}{r_b^2}\right)\right]\frac{\partial^2 x}{\partial t^2} - m_b\left[x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}\right] + m_b g \sin\alpha = 0 \tag{16}$$

In Y axis

$$\left[M_b + \left(\frac{I_b}{r_b^2}\right)\right]\frac{\partial^2 y}{\partial t^2} - m_b\left[y_b\dot{\beta}^2 + y_b\dot{\alpha}\dot{\beta}\right] + m_b g \sin\beta = 0 \tag{17}$$

Relation between angle change in gear and angle change in plate is given by

$$\frac{\sin\alpha}{\sin\Theta} = \frac{2r_{arm}}{L_{px}} \tag{18}$$

Thus

$$\sin\alpha = \frac{2\sin\Theta_x r_{arm}}{L_{px}} \tag{19}$$

$$\sin\beta = \frac{2\sin\Theta_y r_{arm}}{L_{py}} \tag{20}$$

The angular velocity $\frac{d\alpha}{dt}$ and $\frac{d\beta}{dt}$ are approximated to zero for negligible rolling and slip of the ball.

Therefore for

X axis

$$\left[M_b + \left(I_b / r_b^2\right)\right]\frac{\partial^2 x}{\partial t^2} = m_b g \sin\alpha \tag{21}$$

$$\left[M_b + \left(\frac{I_b}{r_b^2}\right)\right]\frac{\partial^2 x}{\partial t^2} = \frac{m_b g \sin\Theta_x r_{arm}}{L_{px}} \tag{22}$$

Similarly for Y axis

$$\left[M_b + \left(\frac{I_b}{r_b^2}\right)\right]\frac{\partial^2 y}{\partial t^2} = \frac{m_b g \sin\Theta_y r_{arm}}{L_{py}} \tag{23}$$

Taking Laplace, we get

$$\frac{x(s)}{\Theta(s)} = \frac{2m_b g r_b^2 r_{arm}}{L_{px}\left(m_b r_b^2 + I_b\right)s^2} = \frac{K_x}{s^2} \tag{24}$$

$$\frac{y(s)}{\Theta(s)} = \frac{2m_b g r_b^2 r_{arm}}{L_{py}\left(m_b r_b^2 + I_b\right)s^2} = \frac{K_y}{s^2} \tag{25}$$

Where K_x and K_y are

$$K_x = \frac{2m_b g r_b^2 r_{arm}}{L_{px}\left(m_b r_b^2 + I_b\right)s^2}$$

$$K_y = \frac{2m_b g r_b^2 r_{arm}}{L_{py}(m_b r_b^2 + I_b)s^2}$$

Here the system constants are

$$m_b = 0.080\ Kg$$
$$g = 9.81 m/\text{sec}^2$$
$$r_b = 0.015 m$$
$$r_{arm} = 0.014 m$$

$$L_{px} = 0.23m$$
$$L_{py} = 0.41m$$
$$I_b = 2m_b r_b^2 / 5$$

Substituting in (24) and (25) we get

$$\frac{x(s)}{\Theta(s)} = \frac{0.476}{s^2} \tag{26}$$

$$\frac{y(s)}{\Theta(s)} = \frac{0.878}{s^2} \tag{27}$$

State Space Equation:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Dx(t)$$

Where $x(t)$ is the state vector,

$$x(t) = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$

here $x, \dot{x}, y, \dot{y}$ represent ball's x co-ordinate, velocity with respect x axis, ball's y co-ordinate , velocity with respect to y-axis.

$y(t)$ is the output vector

$$y(t) = \begin{bmatrix} y1 \\ y2 \end{bmatrix}$$

where y1, y2 output of the system.

$u(t)$ Is the input vector

$$u(t) = \begin{bmatrix} \Theta_x \\ \Theta_y \end{bmatrix}$$

where Θ_x, Θ_y are the inputs to the system.

