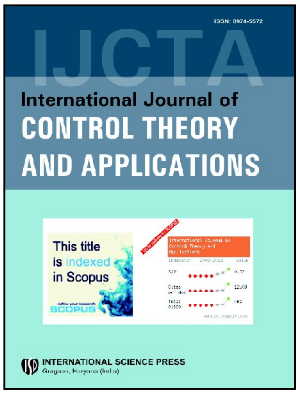

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Study and Improvements of Capacity Bounds of Spatial Multiplexing MIMO Systems with Hardware Impairments

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Abstract: The channel capacity of ideal multiple-input multiple-output (MIMO) channels with high signal-to-noise ratio (SNR) equals the minimum number of transmit and receive antennas operated in spatial multiplexing mode. The MIMO system channels were distorted due to physical transceiver impairments. In this paper we study and prove analytically that in SM MIMO system, *upper limit* of channel capacity is *finite* for any channel variations and SNR. Simulation results show the relative capacity gains of MIMO in the presence of transceiver impairments. Finally multiplexing gain of finite SNR is decreases when add additional constraints and physical impairments limits the asymptotic channel capacity and channel acquisitions.

Keywords: Channel capacity, MIMO, transceiver impairments and SNR

1. INTRODUCTION

In the past decade, multiple input multiple-output (MIMO) system makes use of wireless communication standards. Different modes of MIMO operations are required to optimize the system performance under physical transceiver impairments [1]-[2]. In last decade many number of researchers has been studied MIMO system and motivated towards asymptotic capacity limits in the high signal-to-noise-ration (SNR) regime.

Channel Capacity of MIMO system with channel information at the receiver is given as $M\log_2\left(\frac{S}{N}\right)+\varphi(1)$, with $M=\min\left(N_T, N_R\right)$ where N_T and N_R represents number of transmit and receive antennas respectively [1]. Asymptotic channel gain is M over single antenna channels are known as multiplexing gain or degrees of freedom.

Some ambiguity in applying of these results in cellular networks has recently experienced; the average gain of MIMO system over conventional techniques has been achieved and higher data rates may be reduced due to extra overheads [2]-[3]. The channel consistency duration will limits the resources for channel acquisition [4], thus it makes a fundamental upper limit for network spectral efficiency (SE) regardless of transmitted power and number of base station (BS) antennas.

Further, MIMO system performance is affected by transceiver impairments [5]–[10]. Transceiver impairments such as non linearity's of amplifiers, imbalance of IQ, quantization and phase noise, carrier frequency offset (CFO) etc., affects the physical radio-frequency (RF) transceivers. These impairments are basically neglected in information theory, but we show that SE of MIMO system with high SNR were affected non-negligibly.

In this paper, we studied and an analysis the transceiver impairments [7] of MIMO channel and also shows that upper limit of channel capacity is finite limit at high values of SNR for any channel variations. Hence multiplexing gain of physical MIMO is decreases to zero. Therefore, relative channel capacity of real-time MIMO increases over single-antenna channels.

The organization of this paper as follows: Section –I explains about introduction; MIMO channel model discussed in Section –II; Capacity analysis and multiplexing gains explained in Section –II and IV. Section-V explains simulation results and Section-VI includes concluding remarks.

2. MIMO CHANNEL MODEL

Consider a flat fading MIMO channel, having N_T transmitting antennas and N_R receiving antennas. The received signal $y \in \mathbb{C}^{N_R}$ is given as

$$y = Hx\sqrt{\frac{S}{N}} + n \tag{1}$$

Where $x \in \mathbb{C}^{N_T}$ is the transmitting signal, $H \in \mathbb{C}^{N_R \times N_T}$ is the random channel matrix, *n* is circular-symmetric complex Gaussian noise and SNR is signal- to- noise ratio. The H is multivariate distribution and full rank of H is equal to min (N_T, N_R)- which fundamentally includes any physical channel realizations.

The multiplicative channel variations and additive thermal noise affected only the intended transmitted signal *x*, thus ideal transceiver system is assumed. The influence of impairments is tried to reduce by using different compensation techniques.

