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INSTABILITY OF VOID COLLAPSE FOR INCOMPRESSIBLE THERMO-HYPERELASTIC MATERIALS

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ABSTRACT

The instability problem of void collapse in a spherical shell, composed of an incompressible thermo-hyperelastic material, subject to a uniform radial boundary pressure is examined within the framework of finite elasticity. For all values of the applied pressure, one solution corresponds to the homogeneous state of the shell in which the void is open. For sufficiently large values of the pressure, there is another solution in which the spherical void is collapsed. An explicit expression for the critical pressure at which void collapse occurs is given. Stress distribution after the collapse of void is discussed and the effect of temperature on the collapse of void is studied in the case of the uniform temperature field as well as the non-uniform temperature field. The effect of material constant and the initial size of the void on the collapse of void are discussed, too.

***Key words**:* thermo-hyperelastic material, void collapse, temperature field, instability

1. INTRODUCTION

In recent years, rubber-like materials such as rubber and other polymeric materials are using in a broader and broader range of engineering field. So nonlinear problems for cavitation instability in hyper-elastic materials have attracted much attentions.The occurrence of a cavitation instability has been interpreted either as a bifurcation from a homogeneously stressed solid to a solid involving a void, or as the suddenly rapid growth of a pre-existing void[1]~[5].

A void in a certain nonlinear medium may grow without bound when a cavitation stress limit is reached[6]~[8]. Ball[1] determined the precise sub-class of such incompressible materials that exhibit cavitation instability and gave an explicit expression for the critical load at which the void radius becomes infinite for such materials. For compressible materials, various aspects of the corresponding problem have been considered[1],[3]~[5],[9],[10]. Both for incompressible and compressible isotropic hyper-elastic materials, cavitation solutions don’t always exist. Other aspects of the cavitation instability problems have been investigated include the effects of material inhomogeneity[11]~[14], material anisotropy[15]~[20], finite strain plasticity[21], asymmetric deformation of cavitation[22],[23] and so on.

The “complementary” problem to this concerns the collapse of a void. When a body containing a void is subjected to a pressure, the void may suddenly closed (the void radius becoming zero at a finite value of the applied pressure) when the applied pressure is sufficiently large, which is larger than its critical value. Budiansky, Hutchinson and Slutsky[24] carried out the study of void collapse in the case of power-law viscous solids and pointed out that there are various shapes into which an initially spherical void could asymptotically collapse. Abeyaratne and Hou[25] carried out the examine of the possibility of void collapse in the incompressible solids and determined the class of materials that do exist this phenomenon.

The importance of gaining a theoretical understanding of the thermo-mechanical behavior of rubber was illustrated by the role of the O-ring seals in the Challenger shuttle disaster. Not only the elasticity theory but also extensions of

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the theory to account for inelastic effects, especially for the effect of temperature, are involved for rubber-like materials those are used in the circumstances of high temperature. Thermo-mechanical behavior of rubber-like materials is described as the thermo-hyperelastic model, as considered by Casey[26], Nicholson and Nelson[27], Nicholson and Lin[28] and so on.

The purpose of the present paper is to further investigate the problem of void collapse in the case of the incompressible thermo-hyperelastic materials. The problem of void collapse for a spherical shell composed of the generalized incompressible Gent-Thomas thermo-hyperelastic material, under a uniform radial boundary pressure is investigated. From the condition of incompressibility of the material, the radial symmetric deformation function of the shell is given by means of an undetermined parameter describing the deformation of the void. For all values of the applied pressure, one solution corresponds to the homogeneous state of the shell in which the void is open. For sufficiently large values of the pressure, which is larger than the critical value of the pressure, there is the collapse solution in which the spherical void is collapsed.

The exact analytic relation among the parameter for the void and the pressure as well as an explicit formula to determine the critical pressure is obtained from solving the differential equation satisfied by the deformation function. When the pressure exceeds the critical value, the spherical void may suddenly collapse. The solution depends on the temperature of the material in the case of a uniform temperature field or a non-uniform temperature field.

The bifurcation curves for variation of the void radius and the applied pressure are obtained from numerical calculations based on the analytic solution. For an elevated temperature field, the critical pressure is lower than that at the reference temperature and for a descensional temperature, the critical pressure is larger than that at the reference temperature. The stress distribution subsequent to the collapse as well as the effect of the temperature field on the stress distribution is analyzed. When a void is closed at the critical pressure, the stresses have an obviously catastrophic transition from the homogeneous distribution to the non-homogeneous distribution.

