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Tuning of PID Controllers for First order Stable / Unstable Time Delay Systems with a Zero

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Abstract: PID controllers are designed for First order stable/unstable time delay systems with a zero by using IMC method. In this method first order Pade's approximation is used while deriving PID controller parameters from IMC controller. Simple tuning rules for stable/ unstable First Order Plus Time Delay (FOPTD) system with a positive zero and unstable FOPTD system with a negative zero are given in terms of model parameters. The IMC filter time constant is a tuning parameter and is selected based on the maximum magnitude of the sensitivity function (Ms) value. Simulation results on various transfer function models and on the nonlinear model equations of continuous stirred tank reactor with nonideal mixing carrying out an enzymatic reaction (at two different operating points) are given to show the effectiveness of the proposed method. The proposed controller performs better than the controller designed by the recent literature methods in terms of ISE, IAE and ITAE.

Key words*:* Unstable system, FOPTD system; IMC method; PID controller, positive zero, Negative zero

1. INTRODUCTION

The open loop response of the systems with a positive zero (right half plane zero) is initially in the opposite direction to that of the final steady state. Such systems are called non-minimum phase systems or inverse response systems. Systems with a negative zero show a jump in the direction of the final steady state and they are called as minimum phase systems. The level of a drum boiler to variations in the heating medium flow rate, exit temperature of the tubular reactor for change in inlet reactant temperature, the tray composition of a distillation column for change in vapour flow rate[1], the temperature of the municipal waste incinerator for changes in the inlet load rate [2] are some of the examples for the inverse response systems. Examples for systems with a negative zero are Continuous Stirred Tank Reactor (CSTR) carrying out Van de Vusse reaction and jacketed CSTR carrying out first order exothermic reaction [3].

Methods of designing PI/PID controllers for stable/unstable first/second order inverse response systems are Ziegler Nichols Method [4], stability analysis method [5], Equating coefficient method [6], IMC method [7, 8, 9, 10, 11], synthesis method [6, 12], Optimization method using artificial neural net works [13], extension of Haalman method [14], set point overshoot method [15] and closed loop tuning method [16]. Scali and Rachid [7] have considered stable first order systems with a positive zero and without time delay. Most of the above mentioned methods are applicable to second order systems.

Shamsuzzoha and Skogestard [15] proposed a simple method based on closed loop servo response experiment. Simple correlations are given for PI parameters. The controller gain is a function of overshot and integral time is a function of time to reach the peak.

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Shamsuzzoha [16] have proposed tuning rules for PI/PID controller based on closed loop experiment in which PI/PID controller mode is switched to P controller mode. The controller gain of P controller is adjusted such that 30% overshoot is observed for unit step change in set point. Based on overshoot, time to reach overshoot and relative steady state output change, analytical tuning rules are proposed.

In the present work it is proposed to design PID controller for stable/unstable First Order Plus Time Delay (FOPTD) systems with a positive/negative zero using IMC method. In the IMC method proposed by Padma Sree and Chidambaram [6] for unstable FOPTD and SOPTD systems with a zero, first order Pade's approximation for time delay is used in the process transfer function itself and IMC controller is designed for the approximated process. In the present work, no such approximation is made. IMC controller is designed based on the process transfer function and while deriving PID parameters from the IMC controller first order Pade's approximation is used for time delay.

2. PROPOSED METHOD

2.1 Stable FOPTD system with a positive zero

The process transfer function for the stable FOPTD system with a positive zero is given by

$$G_p = \frac{K_p(1-ps)e^{-Ls}}{(\tau s+1)} \tag{1}$$

The IMC controller is given by controller is obtained from

$$Q = \frac{(\tau s+1)(\gamma s+1)}{K_p(\lambda s+1)^2} \tag{2}$$

The conventional PID by controller is obtained from IMC controller by using the following equation

$$G_c = \frac{Q}{(1-QG_p)} \tag{3}$$

Using the first order Pade's approximation for the time delay $\left((e^{-Ls} = \frac{1-0.5Ls}{1+0.5Ls}),\ G_c \right)$, $G_C$is given by

$$G_c = \frac{\frac{(\tau s+1)(\gamma s+1)}{K_p(\lambda s+1)^2}}{1-\frac{(\gamma s+1)(1-ps)(1-0.5Ls)}{(\lambda s+1)^2(1+0.5Ls)}} \tag{4}$$