From the differential equations (22) and (23) representing the systems in x and y axis:

we get the following state space model

$$\begin{bmatrix} x\cdot \\ x\cdot\cdot \\ y\cdot \\ y\cdot\cdot \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x\cdot \\ x\cdot\cdot \\ y\cdot \\ y\cdot\cdot \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.45627 & 0 \\ 0 & 0 \\ 0 & 0.872 \end{bmatrix} \begin{bmatrix} \Theta_x \\ \Theta_y \end{bmatrix}$$

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x\cdot \\ x\cdot\cdot \\ y\cdot \\ y\cdot\cdot \end{bmatrix} + [0] \begin{bmatrix} \Theta_x \\ \Theta_y \end{bmatrix}$$

where A = state matrix

B = Input matrix

C = Output matrix

D = Feed-forward matrix.

As there is no feed forward path D matrix is equal to zero

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0.45627 & 0 \\ 0 & 0 \\ 0 & 0.872 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = [0]$$

Controllability :

Controllability = $[B: AB: A^2 B: A^3 B]$

The system is controllable as the rank of the controllability matrix is equal to the degrees of freedom i..e 4.

Observability:

$$Q = \left[C : CA : CA^2 : CA^3 \right]^T$$

For the system the rank of the observability matrix is 4 which is equal to degree of freedom which means the system is observable.

4. MECHANICAL DESIGN

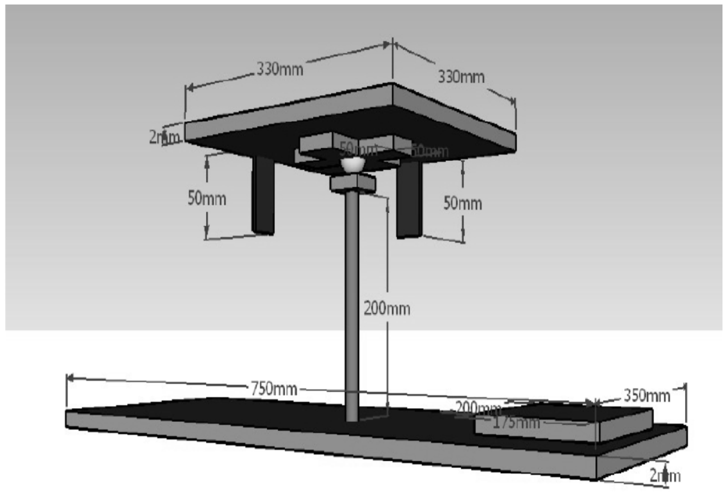

Figure 3: CAD designed model

The mechanical system was modelled using Google Sketch-up. The designed model is shown in Figure[3]. The system consists of a top plate and a bottom support. The top plate is connected to the bottom plate on top of a metallic rod of 300mm. In order to move the plate freely in x and y direction a ball and socket joint was used. The top plate dimension was 19.5 inch diagonally in order to accommodate the 18.5inch touch screen. The servo motor on each direction were connected to the plate with small metal linkage from the servo gear. The electronic components were placed close enough to support the wiring from the touch screen. Figure[4] shows the physical Ball and Plate system.