2. BASIC EQUATIONS

Consider here the finite deformation for a solid spherical shell with inner radius a and outer radius b, composed of the generalized incompressible power-law thermo-hyperelastic material. Assume that the shell is subjected to a uniform radial boundary dead-load p_0 on its outer boundary surface. The undeformed and the deformed configurations of the shell are described by the spherical coordinates systems (R, Θ, Φ) and (r, θ, φ), with the co-origin at the center of the shell. Assume that the deformation function of the shell is radially symmetric, namely,

$$r = r(R),\ \ \theta = \Theta,\ \varphi = \Phi \quad (a \le R \le b,\ 0 \le \Theta \le 2\pi,\ 0 \le \Phi \le \pi) \tag{1}$$

here $r(R)$ is an undetermined function. The principal stretches are given by

$$\lambda_R = \dot{r}(R) = \frac{\mathrm{d}r}{\mathrm{d}R},\ \ \lambda_\Theta = \lambda_\Phi = \frac{r(R)}{R} \tag{2}$$

The strain energy function of the generalized incompressible power-law thermo-hyperelastic material may be

$$W = \begin{cases} \dfrac{\mu}{2n\beta}\left\{\left[1+\beta\left((I_1-3)+\dfrac{2\rho C_3}{\mu}T\ln\left(\dfrac{T}{T_0}\right)+\dfrac{4C_4}{\mu}(T-T_0)(I_1-3)\right)\right]^n-1\right\} & for\ \ n\neq 0 \\ \dfrac{\mu}{2\beta}\log\left[1+\beta\left((I_1-3)+\dfrac{2\rho C_3}{\mu}T\ln\left(\dfrac{T}{T_0}\right)+\dfrac{4C_4}{\mu}(T-T_0)(I_1-3)\right)\right] & for\ \ n=0 \end{cases} \tag{3}$$

In which, $\beta > 0$ and n are material parameters, μ is the infinitesimal shear modulus of the material, $I_1 = \lambda_R^2 + \lambda_\Theta^2 + \lambda_\Phi^2$ is the first invariant of the deformation tensor. Other material parameters, $C_3 = C_e = 1506\ Jkg^{-1} K^{-1}$, $\rho = 950$ kgm^{-3}, $C_4 = -\alpha\lambda$, $\alpha = 6.36 \times 10^{-4}$ K^{-1}, $\lambda = 0.4225MPa$, $T_0 = 300°$ K[29]. Here, C_e is the specific heat at constant strain, α is the volume coefficient of thermo expansion, ρ is the density of the material, T_0 is a reference temperature, λ is the second Lame' coefficient of the material.

Assuming the temperature differences $T = T(R)$ from the stress-free state is passive, stationary and radially symmetric, the heat conduction equation is

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}R^2}+\frac{2}{R}\frac{\mathrm{d}}{\mathrm{d}R}\right)T(R)=0 \tag{4}$$

With the boundary conditions $T(R = a) = T_a$, $T(R = b) = T_b$, integrating (4), we have

$$T(R)=\frac{bT_b-aT_a}{b-a}+\frac{1}{R}\frac{ab(T_a-T_b)}{b-a} \tag{5}$$

If $T_a \neq T_b$, there is a non-uniform temperature field as Eq. (5) for the shell and if $T_a = T_b$, there is a uniform temperature field $T(R) = T_a = T_b$ for the shell.

From the incompressibility condition of the material $\lambda_R \lambda_\Theta \lambda_\Phi = 1$ and Eq. (2), we have

$$r(R)=\left(R^3+c^3-a^3\right)^{\frac{1}{3}} \tag{6}$$

where, $0 \le c \le a$ is an undetermined constant describes the deformation of the void. If $c > 0$, the void remains open and if $c = 0$, a void collapse occurs. Introducing the notation

$$v=v(R)=\lambda_\Theta=\frac{r(R)}{R}=\left(1+\frac{c^3-a^3}{R^3}\right)^{\frac{1}{3}} \tag{7}$$

