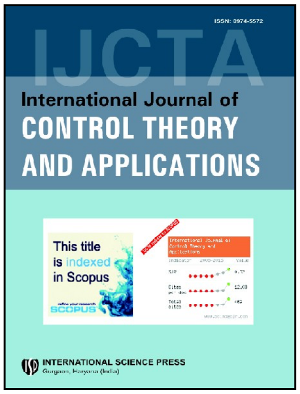

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Stochastic Analysis of Time to Carbohydrate Metabolic Disorder using Exponentiated Exponential Distribution

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Abstract: Diabetic mellitus is a chronic disease of pancreatic origin, characterized by insulin deficiency, subsequent in ability to utilize carbohydrates and excess glucose in the blood due to the secretion of less insulin in the pancreas the glucose level exceeds the threshold level of the human body and the carbohydrate metabolic disorder takes place. In theestimation of the expected time to carbohydrate metabolic disorder, there is an important role for the interval time between damages of the cells. Often expected inter damage times are decreasing order or increasing order forming exponentiated exponential distributions. In this paper the expected time to carbohydrate metabolic disorder its variance are derived with numerical example.

Keywords: Expected time, diabetic mellitus, threshold, exponentiated exponential distribution.

AMS Subject classification: 30c45, 30c80

INTRODUCTION

One of the most dreadful yet controllable disease in our body is diabetes mellitus, which is a common disorder characterized by excess glucose in the blood. The symptoms are developed mostly in middle ages and rarely in young adults and children. Whether the symptoms are mild or severe, excess glucose in the blood can do great harm over the years and can lead to complication with serious damages to the nephropathy in kidneys, coronary heart disease in circulatory system, neuropathy in central nervous system, retinopathy in the eyes and also foot gangrene.

Diabetes mellitus occurs due to the lack of hormone secretion in the pancreas. Pancreas plays a vital role in the metabolism. It lies under the stomach. It is about 23cm long, extending from the duodenum to the Pancreas. The Pancreas has two types of cells named as α and β (or islets of langerhans). Insulin (polypetic) is a hormone produced by special collection of cells in the pancreas known as -cells, the islets of langerhans.

All carbohydrate foods are broken down in the digestive system and absorbed into the blood as glucose. Glucose is then carried to the liver where it is stored as glycogen by the action of insulin. Hence insulin plays a very important part in regulating the level of blood glucose in the blood. Diabetes mellitus is a chronic disease of pancreatic origin, characterized by insulin deficiency, subsequent inability to utilize carbohydrates and excess glucose in the blood.

Stochastic models in the study of diabetes mellitus due to the less secretion of insulin in the pancreas, excess glucose level exceeding the threshold level of the human body causing the carbohydrate metabolic disorder has not been studied so far at any depth by researchers. In the estimation of the expected time to carbohydrate metabolic disorder, there is an important role for the inter arrival times between damages of the cells. Any person who is taking food without any restrictions is likely to become diabetic. However the person cannot avoid food. But there is a possibility that the person can postpone the event of becoming diabetic by taking regular exercise and diet control which may delay the occurrences of further damages to cells. This gives rise to the sequence of random variables of expected interarrival times in increasing order between damages of the cells forming geometric processes. Similarly when a person has irregular habits, the expected interarrival times may be decreasing and forming geometric process. Under the above circumstances the expected time to carbohydrate metabolic disorder and its variance are derived in this chapter.

Generalized exponential distribution discussed by Gupta and kundu [1999] which has an increasing or decreasing failure rate depending on the shape perameter.also gupta and kundu (2001) discussed about exponentiated exponential distribution with two parameters namely scale and shape parameter

The distribution function, $F_E(x, \alpha, \theta) = (1 - e^{-\theta x})^{\alpha}$ $\alpha, \theta, x > 0$

The density function is $f_E(x, \alpha, \theta) = \alpha\theta\, (1 - e^{-\theta x})^{\alpha-1}\, e^{-\theta x}$

The corresponding survival function is $s_E(x, \alpha, \theta) = 1 - (1 - e^{-\theta x})^{\alpha}$

When the shape parameter of the exponentiated exponential distribution ($\alpha = 1$) it represents the exponential distribution. Damodaran and Gopal [2009] stated that, for the simplicity and for a single parameter distribution, the Generalized Exponential Distribution with shape parameter ($\alpha = 2$) was considered. Parthasarathy *et al.*, [2010] discussed when threshold follows Gamma distribution. Further the threshold depicts SCBZ carried out by Sahtyamoorthy and Parthasarathy [2003 [In this direction this paper made an attempt that the threshold hold Exponential Exponential distribution when the shape parameter $\alpha = 3$ for which the mean and variances arederived.