The impact of transmitter impairments is denoted as transmitter distortion $\eta_T \in \mathbb{C}^{N_T}$. Thus (1) can be written as

$$y = H\left(x + \eta_T\right)\sqrt{\frac{S}{N}} + n \tag{2}$$

With the normalized power limits tr(Q) = 1, where $Q = E\left\{xx^H\right\}$. η_T is defined as $\eta_T \sim cN\left(0, \Upsilon_t\left(Q\right)\right)$ with

$$\Upsilon_t = diag\left(v_1\left(q_1\right), \ldots\ldots, v_{N_T}\left(q_{N_T}\right)\right)$$

The transmitter distortion is based on the signal x i.e. variance $v_n(q_n)$ is an increasing function of the signal power q_n at the n[th] transmit antenna and antenna cross correlation Υ_t is neglected.

Above equation (2) represents individual subcarriers, from the OFDM point of view. However some distortion presents between subcarriers, with results variance of transmitted signal $v_n(q_n)$ will be less influenced by q_n. To get range of values we propose

$$v_n(q_n) = \kappa^2 \left((1-\alpha) q_n + \alpha \frac{\sum_{i=1}^{N_T} q_i}{N_T} \right) \tag{3}$$

Where the parameter $\kappa > 0$ is represented as the level of impairments, the parameter $\alpha \in [0,1]$ enables transition from one subcarrier ($\alpha = 0$) to many carriers many ($\alpha = 1$).

3. CHANNEL CAPACITY ANALYSIS

With the channel distribution f_x knows at the transmitter and channel realization H at the receiver, the channel capacity is defined as

$$C_{N_T,N_R}\left(\frac{S}{N}\right) = \sup_{f_x : tr\left(E\left\{xx^H\right\}\right) = tr(Q) = 1} \mathrm{I}(x; y | H) \tag{4}$$

Where f_x is the probability density function of transmitted signal x and $\mathrm{I}(\cdot\cdot|\cdot)$ is the conditional mutual information of desired signal.

Lemma 1: The capacity of MIMO system $C_{N_T,N_R}\left(\frac{S}{N}\right)$ can be given as

$$\sup_{Q: tr(Q)=1} E_H \left\{ \log_2 \det\left(I + \frac{S}{N} HQH^H \left(\frac{S}{N} H \Upsilon_t H^H + I \right)^{-1} \right) \right\}$$

Though above capacity equation is very similar to conventional MIMO system channels but it act very unusual especially in high SNR schemes.

Theorem 1: The capacity asymptotic bound is finite $C_{N_T,N_R}(\infty) = \lim_{\frac{S}{N} \to \infty} C_{N_T,N_R}\left(\frac{S}{N}\right)$ and bounded as

$$M \log_2\left(1 + \frac{1}{\kappa^2}\right) \le C_{N_T,N_R}(\infty) \le M \log_2\left(1 + \frac{N_T}{M\kappa^2}\right) \tag{5}$$

Where $M = \min\left(N_T, N_R\right)$, the lower capacity bound is asymptotically obtained by $Q = \frac{1}{N_T} I$. The two capacity bounds are coinciding if $N_T \le N_R$.

Proof: Consider a full-rank channel matrix H and $H^H H = U_M \Lambda_M U_M^H$ describes eigen decomposition values.

When the mutual information increases with SNR and satisfies as SNR $\to \infty$.

$$\log_2 \det\left(I + \frac{S}{N} HQH^H \left(\frac{S}{N} H\Upsilon_T H^H + I \right)^{-1} \right)$$

$$= \log_2 \det\left(I + \frac{S}{N} U_M^H \left(Q + \Upsilon_T\right) U_M \Lambda_M \right) - \log_2 \det\left(I + \frac{S}{N} U_M^H \Upsilon_T U_M \Lambda_M \right) \to$$

$$\log_2 \det\left(U_M^H \left(Q + \Upsilon_T\right) U_M \Lambda_M\right) - \log_2 \det\left(U_M^H \Upsilon_T U_M \Lambda_M\right)$$

$$= \log_2 \det\left(I + U_M^H Q U_M \left(U_M^H \Upsilon_T U_M\right)^{-1} \right)$$

$$= \log_2 \det\left(I + \Upsilon_T^{-1/2} Q \Upsilon_T^{-H/2} \Pi \Upsilon_T^{H/2} U_M \right) = \sum_{i=1}^{M} \log_2 \left(1 + \mu_i \left(\Upsilon_T^{-1/2} Q \Upsilon_T^{-H/2} \Pi \Upsilon_T^{H/2} U_M \right) \right)$$

From the above theorem, it is clear that channel capacity of Physical MIMO system has finite upper limit in the high SNR regime. Further, the bound in expression (5) gives any channel variations which are characterized by both N_T and impairment level. For $N_T \le N_R$ the limits in express (5) are coincide and when $N_T > N_R$, upper bound enhances with number of transmit antennas. When channel capacity at high SNR obtaining Q in a subspace of N_T and $\min\left(N_R, N_T\right)$, both the bounds are close to each other. These are focused in the following theory.