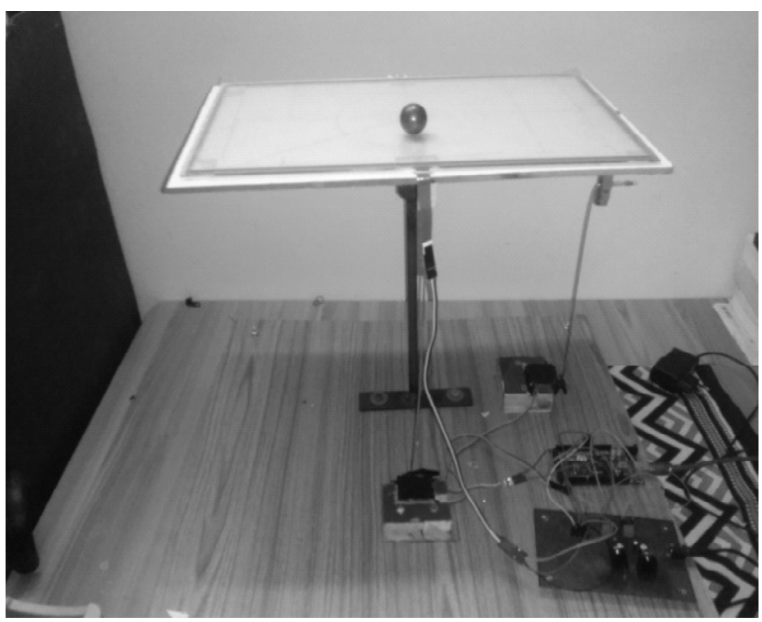

Figure 4: The Physical Ball and Plate System

5. CONTROL OF BALL ON PLATE

The transfer function derived from the mathematical model is considered for the control of X-axis and Y-axis individually. The state space equation for the MIMO system derived from the differential equation provides the matrices of A, B, C and D which prove that the system is controllable and observable with the rank of the controllability matrix and the observability matrix is equal to the degree of freedom which is equal to 4. The ball moves in two axis which adds to 2 degree of freedom and plate is free to rotate in x and y axis which adds 2 more degrees of freedom. Hence the degree of freedom of the system is 4.In order to obtain easier control algorithm the MIMO system is simplified to a simpler combination of 2 SISO systems.

As per the SISO model of the plant in X-axis and Y-axis separately the PID control parameters are derived by the Ziegler Nichols Closed loop method for the real system. As per the transfer function obtained from the Newtonian Laws of Physics the ultimate gain and ultimate period of the system are found using SIMULINK.

Response time: First to design a controller the response time of the system as to be found out to provide satisfactory controller response. For the system the response time was found out by giving a step input that is step change in the servo angle and recording the time at which the input was given and time at which the output produced a change.

For the X-axis the servo motor was moved by an angle of 5° at a time instant of 0 and the ball moved after a time period of 391 milliseconds. The time was measured by a function "millis()" that provides the time value in milliseconds.

Similarly For the Y-axis the servo motor was moved by an angle of 5° at a time instant of 0 and the ball moved after a time period of 393 milliseconds. The time was measured by a function "millis()" that provides the time value in milliseconds. This response time determined is used as sample time to provide necessary control action at the rate to compensate the ball movements.

