

Modeling and Control of Proton Exchange Membrane Fuel Cell

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Abstract: This paper presents the modeling and control problem for Proton Exchange Membrane Fuel cell (PEMFC). Reactant partial pressure, Humidity and Temperature are the main input parameters of the fuel cell system. A static model based on mathematical equations, for PEMFC is developed and a fifth order nonlinear flow model is presented in this paper. Sliding mode control is proposed for fuel flow control. By proper control, the fuel starvation or wastage of fuel is avoided. The effectiveness of the proposed control strategies is checked by using Matlab/Simulink software.

Keywords: Mathematical model, Fifth order nonlinear flow model, Sliding mode control

1. INTRODUCTION

A fuel cell is an efficient energy conversion device which can transform the energy released in chemical reaction directly into electrical energy along with the production of heat and water. The fuel cell reaction is as follows:



Hydrogen powered fuel cells are the source of clean energy and their efficiency is twice as the traditional combustion engines. The main input parameters of FC system are oxygen partial pressure, hydrogen partial pressure, fuel cell temperature, stack current and membrane humidity. [3] To achieve high efficiency and a long life cycle of the fuel cell stack, the reactant gas supply and water and heat subsystems need to be properly controlled. Among these variables, most of the literatures have chosen the reactant flow as the variable to be controlled [4][5][6]. A proportional controller based on the pressure difference was adopted in [4]. The input variable used was hydrogen supply. In [1] air flow was used as the input variable. The control comprises of Humidifier control which was static control based on equations and Hydrogen valve control. The authors of [1] used two PI controllers to control the hydrogen flow and oxygen flow.

2. SYSTEM MODEL

2.1. Mathematical Model of Fuel Cell

The fuel cell output voltage varies with the current density and depends on some parameters such as the operating temperature, reactant partial pressure and membrane water content.

The expression for cell reversible voltage is as follows:

$$E_0 = E_{Rev}^0 + \frac{RT}{nF} \ln \left(\frac{P_{H_2} P_{O_2}^{0.5}}{P_{H_2O}} \right) \quad (2)$$

with the change in reversible voltage at standard pressure and can be expressed as:

$$E_{Rev}^0 = 1.50342 - 9.956 \times 10^{-4}T + 2.5 \times 10^{-7}T^2 \quad (3)$$

T is operating temperature of the fuel cell [K], P_{H_2} , P_{O_2} and P_{H_2O} are respectively the relative partial pressure of the hydrogen, oxygen and water. The unit is atmospheric pressure.

As the name indicates, open-circuit voltage (OCV) E_0 , is the output voltage of a single fuel cell when there is no electrical load. The value of open circuit voltage is lesser than the cell reversible voltage E_{Rev} .

The OCV voltage can be expressed such as:

$$E_0 = E_{Rev}^0 + \frac{RT}{nF} \ln \left(\frac{P_{H_2} P_{O_2}^{0.5}}{P_{H_2O}} \right) + \frac{RT}{\alpha n F} \ln (i_0) \quad (4)$$

where i_0 and α are the exchange current densities and transfer coefficient for oxygen reduction reaction.

A small part of the chemical energy is wasted in the form of heat and the fuel cell reaction is not reversible in practical. Also, the practical value of cell voltage is lower than the ideal value.

The value of output voltage V_{cell} drops from open-circuit voltage, as the fuel cell delivers the current which is less than the open-circuit voltage. This is due to the presence of certain losses. They are activation losses, ohmic losses and concentration losses:

$$V_{cell} = E_0 - V_{act} - V_{ohm} - V_{con}. \quad (5)$$

The activation over potential is expressed as:

$$V_{act} = - (\xi_1 + \xi_2 T + \xi_3 T \ln C_{O_2} + \xi_4 T \ln I) \quad (6)$$

with the parametric coefficients,

$$\xi_1 = -0.948$$

$$\xi_2 = 0.00286 + 0.0002 \ln A_{cell} + 4.3 \times 10^{-5} \ln C_{H_2}$$

$$\xi_3 = 7.6 \times 10^{-5}$$

$$\xi_4 = -1.93 \times 10^{-4}$$

Henry's law is used to determine the concentration of hydrogen at the anode-membrane interface, C_{H_2} , and concentration of the oxygen at cathode- membrane interface, C_{O_2} , is determined by

$$C_{O_2} = 1.97 \times 10^{-7} P_{O_2} \exp (498/T) \quad (7)$$

$$C_{H_2} = 9.174 \times 10^{-7} P_{H_2} \exp (-77/T)$$

The membrane resistance, R_{mem} , electronic resistance of electrodes, and resistance of bipolar plates and end - connections R_e , result in ohmic loss. This is expressed as:

$$V_{ohm} = iR_{ohm} \quad (8)$$

The ohmic resistance is a function of membrane water content σ_m .