Corollary 1*:* We assume that the channel variation is invariant right rotationally. With this case channel capacity is obtained by $Q = \frac{1}{N_T} I$ for any $\frac{S}{N}$ and α. The N_T dimensions of $H^H H$ are distributed isotropically, hence the concavity of E {log det (·)} makes an isotropic covariance matrix optimal.

Corollary 2: Suppose the parameter á is equal to one and H is full rank and deterministic.

Let $H^H H = U_M \Lambda_M U_M^H$ denote a compact Eigen decomposition, where $\Lambda_M = diag\left(\lambda_1, \lambda_2, ..., \lambda_M\right)$ consisting of eigenvalues with non zero and the semi-unitary matrix $U_M \in \mathbb{C}^{N_T \times M}$ includes the corresponding values of eigenvectors. Therefore the capacity is given as

$$C_{N_T,N_R}\left(\frac{S}{N}\right)=\sum_{i=1}^{M}\log_2\left(1+\frac{\frac{S}{N}\lambda_i d_i}{\frac{S}{N}\lambda_i\frac{\kappa^2}{N_T}+1}\right) \tag{6}$$

The capacity of channel is obtained by $Q=U_M diag\left(d_1,d_2,....,d_M\right)U_M^H$.

The capacity upper limit in (5) is asymptotically close for any number of transmit antenna N_T. When $N_T \geq N_R$, the channel capacity limit $M\log_2\left(1+\frac{N_T}{M\kappa^2}\right)$ is improved by increasing the number of transmit antennas N_T, because with a deterministic H selective transmissions is possible in the N_R dimensions of non-zero channel. Therefore lower limit in (5) is always asymptotically possible.

4. MULITPLEXING GAIN

In ideal transceiver, channel capacity is given as $M\log_2\left(\frac{S}{N}\right)+\varphi(1)$. This expression shows that channel capacity increases unlimited with high SNR and linearly proportional with the multiplexing gain $M=\min\left(N_T,N_R\right)$. On the other hand, as per theorem (1), upper bound of channel capacity is finite and it gives a very different multiplexing gain given as

$$m_{\infty}^{classis}=\lim_{SNR\to\infty}\frac{C_{N_T,N_R}\left(\frac{S}{N}\right)}{\log_2\left(\frac{S}{N}\right)}=0 \tag{7}$$

From above equation (7), multiplexing gain is non-zero only by neglecting the impairments. However capacity of a physical MIMO channel can be improved by using multiple antennas with spatial multiplexing. A Relevant practical situation is that $N_T\times N_R$ MIMO channel has relative capacity improvements over the SISO channel.

Theorem 1: Multiplexing gain of finite SNR, $m\left(\frac{S}{N}\right)$, is the ratio of capacities of MIMO system to single input single output (SISO) system at a particular SNR values. Therefore the multiplexing gain is

$$m\left(\frac{S}{N}\right)=\frac{C_{N_T,N_R}\left(\frac{S}{N}\right)}{C_{1,1}\left(\frac{S}{N}\right)} \tag{8}$$

From the above expression, multiplexing gain is defined as ratio of channel capacities of MIMO and single antenna system. The effect of this gain $m\left(\frac{S}{N}\right)$ is discussed in theorem 2.

Theorem 2: Let single antenna system channel is h, the finite-SNR multiplexing gain $m\left(\frac{S}{N}\right)$ for any α values satisfies

$$\frac{E\left\{\|H\|_F^2\right\}}{N_T E\left\{|h|^2\right\}} \leq \lim_{SNR\to\infty} m\left(\frac{S}{N}\right) \leq \frac{E\left\{\|H\|_2^2\right\}}{E\left\{|h|^2\right\}} \tag{9}$$

$$M \leq \lim_{SNR\to\infty} m\left(\frac{S}{N}\right) \leq M\frac{\log_2\left(1+\frac{N_T}{M\kappa^2}\right)}{\log_2\left(1+\frac{1}{\kappa^2}\right)} \tag{10}$$

The upper and lower limits of channel capacities can be obtained for deterministic and right-rotationally invariant channels distributions respectively.