Stochastic Diabetic Mellitus Model

Assumptions

1. At time 0 there is no damage in pancreas.
2. Let U_i be the exponential random time between the occurrences of damages to the pancreas where $U_i = \frac{U_1}{a^{i-1}}$ for $i = 1, 2$...where the pdf U_1 is F (.) And cdf F (.) and $a \geq 1$
3. The number of cells X_i damaged in each instant of damage to the pancreas are independent and identically distributed discrete random variables with probability function $P(X = k) = p_k$ for $k = 1, 2...$ With $p_k > 0$ and $\Sigma_{k=1}^{\infty} p_k$ for k = 1, 2, 3…with $p_k > 0$ and $\Sigma_{k=1}^{\infty} p_k = 1$.
4. When the total number of cells damages Xi + X2+. + X_n exceeds threshold Y then the person is diagnosed as diabetic where Y has exponential distribution with parameter θ
5. The time to i-th damage U_i, the number of damages of cells X_i caused and the threshold Y are independent.

Let T be time for the total number of damaged cells of the pancreas to cross the threshold level of a person to become diabetic. Then the probability that the person is not diagnosed as diabetic up to time t is

P (T > t) = $P(T > t) = \Sigma_{k-1}^{\infty} v_k(t)\,(t)\,P(\Sigma_{i=1}^{k} x_i < y)\ k=1\ i=1$ where v_k (t)) is the probability that exactly k instants of damages have occurred to the pancreas.

3. NOTATIONS

X_i : a continuous random variable denoting the amount of damaged cells due to the exit of persons on the occasion of policy announcement, $i = 1,2,\ldots k$ and $X_i's$ are i.i.d and $X_i = X$ for all i.

Y: a continuous random variable denoting the threshold level having Exponentiated Exponential distribution.

g (.): The probability density functions of X

$g_k(*)$: The k- fold convolution of i.e., p.d.f. of $\Sigma_{i=1}^{k} X_i$

T: a continuous random variable denoting interval time to damagesof the cell.

$g^*(.)$: Laplace transforms of g (.)

g_k (*) : Laplace transform of $g_k(.)$

p(.): The p.d.f. of random threshold level which has Exponentiated Exponential distribution andP (.) is the corresponding c.d.f.

U: a continuous random variable denoting the inter-arrival times between decision epochs.

f (.): p.d.f. of random variable U with corresponding c.d.f. F (.)

$F_k(t)$. : The k-fold convolution functions of F (.)

S (.): The survivor function i.e. P (T > t)

v_k (t) : Probability that there are exactly that k instants of damages have occurred to the pancreas in (0, t]

L (T) = 1 – $S(t)$

4. RESULTS

Here exponentiated exponential distribution with α = 3 is considered

Let Y be the random variable which has the cdf defined as

$$P_E(\mathrm{x},\alpha,\theta) = (1 - e^{-\theta x})^3 \qquad \alpha,\theta,x > 0$$

Therefore it has the density function

$$p_E(\mathrm{x},\alpha,\theta) = 3\theta(1 - e^{-\theta x})$$

The corresponding survival function is

$$s_E(\mathrm{x},\alpha,\theta) = 1-(1 - e^{-\theta x})^3$$

P $(X_1 + X_2 + X_3 + X_4 < Y)$ =P [the system does not fail, after K damages of exits].

Let T be time for the total number of damaged cells of the pancreas to cross the threshold level of a person to become diabetic. Then the probability that the person is not diagnosed as diabetic up to time t is

P (T > t) = $\sum_{k-1}^{\infty} v_k(t)$ P $(\sum_{i=1}^{k} x_i < y)$ k=1 i= 1 where $v_k(t)$) is the probability that exactly k instants of damages have occurred to the pancreas.