Then, the non-zero Cauchy stress components are

$$\begin{aligned}\tau_{rr}(R)&=v^{-2}W_1\left(v^{-2},v,v,T(v)\right)-p(R)\\ \tau_{\theta\theta}(R)&=\tau_{\varphi\varphi}(R)=vW_2\left(v^{-2},v,v,T(v)\right)-p(R)\end{aligned} \tag{8}$$

in which $p(R)$ is the undetermined hydrostatic pressure. The subscript notation on W denotes differentiation with respect to the appropriate argument. The equilibrium equation of the sphere in the absence of body forces is

$$\frac{\mathrm{d}\tau_{rr}}{\mathrm{d}R}+\frac{2\dot{r}(R)}{r(R)}\left[\tau_{rr}-\tau_{\theta\theta}\right]=0 \tag{9}$$

The boundary condition at the outer edge is

$$\tau_{rr}(b)=-p_0\left[\frac{b}{r(b)}\right]^2 \tag{10}$$

The boundary condition at the cavity surface should be

$$c > 0 \quad \tau_{rr}(c) = 0 \tag{11}$$

or

$$c = 0 \quad \tau_{rr}(c) \le 0 \tag{12}$$

3. SOLUTIONS

It is easy to show that the problem always has a trivial solution corresponds to the homogeneous deformation state, namely, $r(R)=(R^3+c^3-a^3)^{\frac{1}{3}}$, $(0<c\le a)$ and the homogeneous stress state, $\tau_{rr}=\tau_{\theta\theta}=\tau_{\varphi\varphi}=p_0\left(1+\dfrac{c^3-a^3}{b^3}\right)^{\frac{1}{3}}$.

In order to seek the collapse solution for $c = 0$, substituting Eq. (8) into Eq. (9) and using the variable transformation $r(R) = Rv(R)$, the equilibrium equation may be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}R}\left[v^{-2}W_1\left(v^{-2},v,v,T\right)-p\left(R\right)\right]+\frac{2v^{-4}}{R}\left(v^{-1}W_1\left(v^{-2},v,v,T\right)-v^2W_2\left(v^{-2},v,v,T\right)\right)=0 \tag{13}$$

Integrating Eq. (13) and substitute it into Eq. (8), we have

$$\tau_{rr}\left(R\right)=-p\left(a\right)-\int_a^R\frac{2v^{-4}}{R}\left(v^{-1}W_1\left(v^{-2},v,v,T\right)-v^2W_2\left(v^{-2},v,v,T\right)\right)\mathrm{d}R \tag{14}$$

Substitute Eq. (14) into Eq. (11)~(12), we have $p(a) = 0$ and

$$p_0=\left(1+\frac{c^3-a^3}{b^3}\right)^{2/3}\int_{v(a)}^{v(b)}\frac{W_v\left(v,T\right)}{v^3-1}\mathrm{d}v \tag{15}$$

Where, $W_v(v, T) = -2(v^{-3}W_1 - W_2)$. The Eq. (15) is an exact analytic relation between the void radius c and the applied pressure p_0. For a given pressure, the parameter c corresponding to various values of it may be obtained. If there exists a root $c = 0$, this means that the void collapses in the shell.

Letting $c = 0$ in Eq.(15), the critical pressure p_{cr} at which the void collapse is given as

$$p_{cr}=\left(1-\frac{a^3}{b^3}\right)^{2/3}\int_0^{\left(1-\frac{a^3}{b^3}\right)^{\frac{1}{3}}}\frac{W_v\left(v,T\right)}{v^3-1}\mathrm{d}v \tag{16}$$

A given material admits the phenomenon of void collapse at a finite pressure if and only if its strain energy function $W(v, T)$ make the integral in Eq. (16) exist. Because material strain energy function satisfies the normalization condition $W(1, T) = 0$, $W_v(1, T) = 0$, the integral is always convergent at the upper limit and so the existence of the integral depends on the behavior of $W(0, T)$. If the value of $W(0, T)$ is finite, then the integral Eq. (16) exists and thus the material admits the phenomenon of void collapse.

Thus if

$$n<0 \tag{17}$$

for the material, the value of the critical pressure p_{cr} given by Eq. (16) will be finite[25].