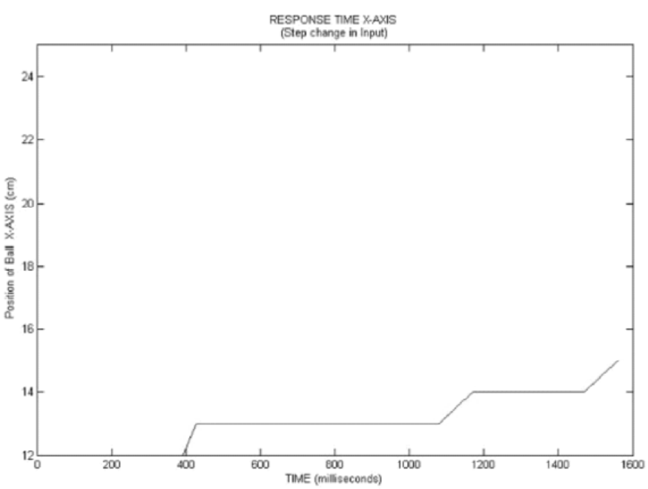

Figure 5: Response time of system X-axis

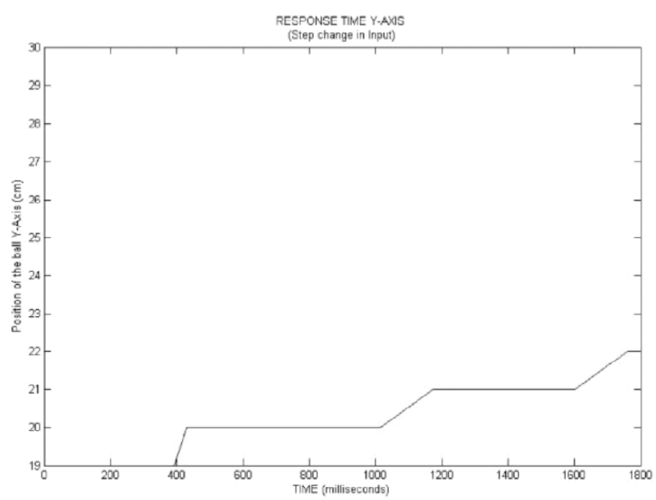

Figure 6: Response time of system Y-axis

Choice of tuning methods

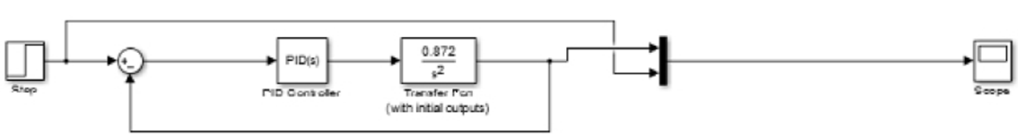

Figure 7: Closed Loop of the system in X-axis

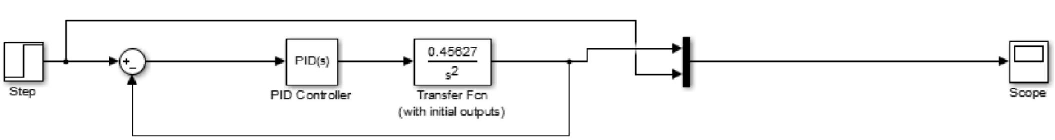

Figure 8: Closed Loop of the system in Y-axis

There are many methods of tuning the PID parameters but the most commonly used one is the Ziegler Nichols tuning method. It involves two methods 1) open Loop tuning and 2)Closed loop tuning method. As the system is open loop unstable the closed loop method was adopted to find the PID parameters

Ziegler Nichols Closed loop tuning: Ziegler Nichols method involves finding the ultimate gain and ultimate period of the system with which the PID parameters are derived.

Table 1
Zeiglar Nichols tuning parameters

controller	K_c	τ_I	τ_D
P	0.5 K_{CU}	–	–
PI	0.45 K_{CU}	$P_U/1.2$	–
PID	0.6 K_{CU}	$P_U/2$	$P_U/8$

For the system considered in X-axis with transfer function

$$\frac{x(s)}{\Theta(s)} = \frac{0.476}{s^2}$$

The ultimate gain K_u was found to be 3.4 and ultimate period P_u was found to be 5. With these values the K_p, K_i and K_d were found by the standard formulae of Ziegler Nichols Method

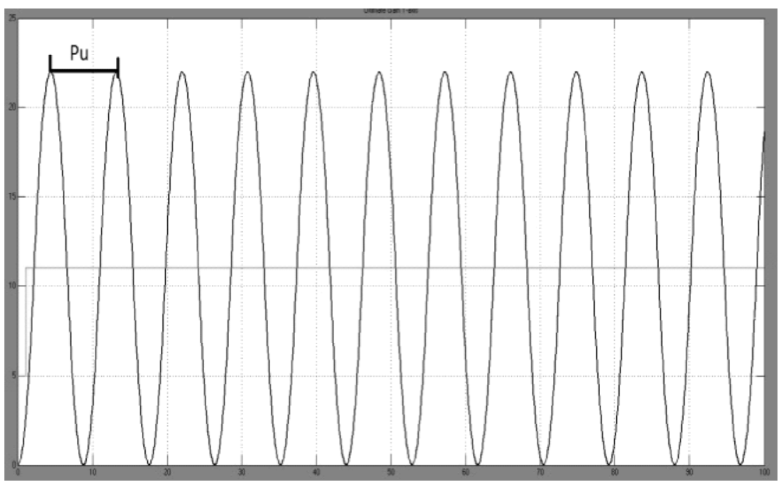

Figure 9: Sustained oscillation for Ku = 3.4 (X Axis)