Let T be time for the total number of damaged cells of the pancreas to cross the threshold level of a person to become diabetic. Then the probability that the person is not diagnosed as diabetic up to time t is

$$P(T > t) = v_k(t)\, P\left(\sum_{i=1}^{\infty} X_i < Y\right)$$

Where (t) is the probability that exactly k instants of damages have occurred to the pancreas.

$$P\left(\sum_{i=1}^{\infty} X_i < y\right)$$

$$= \int_0^{\infty} g_k(x)\bar{P}(x)\,dx$$

$$= \int_0^{\infty} g_k(x)[1-P(x)]dx$$

$$= \int_0^{\infty} g_k(x)[3e^{-\theta x} - 3e^{-2\theta x} - e^{-3\theta x}]dx$$

$$= 3{g^*}_k(\theta) - 3{g^*}_k(2\theta) + {g^*}_k(3\theta)$$

$$= 3[g^*(\theta)]^k - 3[g^*(2\theta)]^k + [g^*(3\theta)]^k$$

The survival function s (t)which is the probability that an individual survives for a time t.

S (*t*) = *p* (*T* > *t*) Probability that the system survives beyond t.

$=\sum_{k=0}^{\infty} p(\textit{there are exactly k damages of cells in } (0,t) *$

$p(\textit{the system does not fail in } (0,t])$ It is also known from renewal theory that

P (exactly k damaged cells in (0, t])

$v_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$

$$=\sum_{k=0}^{\infty} v_k(t)\, p\left(\sum_{i=1}^{\infty} X_i < Y\right)$$

$=3\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(\theta)]^k - 3\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(2\theta)]^k + \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(3\theta)]^k$

Now p (T< $T) = L(t) = 1 - S(t)$

$1-\{3\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(\theta)]^k - 3\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(2\theta)]^k + \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(3\theta)]^k\}$

$=1-\{3(F_0(t) - F_{0+1}(t)]([g^*(\theta)]^0(\sum_{k=1}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(\theta)]^{k-1}$

$-3(F_0(t) - F_{0+1}(t))[g^*(2\theta)]^0(\sum_{k=1}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(2\theta)]^{k-1}$

$+(F_0(t) - F_{0+1}(t))[g^*(3\theta)]^0(\sum_{k=1}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(3\theta)]^{k-1}$

$=1-\{3(1-(1-g^*(\theta))\ (\sum_{k=1}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(\theta)]^{k-1}$

$+\ 3(1-(1-g^*(2\theta))\ \ (\sum_{k=1}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(2\theta)]^{k-1} - 1-(1-g^*(3\theta))\ (\sum_{k=1}^{\infty}[F_k(t) - F_{k+1}(t)]([g^*(3\theta)]^{k-1}$

$=3(1-g^*(\theta)\sum_{k=1}^{\infty} v_k(t))[g^*(\theta)]^{k-1} - 3(1-g^*(2\theta)\sum_{k=1}^{\infty} v_k(t))([g^*(2\theta)]^{k-1} + 1-g^*(3\theta)\sum_{k=1}^{\infty} v_k(t))]([g^*(3\theta)]^{k-1}$

Taking Laplace transform of L (t), we get

L (L(S)) $=L^*(S)=\frac{3[1-g^*(\theta)v^*(S)}{[1-g^*(\theta)v^*(S)} - 3\frac{3[1-g^*(2\theta)v^*(S)}{[1-g^*(2\theta)v^*(S)} + \frac{[1-g^*(3\theta)v^*(S)}{[1-g^*(3\theta)v^*(S)}$

Where $[\,v^*(s)]^k$ is Laplace transform of $v_k(t)$ since the inter arrival time arei.i.d.

$v^*(s)=\int_0^\infty \exp(-sx)\,\theta\exp(-\theta x)\,dx=\frac{\theta}{\theta+s}$ let the random variable U denoting inter arrival time which follows exponential with parameter θ

The above equations can be written as

$$\frac{3[1-g^*(\theta)\frac{c}{c+s}]}{[1-g^*(\theta)\frac{c}{c+s}]} - 3\frac{3[1-g^*(2\theta)\frac{c}{c+s}]}{[1-g^*(2\theta)\frac{c}{c+s}]} + \frac{[1-g^*(3\theta)\frac{c}{c+s}]}{[1-g^*(3\theta)\frac{c}{c+s}]}$$

$$=3c\,[1-g^*(\theta)][c+s-g^{\wedge}*\,(\theta)]^{-1} - 3c\,[1\text{-}g^*(2\theta)][c+s-g^{\wedge}*\,(2\theta)]^{-1} + c\,[1\text{-}g^*(3\theta)][c+s-g^{\wedge}*\,(3\theta)]^{-1}$$

$$E(T) = -\frac{d}{ds}[L^*(S)] \quad \text{given } s=0$$

$$E(T^2) = \frac{d^2}{ds^2}(L^*(S)]$$

From which V (T) can be obtained.