4. RESULTS OF VOID COLLAPSE

Operating numerical computation on Eq. (16) yields the critical value p_{cr} of the pressure for the thermo-hyperelastic material under a given temperature field. Critical values p_{cr} of the pressure for the thermo-hyperelastic material under some given temperature fields are summarized in Table 1. Curve for variation of the critical pressure p_{cr} with the original radius of the void are shown in Fig. 1. At the same time, operating numerical computation on Eq. (15) yields the relationship $c = c(p_0)$ between the dimensionless void radius c/b and the applied pressure p_0. Curves for the thermo-hyperelastic material under some given elevated and lowered uniform temperature fields are shown in Fig. 2 and Fig. 3, respectively. Curves for the thermo-hyperelastic material under some given lowered and elevated non-uniform temperature fields are shown in Fig. 4 and Fig. 5, respectively. At the same time, curves corresponding to different values of the hardening exponent n and different original radius of the void are shown in Fig. 6 and Fig. 7, respectively.

If $p < p_{cr}$, there is a unique solution for Eq. (15), that is, $c > 0$, so the only possible configuration is one in which the void is open under a homogeneous deformation state. If $p \geq p_{cr}$, there is another solution for Eq. (15) with $c = 0$, so there exists the collapse solution. When $p > p_{cr}$, the void almost keep unchanged but when $p \geq p_{cr}$, the void suddenly collapsed.

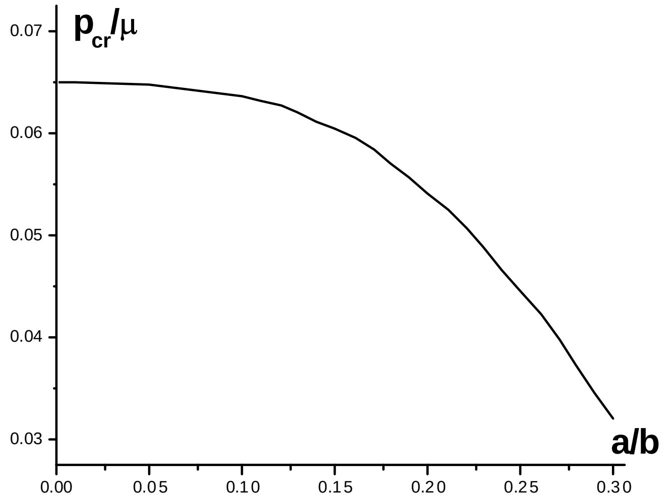

Figure 1: Variation of the Critical Pressure with the Original Radius of the Void

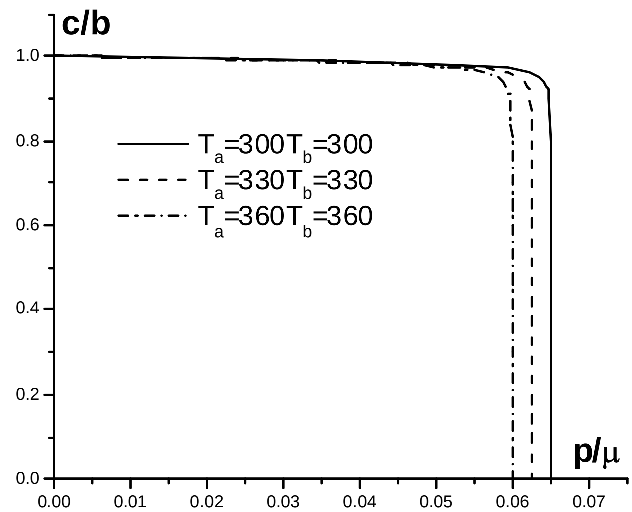

Figure 2: Variation of the Void radius with the Applied Pressure for Uniform Elevated Temperature Field

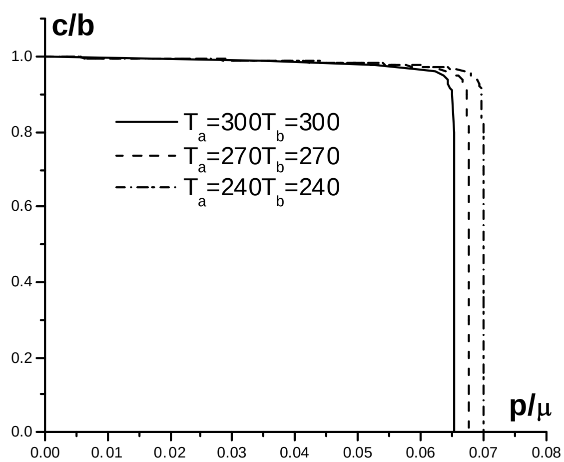

Figure 3: Variation of the Void Radius with the Applied Pressure for Uniform Descensional Temperature Field