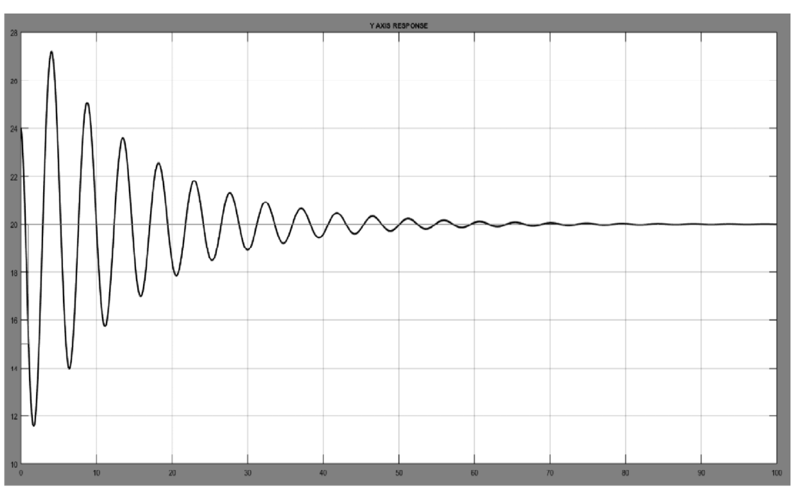

Figure 10: Closed loop response of simulated system (X-Axis)

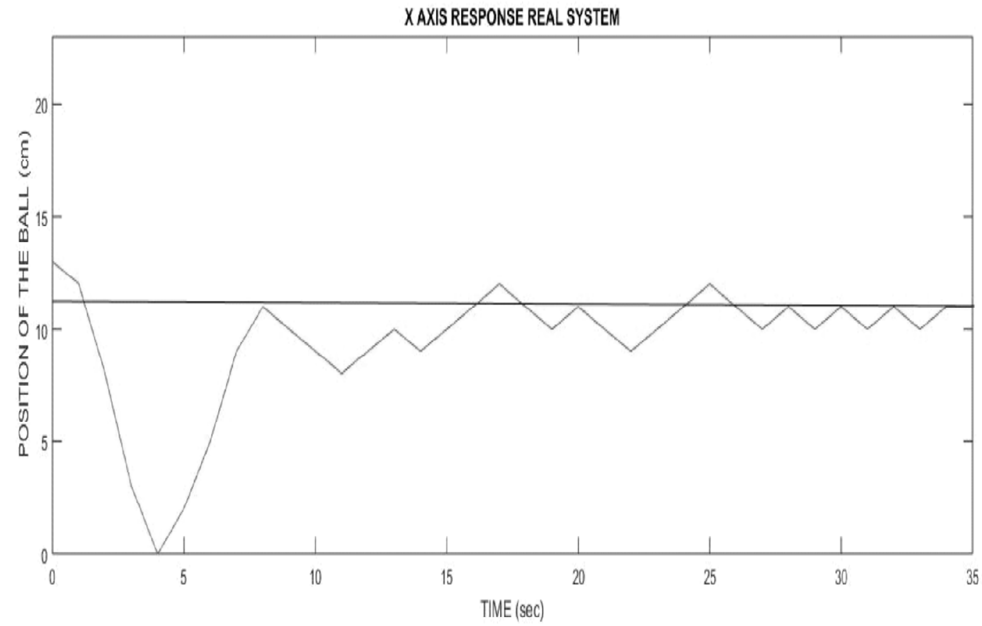

Figure 11: Closed loop response of real system in X

For the system considered in Y-axis with transfer function

$$\frac{y(s)}{\Theta(s)} = \frac{0.878}{s^2}$$

The ultimate gain K_u was found to be 3.4 and ultimate period P_u was found to be 3.75. With these values the K_p, K_i and K_d were found by the standard formulae of Ziegler Nichols Method

With the response of the real system in close context compared to that of the system simulated in the SIMULINK with settling time of 36 seconds for the above PID parameters derived.