$$E(T) = -\frac{d}{ds}[L^*(S)] = -\frac{d}{ds}[-3c\frac{(1-g^*(\theta))}{c+S-Cg^*\theta} - 3c\frac{(1-g^*(2\theta)}{c+S-Cg^*2\theta} + c\,(\frac{(1-g^*(3\theta)}{c+S-Cg^*3\theta}]$$

$$=\frac{3}{c(1-g^*(\theta))} - \frac{3}{c(1-g^*(2\theta))} + \frac{1}{c(1-g^*(3\theta)}$$

$$E(T^2) = \frac{d}{ds}[\frac{3}{c(1-g^*(\theta)} - \frac{3}{c(1-g^*(2\theta)} + \frac{1}{c(1-g^*(3\theta)}] = \frac{d}{ds}[-\frac{3c\,[1-g^*(\theta)]}{[c+s-g^{\wedge}*(\theta)c]^2} + \frac{3c\,[1-g^*(2\theta)}{[c+s-g^*(2\theta)c]^2} - \frac{3c\,[1-g^*(3\theta)}{[c+s-g^*(3\theta)c]^2}]$$

$$=\frac{6}{c^2([1-g^*(\theta)]^2} - \frac{6}{c^2([1-g^*(2\theta)]^2} + \frac{2}{c^2([1-g^*(3\theta)]^2}$$

$$V(T) = \frac{6}{c^2([1-g^*(\theta)]^2} - \frac{6}{c^2([1-g^*(2\theta)]^2} + \frac{2}{c^2([1-g^*(3\theta)]^2}$$

$$-\ [\frac{3}{c(1-g^*(\theta))} - \frac{3}{c(1-g^*(2\theta))} + \frac{1}{c(1-g^*(3\theta)}]^2$$

$g^*(.)\sim\exp(\mu)$

$$g^*(\theta) = \frac{\mu}{\mu+\theta},\quad g^*(2\theta) = \frac{\mu}{\mu+2\theta},\quad g^*(3\theta) = \frac{\mu}{\mu+3\theta},$$

$$E(T) = \frac{3}{c(1-\frac{\mu}{\mu+\theta})} - \frac{3}{c(1-\frac{\mu}{\mu+2\theta})} + \frac{1}{c(1-\frac{\mu}{\mu+3\theta})}$$

$$V(T) = E(T^2) - E(T)^2$$

$$=\frac{6}{c^2([1-g^*(\theta)]^2}-\frac{6}{c^2([1-g^*(2\theta)]^2}+\frac{2}{c^2([1-g^*(3\theta)]^2}-\frac{1}{C^2}[\frac{3}{(1-g^*(\theta))}-\frac{3}{(1-g^*(2\theta))}+\frac{1}{(1-g^*(3\theta)}]$$

$(3\mu+6\theta)/2\theta+(\mu+3\theta)/3\theta]^2$On Simplification

5. NUMERICAL ILLUSTRATION

From table 1 and the corresponding figure1 we could observethe difference in the values E(T) of and V(T) when the threshold distribution has Exponentiated exponential distribution with$\mu=0.4$ and $\theta=0.2$ as c as increases as well as V (T) decreases.

From table 2 and the corresponding figure 2 we could observe

The difference in the values of E (T) and V (T) and when the threshold distribution has Exponentiated exponential distribution. If parameter value is increased by μ = 0.7 and θ = 0.4 as C increases E (T) and V (T) decreases. In both the cases the behavior is found to be same.