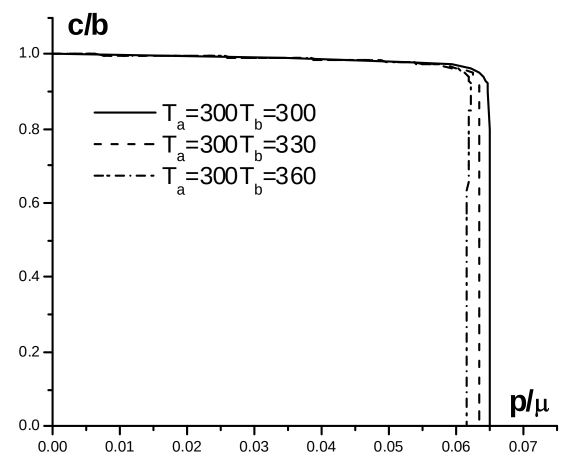

Figure 4: Variation of the Void Radius with the Applied Pressure for Non-uniform Elevated Temperature Field

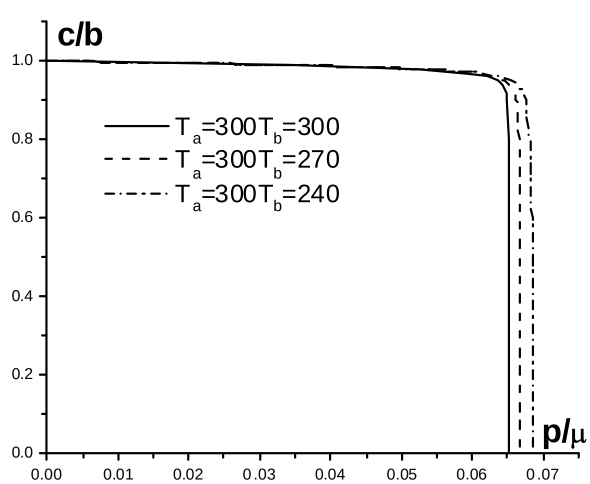

Figure 5: Variation of the Void Radius with the Applied Pressure for Non-uniform Descensional Temperature Field

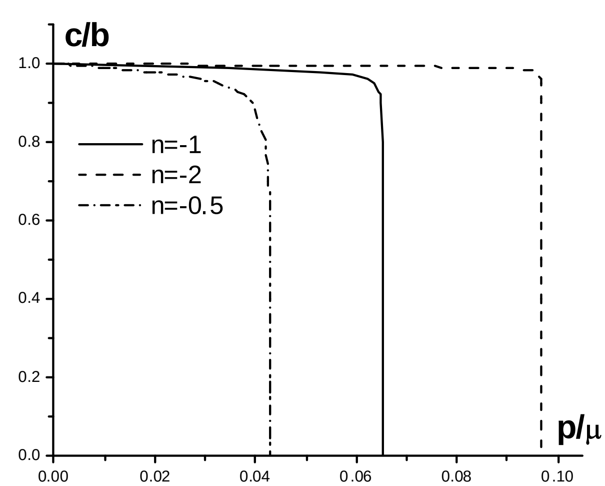

Figure 6: Variation of the Void Radius with the Applied Pressure for the Hardening Exponent *n*

Table 1
Critical Values of the Pressure for Some Given Temperature Fields

$T_{a(K)}$	360	300	330	300	300	300	270	300	240
$T_{b(K)}$	360	360	330	330	300	270	270	240	240
p_{cr}	0.06	0.0616	0.0626	0.0634	0.065	0.0685	0.0675	0.0685	0.07

It shows that the critical pressure p_{cr} increases with the decreasing of temperature and decreases with the increasing of temperature both for uniform and non-uniform temperature field. The effect of uniform temperature field is larger than that of non-uniform temperature field with the same temperature on the outer boundary. That is to say the material will be softened if the temperature is increased and the material will be hardened if the temperature is decreased.

It also shows that the critical pressure p_{cr} decreases with the increasing value of the original radius of the void. Thus it is easier for a shell to occur void collapse with a larger void. At the same time, the critical pressure p_{cr} increases with the increasing value of the hardening exponent n. That is to say it is more difficult to for the collapse of void in a more hardening material.