The system settled in about 36 seconds for the real system which was very much slow to compensate for external disturbances.

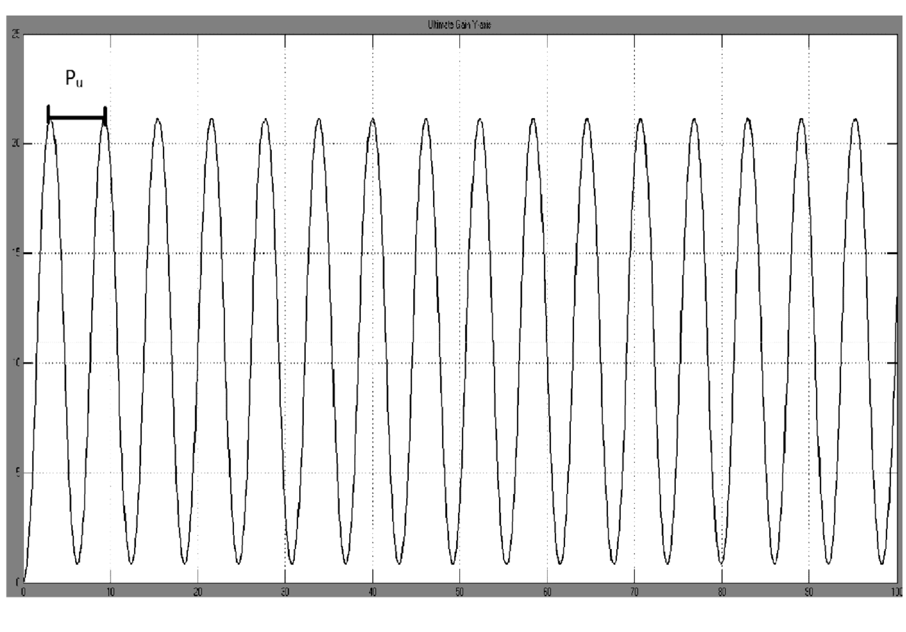

Figure 12: Sustained oscillation for Ku = 3.4 (Y Axis)

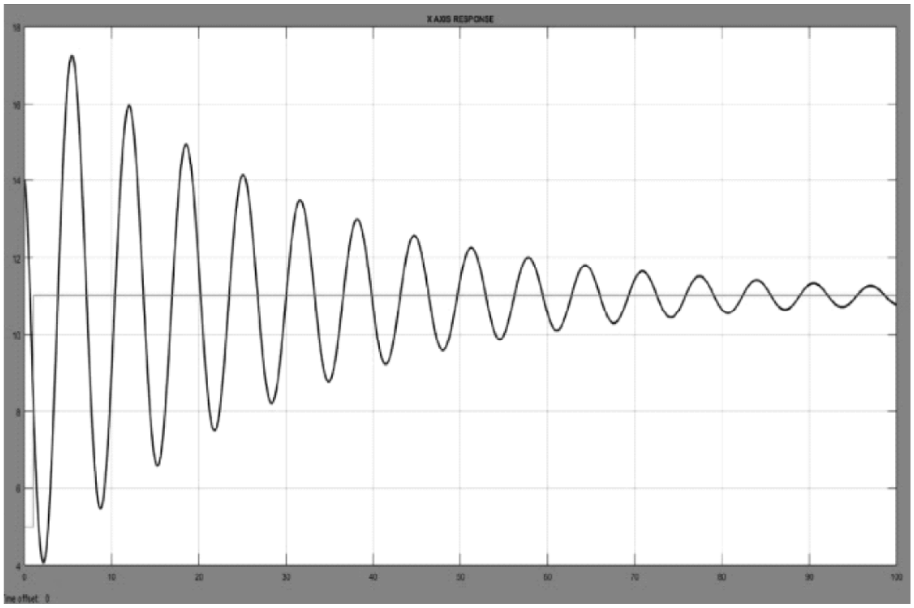

Figure 13: Closed loop response of simulated system (Y-Axis)