Rearranging Eq (4), the following equation is obtained.

$$G_c = \frac{(\tau s+1)(\gamma s+1)(1+0.5Ls)}{K_p[(\lambda s+1)^2(1+0.5Ls)-(\gamma s+1)(1-ps)(1+0.5Ls)]} \tag{5}$$

The denominator of Eq (5) is written as

$$K_p bs(\tau s+1)(\alpha s+1) \tag{6}$$

Where $b = 2\lambda + p + L - \gamma$

Equating the corresponding coefficients of s^2 and s in Eq (6) and the denominator of Eq (5), α and γ are obtained as

$$\alpha = \frac{0.5L\lambda^2 - 0.5Lp\gamma}{(2\lambda + L + p - \gamma)\tau} \tag{7}$$

$$\gamma = \frac{[0.5L\lambda^2 - \tau(L\lambda + \lambda^2 - 0.5Lp - \tau(2\lambda + L + p))]}{\tau^2 + 0.5L\tau + p\tau + 0.5Lp} \tag{8}$$

Substituting Eq (6) for denominator in Eq (5) and rearranging, the following equation for G_c is obtained. $G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1}{(\alpha s + 1)}$ (9)

This is a PID controller with a first order filter, here

$$K_c = \frac{(0.5L + \gamma)}{K_p(2\lambda + L + p - \gamma)} \tag{10}$$

$$\tau_I = 0.5L + \gamma \tag{11}$$

$$\tau_D = \frac{0.5L\gamma}{0.5L + \gamma} \tag{12}$$

2.2 Unstable FOPTD system with a positive zero

The process transfer function for the unstable FOPTD system with a positive zero is given by

$$G_p = \frac{K_p(1 - ps)e^{-Ls}}{(\tau s - 1)} \tag{13}$$

The IMC controller is given by controller is obtained from

$$Q = \frac{(\tau s - 1)(\gamma s + 1)}{K_p(\lambda s + 1)^3} \tag{14}$$

The conventional PID by controller is obtained from IMC controller by using Eq (3).Substituting Eq (13), Eq (14) in Eq (3) with the first order Pade's approximation for the time delay $\left(e^{-Ls} = \frac{1 - 0.5Ls}{1 + 0.5Ls}\right)$, the following equation is obtained.

$$G_c = \frac{(\tau s - 1)(\delta s + 1)(1 + 0.5Ls)}{K_p[(\lambda s + 1)^3(1 + 0.5Ls) - (1 - ps)(1 - 0.5Ls)(\delta s + 1)]} \tag{15}$$

The denominator of Eq (15) is written as

$$K_p x s(\tau s - 1)(\alpha_1 s^2 + \alpha_2 s + 1) \tag{16}$$

Where $x = 3\lambda + p + L - \delta$

Equating the corresponding coefficients of s^3,s^2 and s in Eq (16) and the denominator of Eq (15), α_1, α_2 and δ are obtained as

$$\delta = \frac{-(\lambda^3 + 1.5L\lambda^2)\tau + 0.5L\lambda^3 - \tau^2(3\lambda^2 + 1.5L\lambda - 0.5Lp) - \tau^3(3\lambda + L + p)}{-\tau^3 - 0.5Lp\tau + (p + 0.5L)\tau^2} \tag{17}$$

$$\alpha_1 = \frac{-0.5L\lambda^3}{(3\lambda+L+p-\delta)\tau} \tag{18}$$

$$\alpha_2 = \tau + \frac{(3\lambda^2+1.5L\lambda+\gamma(p+0.5L)-0.5Lp)}{(3\lambda+L+p-\delta)} \tag{19}$$

Substituting Eq (16) for denominator in Eq (15) and rearranging, the following equation for G_c is obtained.

$$G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1}{(\alpha_1 s^2+\alpha_2 s+1)} \tag{20}$$