5. RESULTS OF STRESS CONTRIBUTION

When $p > p_{cr}$, the void will collapse suddenly. The corresponding principal stresses after the collapse of void are

$$\begin{aligned} \tau_{rr}(R) &= -p_0 v^{-2}(b) + \int_{v(R)}^{v(b)} \frac{W_v(v,T)}{v^3-1}\,\mathrm{d}v \\ \tau_{\theta\theta}(R) &= \tau_{\varphi\varphi}(R) = \frac{v}{2}W_v(v,T) + \tau_{rr}(R) \end{aligned} \tag{18}$$

When $p \geq p_{cr}$, the principal stresses obtained from Eq. (18) for the reference temperature and a given uniform elevated temperature field are shown in Fig. 8 and Fig. 9, respectively. It can be seen that the radial stress τ_{rr} is zero at the void surface and decreases rapidly with the increasing radius and approaches an asymptotic value in the region far from the void. The graph declines monotonically through the shell. On the other hand, the circumferential stress $\tau_{\theta\theta}$ is zero at the void surface and decreases rapidly with the increasing radius and approaches the same asymptotic value in the region far from the void. But the graph does not decline monotonically through the shell. One may see that in the region far from the void ($R \geq 0.2b$), there is a homogeneous stress state.

At the same time, if the void has collapsed, stress distribution does not have obvious different among different temperature fields under the same pressure and the homogeneous stress state in the region far from the void is almost the same as others.

The stresses corresponding to the homogeneous solution are given by the homogeneous state of stress $\tau_{rr} = \tau_{\theta\theta} = \tau_{\varphi\varphi} = p_0\left(1+\dfrac{c^3-a^3}{b^3}\right)^{\frac{1}{3}}$, when $p < p_{cr}$. Thus, when a void is closed when $p = p_{cr}$, the stresses have an obviously catastrophic transition from the homogeneous distribution to the non-homogeneous distribution.

6. ENERGY COMPARISON

For a spherical shell composed of the incompressible thermo-hyperelastic material, under a uniform radial boundary pressure, one solution corresponds to the homogeneous state of the shell in which the void is open always exist for all values of the pressure. For sufficiently large values of the pressure, there is the collapse solution in which the spherical void is collapsed. So it is necessary to compare the potential energies corresponding to each solution.

Apparently, for the trivial solution, the potential energy of the shell is $E_0 = 0$.

For the collapse solution, the potential energy of the shell is

$$E=\int_V WdV-\int_A p_0\left(b-\left(b^3+c^3-a^3\right)^{\frac{1}{3}}\right)dA$$
$$=4\pi\int_0^{\left(b^3+c^3-a^3\right)^{\frac{1}{3}}} R^2WdR-4\pi b^2 p_0\left(1-\left(\frac{b^3+c^3-a^3}{b^3}\right)^{\frac{1}{3}}\right) \tag{19}$$

Under a given temperature field, substituting Eq. (3) and Eq. (15) into Eq. (19), we will yield the potential energy of the shell for the collapse solution. Numerical result under the reference temperature is shown in Fig. 10. It is shown that when $p_0 \geq p_{cr}$, the potential energy of the shell corresponds to the collapse solution is always lower than that corresponds to the trivial solution. So the collapse solution is stable when $p_0 \geq p_{cr}$ and the phenomenon of void collapse does exist in the shell.

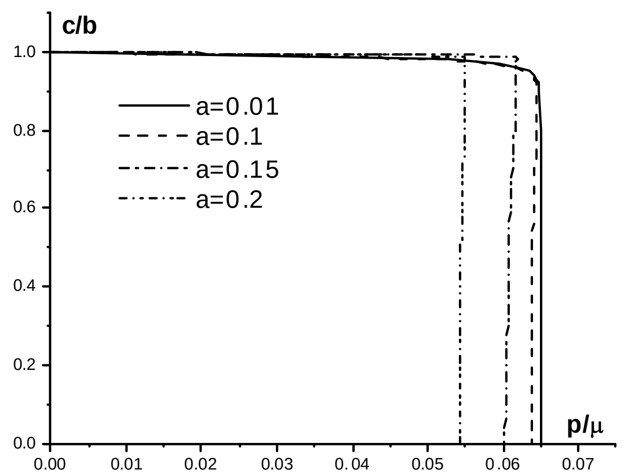

Figure 7: Variation of the Void Radius with the Applied Pressure for the Original Radius of the Void