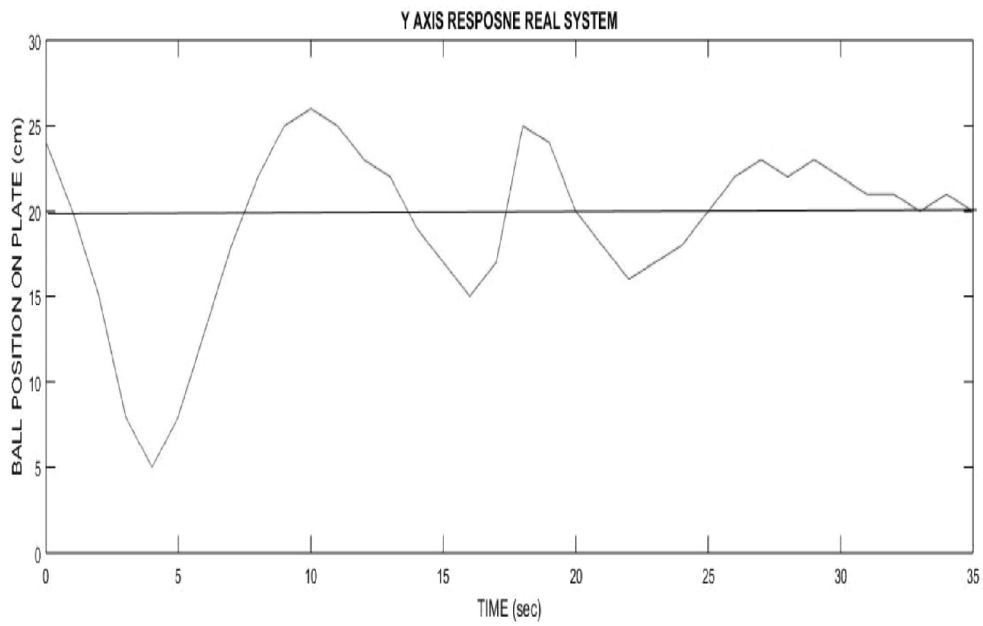

Figure 14: Closed loop response of real system in Y

In order to provide a system that is more resistant to disturbances the parameters derived were manually adjusted so that the settling time was much faster. The final PID parameters were found to be Kp = 2 ; Ki = 0.5 and Kd = 2 that provided satisfactory settling time of 4.6 seconds.

Trajectory Tracking:

Rectangle:

The rectangle trajectory was achieved with the variation of set-point every 6seconds in a rectangular fashion. The rectangle trajectory was achieved with error at the two diagonal points since the plate a small deformity in the surface at these points. The above figure shows the real system's rectangular ,trajectory tracking. The rectangular co-ordinates are (7,26), (14,26), (14,13) and (7,13).