This is a PID controller with a second order filter, where

$$K_c = \frac{-(0.5L+\delta)}{K_p(3\lambda+L+p-\delta)} \tag{21}$$

$$\tau_I = 0.5L+\delta \tag{22}$$

$$\tau_D = \frac{0.5L\delta}{0.5L+\delta} \tag{23}$$

2.3 Unstable FOPTD system with a negative zero

The process transfer function for the unstable FOPTD system with a negative zero is given by

$$G_p = \frac{K_p(1+ps)e^{-Ls}}{(\tau s-1)} \tag{24}$$

The IMC controller is given by controller is obtained from

$$Q = \frac{(\tau s-1)(\gamma s+1)}{K_p(1+ps)(\lambda s+1)^2} \tag{25}$$

The conventional PID by controller is obtained from IMC controller by using Eq (3). Substituting Eq (24), Eq (25) in Eq (3) with the first order Pade's approximation for the time delay $\left(e^{-Ls} = \frac{1-0.5Ls}{1+0.5Ls}\right)$, the following equation is obtained.

$$G_c = \frac{(\tau s-1)(\sigma s+1)(1+0.5Ls)}{K_p(1+ps)[(\lambda s+1)^2(1+0.5Ls)-(1-0.5Ls)(\sigma s+1)]} \tag{26}$$

The denominator of Eq (26) is written as

$$K_p ys(\tau s-1)(\beta s+1)(1+ps) \tag{27}$$

Where $y = 2\lambda + L - \sigma$

Equating the corresponding coefficients of s^2 and s in Eq (27) and the denominator of Eq (26), σ and β are obtained as

$$\sigma = \frac{0.5L\lambda^2+\tau^2(2\lambda+L)+\tau(\lambda^2+\lambda L)}{\tau^2-0.5L\tau} \tag{28}$$

$$\beta = \frac{-0.5L\lambda^2}{(2\lambda+L-\sigma)\tau} \tag{29}$$

Substituting Eq (27) for denominator in Eq (26) and rearranging the following equation for G_c isobtained.

$$G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\frac{1}{(1+ps)(\beta s+1)} \tag{30}$$

This is a PID controller with a second order filter, where

$$K_c = \frac{-(0.5L+\sigma)}{K_p(2\lambda+L-\sigma)} \tag{31}$$

$$\tau_I = 0.5L + \sigma \tag{32}$$

$$\tau_{D} = \frac{0.5L\sigma}{0.5L+\sigma} \tag{33}$$

3. SIMULATION STUDIES

Simulation studies on stable/unstable FOPTD systems with a positive zero, unstable FOPTD system with a negative zero and for the nonlinear model equations of CSTR with nonideal mixing carrying out an enzymatic reaction are given in this section. The performance of the proposed controller is compared with the recently reported methods in terms of Integral Square Error (ISE), Integral Absolute error (IAE) and Integral Time weighted Absolute Error (ITAE). The smoothness of the controller is measured in terms of Total Variation (TV) [16]. The PID parameters along with filter parameters and performance indices for the case studies are reported in Table 1 and Table 2 for various methods.

3.1 Stable FOPTD system with a positive zero

The transfer function model $\frac{1-s}{s+1}e^{-0.2s}$ [16] is considered. The PID settings using the present method and the literature reported methods [15, 16] are given in Table 1. The performance of the proposed controller for unit step change in the set point at time t=0 and the unit step disturbance in load at time t=100 is shown in Figure 1. The performance of the proposed controller is superior compared to literature reported methods. The response of the literature reported methods is oscillatory with large overshoot. The performance of the controllers in terms of ISE, IAE, ITAE and TV are reported in Table 2 for both servo and regulatory problems. The Ms value for the proposed controller is less compared to the controllers designed by set point overshoot method [15] and closed loop tuning method [16] indicating that the proposed controller is robust to parametric uncertainties (refer to Table 1).