$$R_{ohm} = \frac{t_m}{\sigma_m} \quad (9)$$

where t_m represents the thickness of the membrane, and σ_m represents the conductivity of membrane which is related to the membrane's water content λ_m . The value of λ_m lies within 0 and 14, which indicates a relative humidity of 0 % to 100 % respectively.

The membrane conductivity is represented as:

$$\sigma_m = b_1 \exp \left(b_2 \left(\frac{1}{303} - \frac{1}{T} \right) \right) \quad (10)$$

where b_2 is a constant and b_1 depends on the membrane water content λ_m , and can be expressed as follow:

$$b_1 = b_{11} \lambda_m - b_{12} \quad (11)$$

where b_{11} and b_{12} are constants, and are empirically calculated.

As the reaction proceeds, the concentration of reactants varies along the electrode-membrane assembly. This results in concentration loss, which can be expressed as:

$$V_{con} = \frac{RT}{nF} \ln \left(1 - \frac{I}{I_m} \right) \quad (12)$$

where I_m represents the limiting current density of the cell. It denotes the maximum value of current density that a cell can deliver.

2.2. Fifth Order Nonlinear Gas Flow Model

The assumptions taken in the development of the detailed dynamic nonlinear model for the PEMFC are given as:

1. The operating temperature of the stack is considered as constant, as the stack temperature has slow operating characteristics.
2. Both anode and cathode inputs are well humidified.
3. Continuous and adequate supply of gases is ensured for the faster response of the FC system.
4. 100% membrane humidity is ensured by proper water management.
5. The anode input is pure hydrogen (99.99 %) and the input to the cathode is air mixed with oxygen and nitrogen in a ratio of 21: 79.

The gases are assumed to be ideal and thus ideal gas law and mole conservation rule are used to derive the flow model. The partial pressures of hydrogen and water on the anode, and the partial pressures of oxygen, nitrogen, and water on the cathode are defined as state variables of the PEMFC. The state variable, $x(t)$ is defined as:

$$x(t) = [P_{H_2}(t) P_{H_2O_A}(t) P_{O_2}(t) P_{N_2}(t) P_{H_2O_C}(t)]^T = [x_1(t) x_2(t) x_3(t) x_4(t) x_5(t)]^T \quad (13)$$

The state model is given as:

$$\begin{aligned} \dot{x}_1 &= \frac{RT \lambda_{H_2}}{V_A} \left(Y_{H_2} - \frac{x_1}{x_1 + x_2} \right) k_a u_a + \frac{RT C_2}{V_A} \left(\frac{x_1}{x_1 + x_2} - 1 \right) I \\ \dot{x}_2 &= \frac{RT \lambda_{H_2}}{V_A} \left(\frac{\varphi_a P_{VS}}{x_1 + x_2 - \varphi_a P_{VS}} - \frac{x_1}{x_1 + x_2} \right) k_a u_a + \frac{RT C_2}{V_A} \left(\frac{x_1}{x_1 + x_2} - 1 \right) I \end{aligned}$$

$$\begin{aligned}
\dot{x}_3 &= \frac{RT\lambda_{air}}{V_C} \left(Y_{O_2} - \frac{x_3}{x_3+x_4+x_5} \right) k_c u_c + \frac{RTC_1}{2V_C} \left(\frac{x_3}{x_3+x_4+x_5} - 1 \right) I \\
\dot{x}_4 &= \frac{RT\lambda_{air}}{V_C} \left(Y_{N_2} - \frac{x_4}{x_3+x_4+x_5} \right) k_c u_c \\
\dot{x}_5 &= \frac{RT\lambda_{air}}{V_C} \left(\frac{\phi_c P_{vs}}{x_3+x_4+x_5 - \phi_c P_{vs}} - \frac{x_5}{x_3+x_4+x_5} \right) k_c u_c + \frac{RTC_2}{V_C} \left(\frac{C_2}{C_4} \left(1 - \frac{x_5}{x_3+x_4+x_5} \right) - 1 - \frac{x_5}{x_3+x_4+x_5} \right) I
\end{aligned} \tag{14}$$