Proof: Using Taylor approximation low-SNR is achieved: $Q=\frac{1}{N_T}I$ allows the lower limit, while the optimal allows the upper limit.

5. NUMERICAL AND SIMULATION RESULTS

Consider a MIMO system with equal number of transmit and receive antennas (i.e.,), and variable SNR. Figure 1 show average channel capacity of various deterministic channels [11]. The level of physical impairments is varied from 0.05 to 0.1 such that. At low and medium SNR values, both the ideal and physical transceivers act similarly, but behave differently in high SNR values shown in Figure 1. Although channel capacity increases

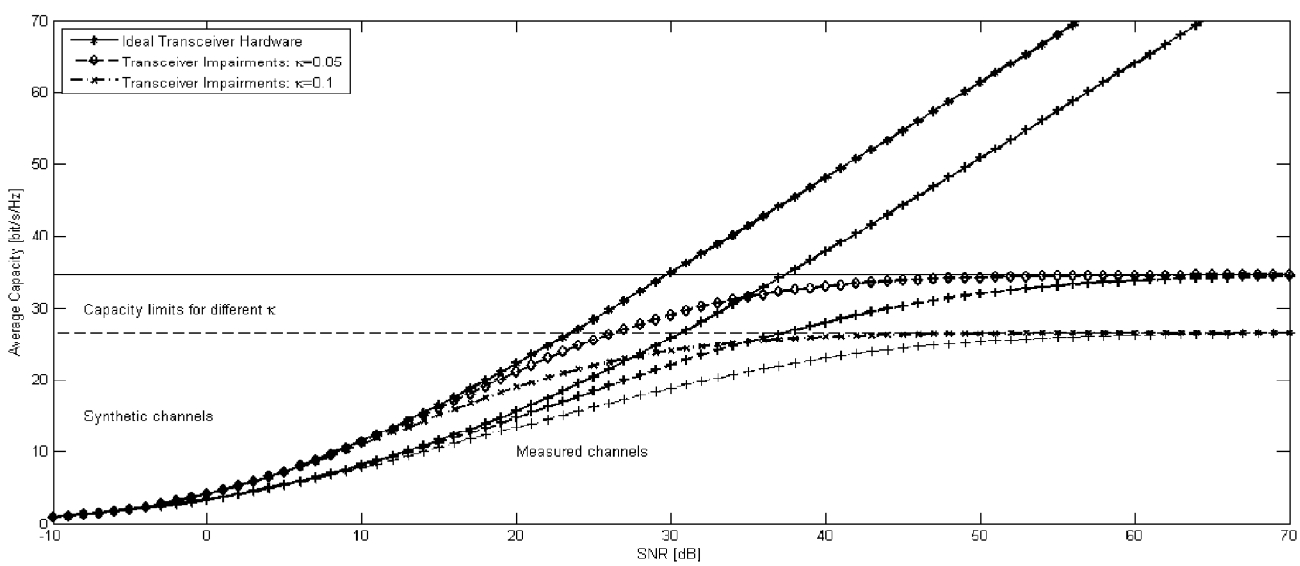

Figure 1: Average channel Capacity of MIMO system

unlimitedly for ideal cases, the capacity with transceiver impairments approaches to $C_{4,4}(\infty) = 4\log_2\left(1+\frac{1}{\kappa^2}\right)$.
The variation of channel capacity for correlated and uncorrelated channels are vanishes asymptotically. Therefore, capacity limit is decided only by the level of impairment κ.

Next we consider, $N_T \in \{4, 12\}$ with number of receiver antennas $N_R = 4$, level of impairment $\kappa = 0.05$ different α values and deterministic and uncorrelated Rayleigh fading channel distributions as shown in figure 2.