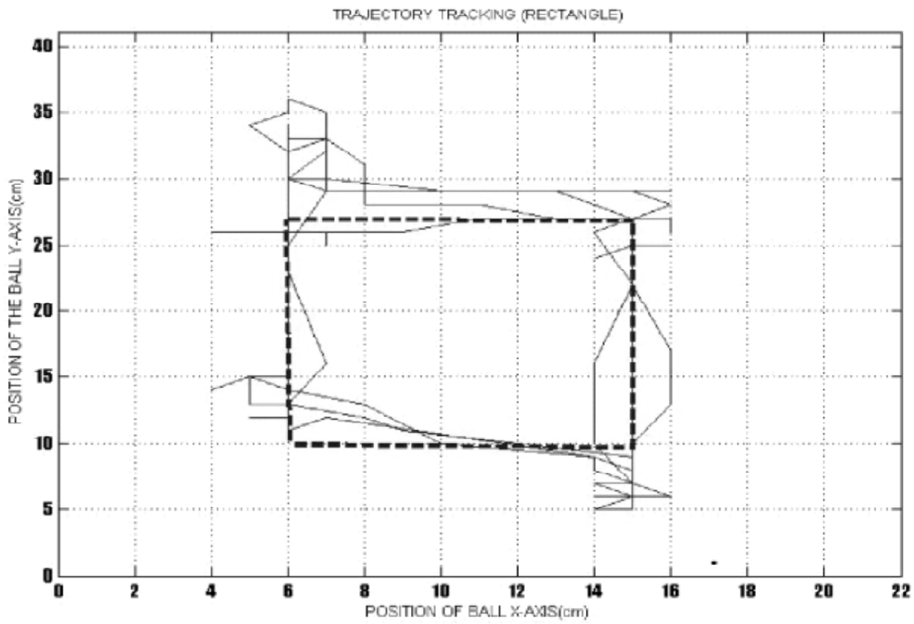

Figure 15: Rectangular Trajectory tracking

Circle:

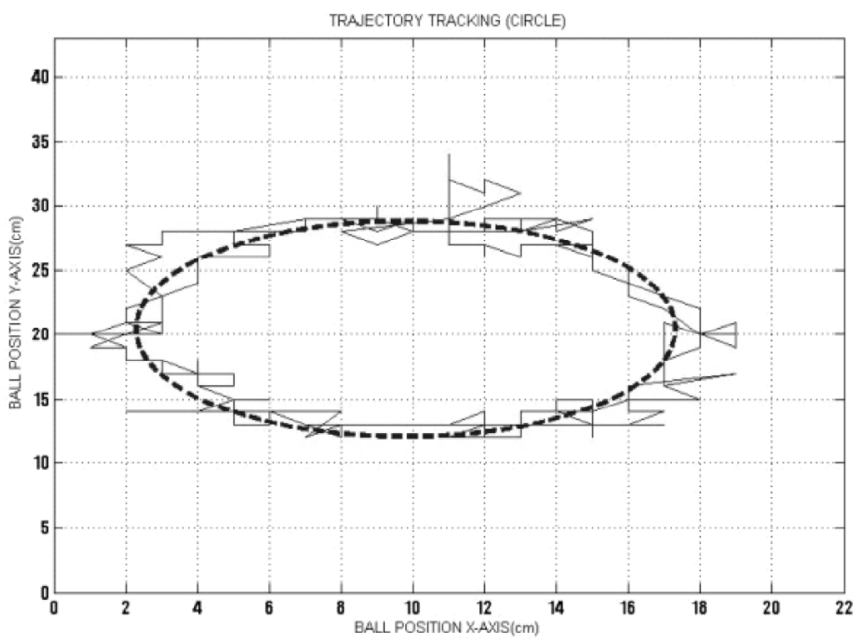

Figure 16: Circular trajectory tracking

The circle trajectory was achieved by changing the set-point across 33 points in a circle with a delay of 3 seconds for the ball to settle. The above figure shows the real time trajectory tracking in a circle. The circle could be completed in 99 seconds that is about 3.63^{o}/sec.

6. CONCLUSION

Ball and plate system is a classic non-linear example for modern control system applications. The paper describes overall design, mathematical modelling, components used and control algorithm design of the system. It shows desirable performance with least possible error and smallest settling time. This setup will help students to learn real time working of systems like flight simulator. Damped oscillation method, Zeiglar Nichols closed loop method of tuning to find the parameters which are widely used in industries can be learnt from the system.

Acknowledgments

The authors would like to thank all their colleagues for their continuous encouragement and support. Also we thank all the faculty members for their ideas and discussions.

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