Table 1

PID settings

Case study	*Method*	K_c	τ_I	τ_D	α_1	α_2	*Ms*
1	Proposed	0.3021	0.9977	0.09		0.0409	2.8294
	SOM	0.314	0.418	0	0	0	3.91
	Shamsuzzoha	0.433	0.432	0	0	0	12.352
2	Proposed	1.6663	6.0644	0.1224	0.0074	0.038	4.03
	Lit-IMC	1.5673	8.01	0.123		0.052	10.49

3	Proposed	-4.8826	-89.403	0.05	-0.0097	-0.2597	11.8789
	Lit-IMC	-5.0324	-129.94	0.05	0	-0.3324	12.7476
4	Proposed	1.7561	140.0987	9.2862	42.942	14.99	2.286
	Lit-IMC	1.9053	160.76	9.378		11.133	2.192

Lit-IMC: Padma Sree and Chidambaram (2004, 2006), SOM: [15], Shamsuzzoha: [16]

Table 2
Performance comparison

Case study	*Method*	*Servo Problem*				*Regulatory problem*			
		ISE	*IAE*	*ITAE*	*TV*	*ISE*	*IAE*	*ITAE*	*TV*
1	Proposed	2.58	3.3075	8.7448	1.012	2.56	4.078	122.3278	3.614
	SOM	3.52	4.8551	19.682	3.575	3.55	5.272	553.29	4.455
	Shamsuzzoha	9.24	12.973	144.71	14.15	9.36	13.36	1.496×10^4	15.41
2	Proposed	7.55	5.4773	12.129	11.7	4.17	3.735	83.496	6.385
	Lit-IMC	9.75	6.8836	18.889	13.29	6.37	5.207	118.7835	16.86
3	Proposed	3.65	11.972	239.14	7.602	0.2	2.676	1120.1	2.025
	Lit-IMC	4.85	16.173	440.48	7.206	0.25	3.52	1496.6	1.885
4	Proposed	0.48	9.1019	869.4	0.421	0.41	8.936	10^4	0.221
	Lit-IMC	0.38	8.3748	939.61	0.404	0.36	9.45	1.0797×10^4	0.257

Lit-IMC: Padma Sree and Chidambaram (2004, 2006), SOM: [15], Shamsuzzoha: [16]

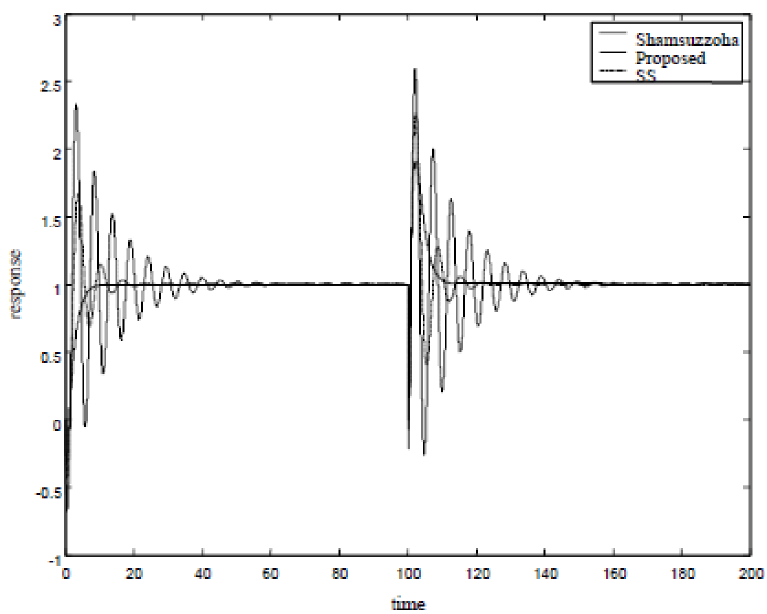

Figure 1: Performance comparison for the Case study 1 $\frac{1-s}{s+1}e^{-0.2s}$

3.2 Unstable FOPTD system with a positive zero

The transfer function model $\frac{1-0.25s}{s-1}e^{-0.25s}$ [6] is considered. The PID settings using the present method and the IMC method [6] are given in Table 1. The performance of the proposed controller for unit step change in the set point at time t=0 and the unit step disturbance in load at time t=20 is shown in Figure 2.

The performance of the proposed controller is better compared to the controller designed by IMC method. The response of the controller designed by IMC method [6] is sluggish. The performance of the controllers in terms of ISE, IAE, ITAE and TV are reported in Table 2 for both servo and regulatory problems. The Ms value for the proposed controller is less compared to the controller designed by IMC method indicating that the proposed controller is robust to parametric uncertainties (refer to Table 1).