Table 1
by μ = 0.4 and θ = 0.2 C = 1, 2, 3…10

C	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
E(T)	**0.12**	**0.06**	**0.04**	**0.03**	**0.024**	**0.02**	**.017**	**.015**	**.013**	**.012**
V(T)	**4.75**	**1.19**	**0.53**	**0.30**	**0.19**	**0.13**	**0.10**	**0.07**	**0.06**	**0.05**

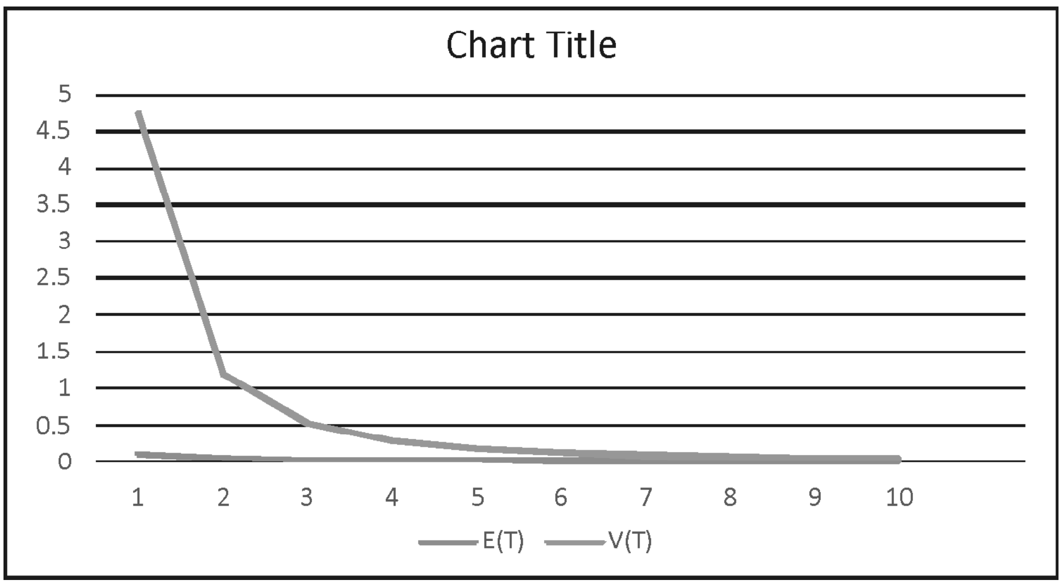
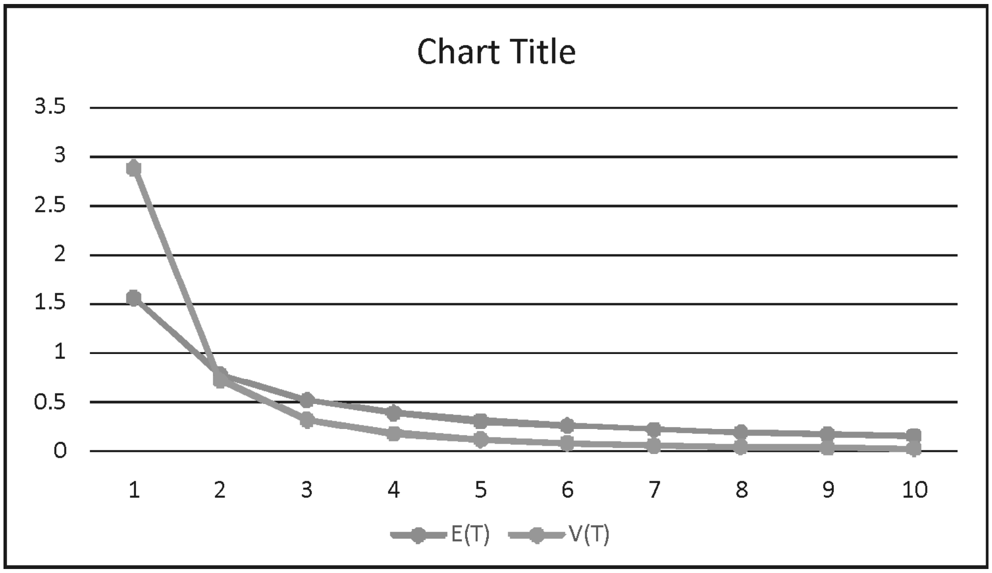


Table 2
μ = 0.7, θ = 0.4 c = 1, 2…10

c	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
E(T)	1.56	0.78	0.52	0.39	0.312	0.26	0.222	0.195	0.173	0.156
V(T)	2.88	0.72	0.32	0.18	0.118	0.08	0.058	0.045	0.035	0.028



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