The model output variables are:

$$y(t) = \begin{bmatrix} P_{H_2}(t) \\ P_{O_2}(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_3(t) \end{bmatrix} \tag{15}$$

with the control inputs as:

$$u(t) = [u_a(t) \ u_c(t)]^T \tag{16}$$

where $u_a(t)$ and $u_c(t)$ are the anode side control input and cathode side control input respectively.

3. SLIDING MODE CONTROL FOR THE NONLINEAR PEMFC MODEL

In this section the sliding mode surface is designed and a suitable control law for the output $y(t)$ to track a desired value, y^{ref} is found. The sliding surface is defined as:

$$s(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = e(t) = y(t) - y^{ref} = 0 \tag{17}$$

$e_i(t)$ is the difference between the output $y_i(t)$ and y^{ref} . $s(t) = 0$ indicates that the pressure difference is zero. The equation for $s(t)$ contains the expressions for the control inputs. It is shown as:

$$s(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = y(t) = \begin{bmatrix} x_1(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} \frac{RT\lambda_{H_2}}{V_A} \left(Y_{H_2} - \frac{x_1}{x_1+x_2} \right) k_a u_a + \frac{RTC_1}{V_A} \left(\frac{x_1}{x_1+x_2} - 1 \right) I \\ \frac{RT\lambda_{air}}{V_C} \left(Y_{O_2} - \frac{x_3}{x_3+x_4+x_5} \right) k_c u_c + \frac{RTC_1}{2V_C} \left(\frac{x_3}{x_3+x_4+x_5} - 1 \right) I \end{bmatrix} \tag{18}$$

Note that

$$Y_{H_2} - \frac{x_1(t)}{x_1(t) + x_2(t)} > 0 \quad Y_{O_2} - \frac{x_3(t)}{x_3(t) + x_4(t) + x_5(t)} > 0$$

It is assumed that, the load changes are bounded to the region:

$$0 < I_{min} < I(t) < I_{max} \tag{19}$$

In order to satisfy the Lyapunov stability condition, the sliding mode control laws must satisfy the following sliding mode condition:

$$s_i(t) \dot{s}_i(t) < 0, i = 1, 2. \tag{20}$$

From (18) - (20),

$$s_1(t) = \begin{cases} \frac{RTC_1}{V_A} \left(1 - \frac{x_1}{x_1+x_2} \right) (I_{min} - I(t)) - \frac{RT\lambda_{H_2}}{V_A} \left(Y_{H_2} - \frac{x_1}{x_1+x_2} \right) k_a \sigma_1 \text{sat}(s_1(t)) < 0 & \text{if } s_1(t) > 0 \\ \frac{RTC_1}{V_A} \left(1 - \frac{x_1}{x_1+x_2} \right) (I_{max} - I(t)) - \frac{RT\lambda_{H_2}}{V_A} \left(Y_{H_2} - \frac{x_1}{x_1+x_2} \right) k_a \sigma_2 \text{sat}(s_1(t)) > 0 & \text{if } s_1(t) < 0 \end{cases} \tag{21}$$

$$s_2(t) = \begin{cases} \frac{RTC_1}{2V_C} \left(1 - \frac{x_3}{x_3 + x_4 + x_5}\right) (I_{min} - I(t)) - \frac{RT\lambda_{air}}{V_C} \left(Y_{O_2} - \frac{x_3}{x_3 + x_4 + x_5}\right) k_c \sigma_3 \text{sat}(s_2(t)) < 0 & \text{if } s_2(t) > 0 \\ \frac{RTC_1}{2V_C} \left(1 - \frac{x_3}{x_3 + x_4 + x_5}\right) (I_{max} - I(t)) - \frac{RT\lambda_{air}}{V_C} \left(Y_{O_2} - \frac{x_3}{x_3 + x_4 + x_5}\right) k_c \sigma_4 \text{sat}(s_2(t)) > 0 & \text{if } s_2(t) < 0 \end{cases}$$

$$\text{Since, } \frac{RTC_1}{V_A} \left(\frac{x_1}{x_1 + x_2} - 1\right) I = -\frac{RTC_1}{V_A} \frac{x_1}{x_1 + x_2} I < 0$$

$$\frac{RTC_1}{2V_C} \left(\frac{x_3}{x_3 + x_4 + x_5} - 1\right) I = -\frac{RTC_1}{2V_C} \frac{x_4 + x_5}{x_3 + x_4 + x_5} I < 0$$