3.3 Unstable FOPTD system with a dominant positive zero: Nonideal CSTR carrying out an enzymatic reaction

The mathematical model equation for a CSTR with nonideal mixing is given by [18] as:

$$\frac{dc}{dt} = \frac{nQ}{mV}\left(c_f - c\right) - \frac{k_1 c}{(1+k_2 c)^2} \tag{35}$$

$$nc + (1-n)c_f = c_e \tag{36}$$

Here the nonideal mixing is described by Cholette's model. Here *n* is the fraction of the reactant feed that enters the zone of perfect mixing and *m* is the fraction of the total volume of the reactor where the reaction occurs [i.e. (1-*m*) fraction of the volume is a dead zone]. c is the concentration of the reactant in the well mixed zone and c_e is the concentration of the reactant in the exit stream. The controlled variable is c_e and the manipulated variable is c_f. For the present simulation study n=m=0.75, k_1=10 s^{-1}, k_2 =10 l/mol; V =1 l is considered.

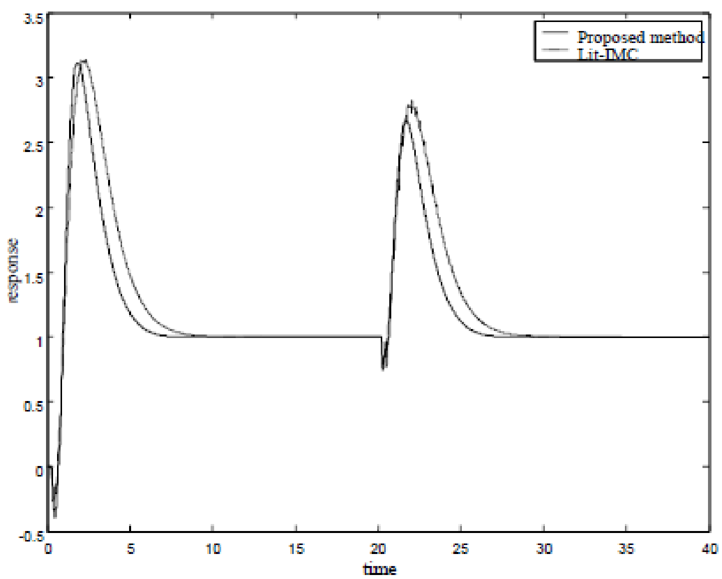

Figure 2: Performance comparison for the Case study 2 $\frac{1-0.25s}{s-1}e^{-0.25s}$

The particular reaction rate form $\frac{-k_1 c}{(1+k_2 c)^2}$ has been extensively studied [19]. For c_f = 6.484 mol/l, c_e = 1.8 mol/l and c=0.2387 mol/l, the linearization of the nonlinear equation around this nominal operating point gives the transfer function model as:

$$\frac{\Delta c_e(s)}{\Delta c_{f(s)}} = \frac{-0.1727(1-4.473s)}{(3.1s-1)} \tag{37}$$

A measurement time delay of 0.1s is considered along with the above transfer function model. The PID settings using the present method and the IMC method [6] are given in Table 1. For dominant positive zero unstable system (i.e. the zero is closer to the origin compared to the pole), both the integral time and filter time constant are negative. That is the controller is an unstable one. Cordrons et al. [20] discussed the occurrence of unstable non-minimum phase controller and associated identification problem. The performance of the proposed controller for a step change of 0.1 in the set point (1.8 to 1.9) at time t=0 and the step disturbance in feed concentration from 6.484 mol/l to 6.6 mol/l at time t=400 using the nonlinear model equations is shown in Figure 3. The performance of the proposed controller is better compared to IMC method. The response of the controller designed by the reported IMC method is sluggish. The performance of the controllers in terms of ISE, IAE, ITAE and TV are reported in Table 2 for both servo and regulatory problems. The Ms value for the proposed controller is less compared to the controller designed by IMC method indicating that the proposed controller is robust to parametric uncertainties.