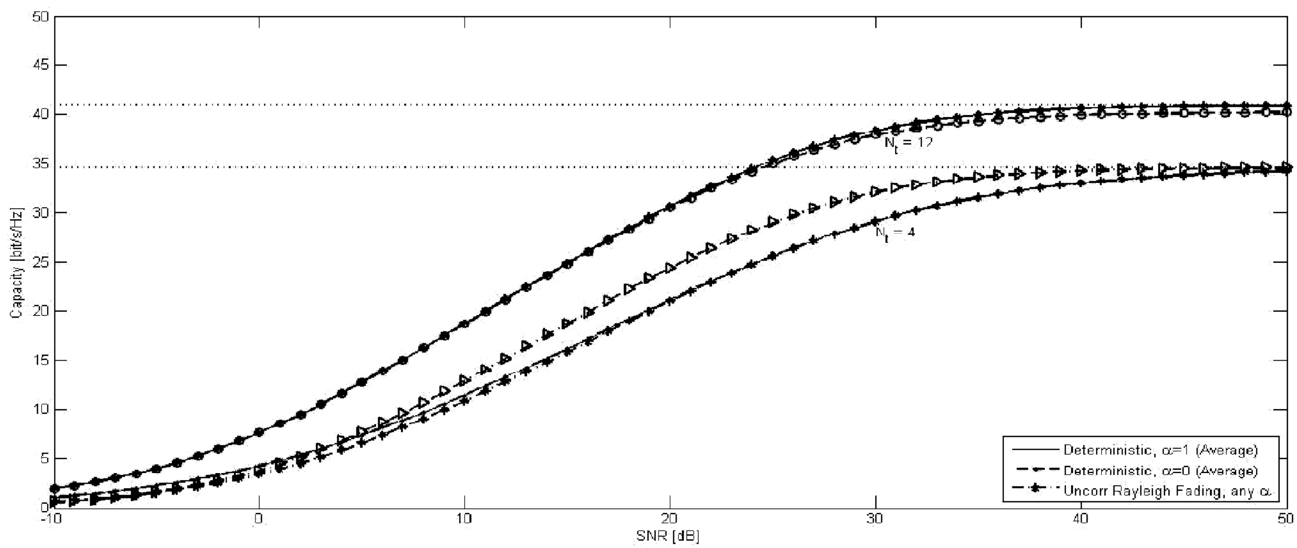

Figure 2: MIMO channel capacity with $N_R = 4$ and impairment, different N_T, channel distributions, and á value

We show $\alpha \in \{0, 1\}$ for deterministic channels and any α for Rayleigh fading channel distributions. Both the channels have same performance and have the same capacity limit for $N_T = 4$. As N_T increases channel capacity limit fall of rapidly for the random distribution, however this limit is unchanged. On the other hand, for the deterministic distribution capacity limits increases with N_T.

In Figure 2, there is an average SNR where channel capacity have almost same slope M as obtained asymptotically for ideal physical transceivers.

This phenomenon is experienced in cellular communication networks due to limited coherent time. However we show its presence in physical MIMO channel due to transceiver impairments.

Figure 3 shows the multiplexing gain of finite-SNR for rayleigh fading uncorrelated channels and Figure 4 shows the multiplexing gain of finite-SNR for deterministic channels with various transmit antennas and fixed receive antennas i.e., $N_R \in \{4, 8, 12\}$, $N_R = 4$, with $\kappa = 0.05$, $\alpha = 1$.

Although channel capacity is different for ideal and physical transceiver, multiplexing gain with finite SNR is unexpectedly similar.

6. CONCLUSION

Channel capacity of physical MIMO channel is saturated with high SNR and this finite limit is does not depends on channel variations. In this paper simulation results explained that transceiver impairment distortion is directly proportional to transmitted signal power. However the MIMO system channel capacity grows linearly with $M = \min(N_T, N_R)$. Further, simulation results are shows the physical system multiplexing gain of MIMO systems.