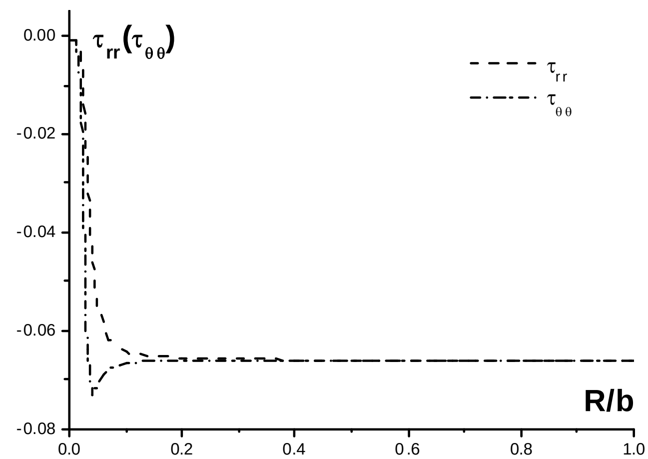

Figure 8: Stress Distribution under an Uniform Temperature Field

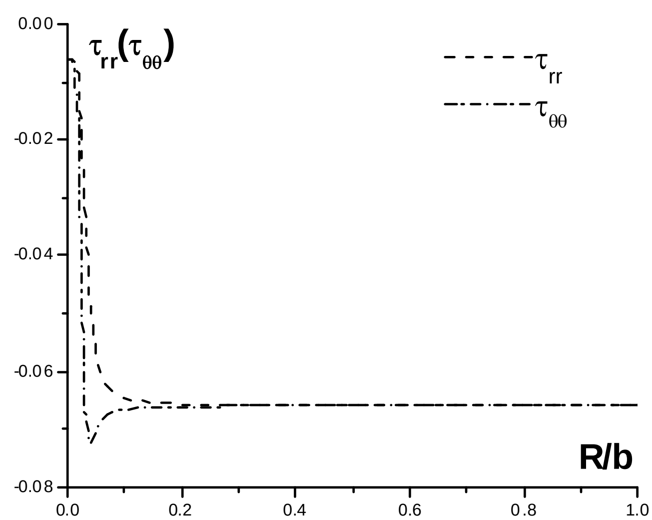

Figure 9: Stress Distribution under an Uniform Temperature Field

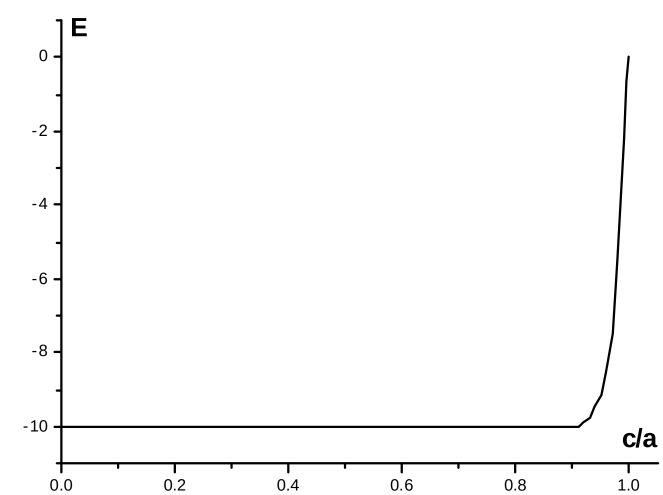

Figure 10: Energy Curve under the Renference Temperature Field

7. CONCLUSION

The bifurcation problem of void collapse for incompressible thermo-hyperelastic materials under a uniform radial boundary pressure and a given temperature field is studied. For small values of the pressure, one solution corresponds to a homogeneous state exists. However, for sufficiently large values of the pressure, there is the collapse solution in which the spherical void is collapsed. It is apparent that the effect of temperature on the collapse of void is obvious.The critical pressure increases with the decreasing of temperature and decreases with the increasing of temperature both for uniform and non-uniform temperature field. At the same time, the critical pressure decreases with the increasing value of the original radius of the void but increases with the increasing value of the hardening exponent n of the material.

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