3.4 Unstable FOPTD System with a Negative Zero: Nonideal CSTR Carrying out an Enzymatic Reaction

The mathematical model equation for a CSTR with nonideal mixing is given by [18] are considered (refer to section 4.3). For c_f = 3.288 mol/l, c_e = 1.8 mol/l and c=1.304 mol/l, *t*he linearization of the nonlinear equation around this nominal operating point gives the transfer function model as:

$$\frac{\Delta c_e(s)}{\Delta c_{f(s)}} = \frac{2.21(1+11.133s)}{(98.3s-1)} \tag{38}$$

A measurement time delay of 20 s is considered along with the above transfer function model. The PID settings using the present method and the IMC method [6] are given in Table 1.

The performance of the proposed controller for a step change of 0.1 in the set point (1.8 to 1.9) at time t=0 and the step disturbance in feed concentration from 3.288 mol/l to 3.4 mol/l at time t=1000 using the nonlinear model equations is shown in Figure 4. The performance of the proposed controller is almost similar to the controller designed by IMC method. The performance of the controllers in terms of ISE, IAE, ITAE and TV are reported in Table 2 for both servo and regulatory problems. The Ms value for the proposed controller is comparable with the controller designed by IMC method indicating that the proposed controller is robust to parametric uncertainties.

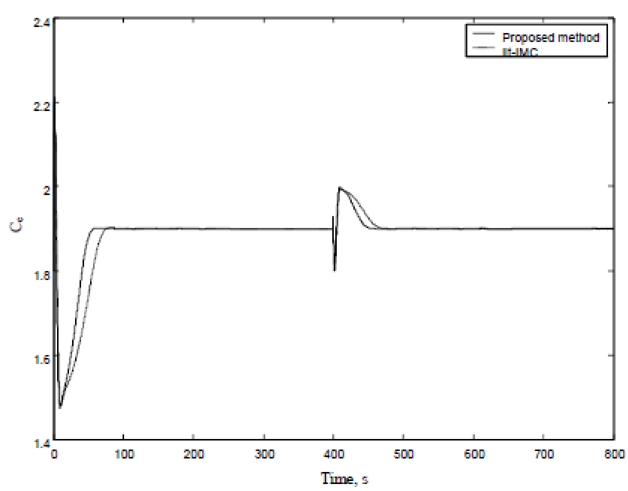

Figure 3: Performance comparison for the nonideal CSTR with feed concentration C_f=6.484mol/l

4. CONCLUSIONS

PID controllers are designed for stable/unstable FOPTD system with a positive zero and unstable FOPTD system with a negative zero by IMC method. Simulation results on various transfer function models and on the non-linear model equations of CSTR carrying out an enzymatic reaction (at two different feed concentrations) show that the proposed controllers perform better than the literature reported methods. For stable FOPTD system with a positive zero, the performance and robustness of the controller designed by the proposed method is superior to that of the recently reported methods. The performance of the proposed controller is better than the performance of the controller designed by the literature reported IMC method for unstable FOPTD system with a positive/negative zero. The proposed controllers are robust to parametric uncertainties as they give less value of Ms compared to recently reported methods.

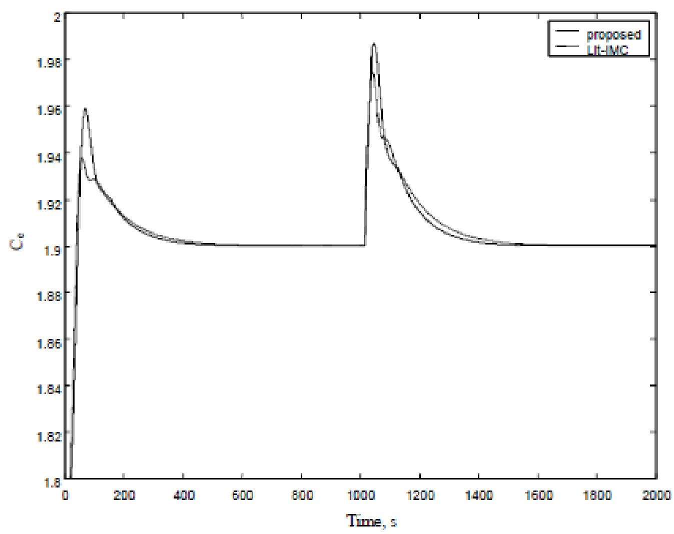

Figure 4: Performance comparison for the nonideal CSTR with feed concentration C_f=3.288mol/l (Nonlinear model simulation)

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