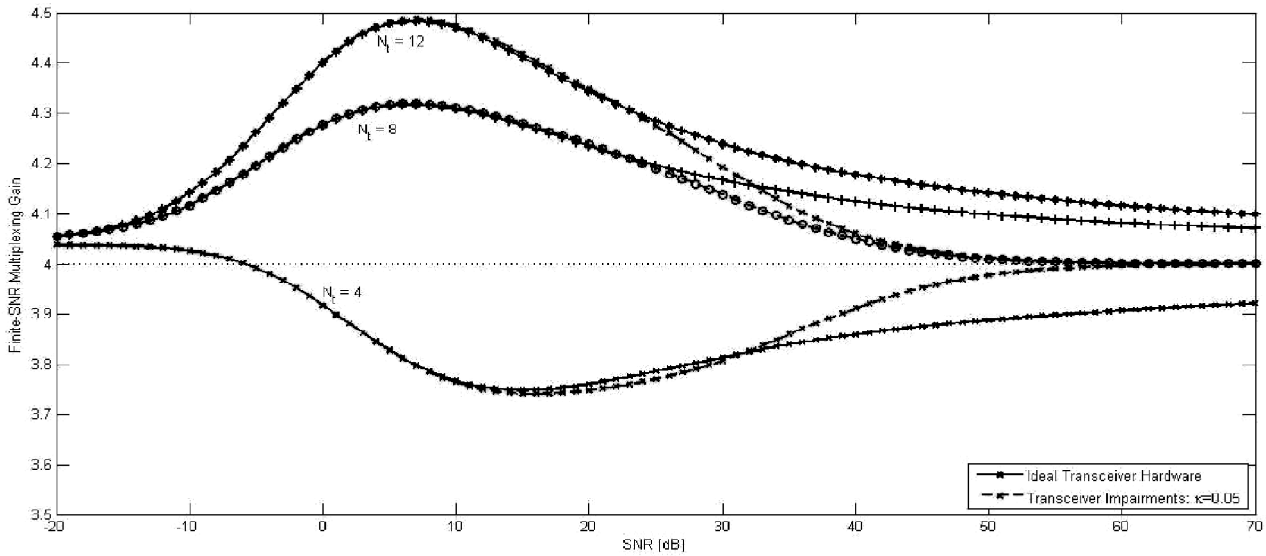

Figure 3: Multiplexing gain of finite SNR for Rayleigh fading channel with $N_R = 4$ and $N_T \geq 4$

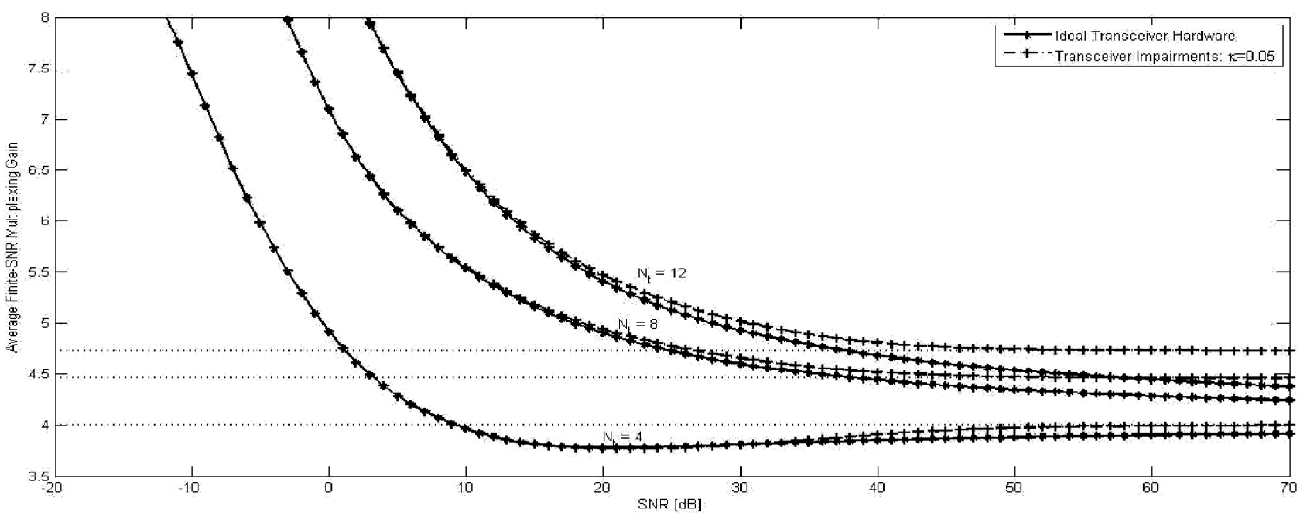

Figure 4: Deterministic channel finite SNR multiplexing gain

Finally multiplexing gain of finite SNR reduces when add additional constraint and impairments that limits the asymptotic capacity and channel acquisition performance.

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