Note that $I(t) - I_{min} > 0$ and $I_{max} - I(t) > 0$.

The control laws are designed as:

$$u_a(t) = \begin{cases} -\frac{C_1 \left(\frac{x_1}{x_1 + x_2} - 1\right) I_{min}}{\lambda_{H_2} \left(Y_{H_2} - \frac{x_1}{x_1 + x_2}\right) k_a} - \sigma_1 \text{sat}(s_1(t)) & \text{if } s_1(t) > 0 \\ -\frac{C_1 \left(\frac{x_1}{x_1 + x_2} - 1\right) I_{max}}{\lambda_{H_2} \left(Y_{H_2} - \frac{x_1}{x_1 + x_2}\right) k_a} - \sigma_2 \text{sat}(s_1(t)) & \text{if } s_1(t) < 0 \end{cases} \quad (22)$$

and

$$u_c(t) = \begin{cases} -\frac{C_1 \left(\frac{x_3}{x_3 + x_4 + x_5} - 1\right) I_{min}}{2\lambda_{air} \left(Y_{O_2} - \frac{x_3}{x_3 + x_4 + x_5}\right) k_a} - \sigma_3 \text{sat}(s_2(t)) & \text{if } s_2(t) > 0 \\ -\frac{C_1 \left(\frac{x_3}{x_3 + x_4 + x_5} - 1\right) I_{max}}{2\lambda_{air} \left(Y_{O_2} - \frac{x_3}{x_3 + x_4 + x_5}\right) k_a} - \sigma_4 \text{sat}(s_2(t)) & \text{if } s_2(t) < 0 \end{cases} \quad (23)$$

where $\sigma_i > 0, i = 1, 2, 3, 4$ are used to adjust the speed of reaching the sliding manifold. Since $u_a(t)$ and $u_c(t)$ are being the flow rates, values of σ_1 and σ_3 should give $u_a(t) > 0$ and $u_c(t) > 0$. σ_2 and σ_4 are chosen depending on the upper limits of $u_a(t)$ (u_{amax}) and $u_c(t)$ (u_{cmax}).

4. SIMULATION RESULTS

The parameters of the fuel cell system are given in Table 1. The control objective is formulated as to keep the partial pressure of reactants (hydrogen and oxygen) as desired, in order to ensure proper fuel usage with control inputs limited to: 0 slpm < input flow rate of hydrogen or air ([slpm]) < 10 slpm. The desired value of the reactant partial pressure is selected as 3 atm for both hydrogen and oxygen.

Figure 1 gives the disturbance current waveform. The simulation results for hydrogen and oxygen partial pressures are given in Figures 2 and 3. From Figures 2 and 3, it can be seen that the partial pressures are maintained to the desired value, i.e., 3 atm, even with the abrupt changes in the input current, and it takes 2.5s to reach the steady-state values. As fuel cell has a slow start, a time delay of 2.5s has an ignorable impact on fuel cell working.

Table 1
Parameters of FC system

<i>Parameter</i>	<i>Value</i>
Operating temperature, T	353 [K]
Number of cells, N	35
Anode volume, V_A	0.005 [m^3]
Cathode volume, V_C	0.010 [m^3]
Anode conversion factor, k_a	7.034×10^{-4} [mol s^{-1}]
Cathode conversion factor, k_c	7.036×10^{-4} [mol s^{-1}]
Fuel cell area, A	232×10^{-4} [m^2]
Saturation pressure, P_{vs}	0.3158 [atm]
Y_{O_2}	0.2095
Y_{N_2}	0.7808
Y_{H_2}	0.9999
$C_1 = NA/2F$	4.21×10^{-6} [$m^2 \text{ mol } A^{-1} S^{-1}$]
$C_2 = 1.2384 NA/2F$	1.07×10^{-5} [$m^2 \text{ mol } A^{-1} S^{-1}$]
Stoichiometric constant for hydrogen, λ_{H_2}	2
Stoichiometric constant for air, λ_{air}	2.5
Relative humidity at anode side, $\lambda\phi_a$	0.8
Relative humidity at cathode side, ϕ_c	0.9
Faraday constant, F	96485 [$A \text{ s } mol^{-1}$]

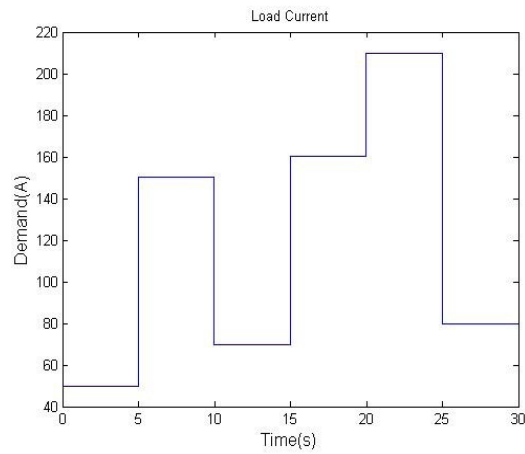


Figure 1: Fuel cells current

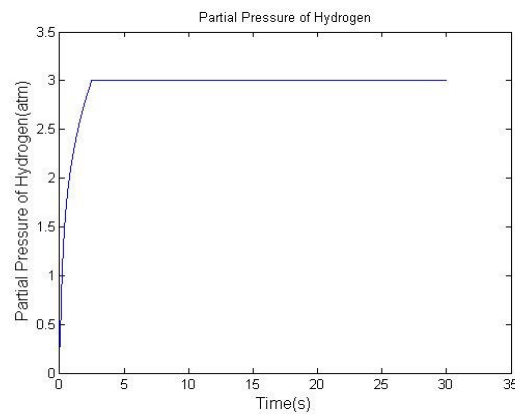


Figure 2: Variation of hydrogen partial pressure: Using sliding mode control.

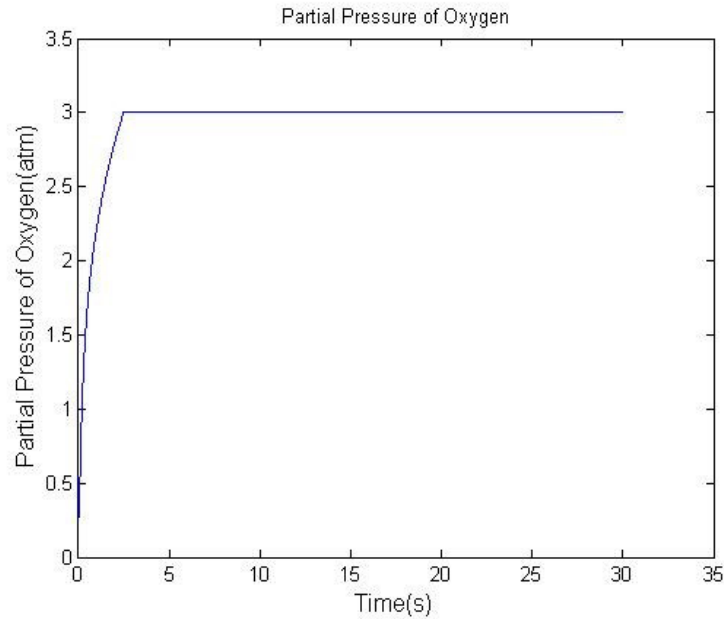


Figure 3: Variation of oxygen partial pressure: Using sliding mode control.

5. CONCLUSION

In this paper, a mathematical model is proposed to simulate the steady state and the transient phenomena in a PEMFC system. A detailed, i.e. fifth order nonlinear flow model is also presented in this paper and sliding mode control law is used. The chattering phenomenon is fully suppressed and the sudden changes in fuel cell current have no influence on the reactant partial pressure. The effectiveness of the proposed control strategy for Proton Exchange Membrane Fuel Cell system was verified through simulations carried out in the Matlab/Simulink software. The results show that the proposed control provide faster response ensuring proper usage of fuel.

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