

# International Journal of Economic Research 

ISSN : 0972-9380
available at http: www. serialsjournal. com
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Volume 14 - Number 8 • 2017

# Optimal Pricing: Theory and Application to Publicly Supplied Bus Transport Services 

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#### Abstract

Optimal pricing facilitates attainment of specific goals. The optimum price to achieve profit maximization may differ from the one needed to maximize welfare or to ensure the highest revenue. Profit maximization is the traditional motivation of the private firms. In case of private monopoly supplier, it is almost certain that the profit maximizing price will result in charges above marginal and average cost. That's why, traditionally in most countries, public sector firms are involved in supplying essential goods and services and operate in an industry that experiences natural monopoly characteristics where it is cost effective to have a single firm to supply the whole market. Pricing in public sector firms is as important as it is in private sector since prices are an instrument for coordinating supply and demand of goods and services. This paper discusses the theory of public sector pricing and its application to publicly supplied bus transport services using a case study of state transport undertakings in India. The paper focuses on extended version of the Boiteux (1956) model following the analysis in line with Bös (1994). Although, implementation of the general extended Boiteux model presents a challenge with respect to information and computation, paper shows that some simplifying assumptions can be made to find out optimal prices for publicly supplied bus transport services.


Keywords: Optimal Pricing; Boiteux Model; Bus Transport; STUs; India
JEL Classification: D24, D69, H42, L32, L92, R48

## 1. INTRODUCTION

Prices are an instrument for coordinating supply and demand of goods and services. Hence prices need to be set not only by private entrepreneurs, but also by public utility managers. Traditionally, in most countries, public prices have been prevalent in the industries closely associated with supplying essential goods and services.

The theory of public sector pricing arises not from the welfare economics but rather from the theory of taxation and utility regulation. The early literature (Ramsey, 1927; Hotelling, 1938; Manne, 1952; Boiteux, 1956) addressed the problem in terms of optimal taxation to be raised by the government to cover the deficits of public utilities. It was believed that public utilities are subject to increasing returns to scale and hence marginal cost pricing will not be able to cover full cost.

Recent theoretical and empirical studies on public sector pricing are primarily based on Boiteux' model which was developed by M. Boiteux. He was manager of the nationalized French electricity industry when he published his seminal work in 1956 on management of public monopolies subject to budgetary constraints. Building on Ramsey's work, he introduced a general equilibrium approach to the problem and thus took into account the interdependence between public sector and the rest of the economy, a consideration that had been missing in the then existing literature on the theory of taxation and public utility pricing. The Boiteux model (extensively reviewed by Drèze (1964) and Rees (1968)) analyzes the public-sector pricing problem in the context of many-person, many-goods, and many factors in which all other sectors are perfectly competitive and the government has the ability to redistribute endowment to achieve distributional objectives. Rees (1968) and Hagen (1979) analyze the implications of the welfare maximization criterion when deregulated private sector is not working under perfectly competitive environment. Feldstein (1972) investigates optimal differential pricing rules for public-sector firms that sell intermediate goods as inputs to producers rather than as final goods to consumers. To deal with markets in disequilibrium, Drèze (1984) developed pricing mechanism for the public-sector subject to a budget constraint, in an economy where private sector experiences excess supply of labor and of commodities.

The Boiteux model is the basis for much of the public-sector pricing theory developed since 1956. This paper focuses on extended version of the Boiteux model following the analysis in line with Bös (1994). The extended version model incorporates subsequent extensions that deal with various restrictions and make it more generally applicable. We first present the general extended Boiteux model and derive the marginal conditions that are needed to compute the optimal prices. The solution equation has five terms representing the efficiency and equity effects of 'second best' pricing policies. To interpret their economic relationship, we present a brief summary of how extensions of basic Boiteux model have contributed to the development of public-sector economic theory in recent years.

Implementation of the general extended Boiteux model presents a challenge with respect to information and computation. Although, there may be practical applications where all the conditions of the model are relevant, but often many of its restrictions do not apply in specific cases so that the informational and computational requirements become less severe. For example, some simplifying assumptions can be made in the case of pricing of publicly supplied bus transport services.

## 2. THE MODEL

Bus transport services can be characterized as private goods, which are often supplied by public sector. A good is characterized as a private good if its consumption across consumers is competitive. The basic principles of pricing of publicly supplied private goods can be sketched in a theoretical full-information approach to price setting. We consider a welfare-maximizing public enterprise that does not pursue any other political or bureaucratic objectives, such as winning votes or increasing size. The management of
public enterprise maximizes its social welfare function subject to the market clearing conditions, public enterprise's technology, and a revenue-cost constraint.

### 2.1. The Objective Function

We assume that management of public enterprise is interested in maximizing a Paretian welfare function of the form:

$$
\begin{equation*}
W\left(v^{1}, v^{2}, \ldots . ., v^{H}\right) ; \quad \partial W / \partial v^{h} \geq 0 ; \quad h=1,2, \ldots ., H \tag{1}
\end{equation*}
$$

where $v^{b}$ is the indirect utility function of the $\mathrm{h}_{\mathrm{th}}$ consumer.
Here we consider an economy with $\mathrm{n}+1$ private goods. They are sold at prices $p, i \in I, I=\{0,1, \ldots$, $n\}$. Labor is used as numeraire and its price $p_{0}$ equals 1 . There are $H$ consumers. Their consumption plans are $x^{h}=\left(x_{0}^{h}, \ldots ., x_{n}^{h}\right)$, where positive quantities denote net demand, negative quantities net supply. The individual consumption plans result from maximizing the (strictly increasing and quasi-concave) utility function subject to the individual's budget constraint, given lump-sum income $r^{b}$.

In order to facilitate the economic interpretation of the marginal condition on which the analysis centers, the welfare function is defined over the 'budget space' rather than the 'commodity space'; that is, the welfare function is maximized with respect to prices rather than quantities. That's why, we deal in terms of individual indirect utility functions $\nu^{b}\left(p, r^{b}\right)$ and Marshallian demand functions $x_{i}^{b}\left(p, r^{b}\right)$. The consumers' optimum utility depends on vector of prices $p=\left(p_{0}, \ldots, p_{n}\right)$ and on exogenously given, non-labor, lumpsum income $r^{r}$.

### 2.2. Constraints

### 2.2.1. Market clearing conditions

The market clearing conditions link the public enterprise to the rest of the economy. The model assumes that there are $J$ private unregulated firms, $j=1, \ldots, J$, and one public enterprise. The production plans of the private firms are $y^{j}=\left(y_{0}^{j}, \ldots, y_{n}^{j}\right)$, for the public enterprise they are $z=\left(z_{0}, \ldots, z_{n}\right)$. The positive quantities are net outputs and negative quantities are net inputs. Any firm's output is either used for consumption or as an input for its own or other firms' production. Consumers supply labor to private and public sector firms; they buy goods from private firms and from the public sector. The model assumes that supply and demand is in equilibrium according to the market clearing conditions:

$$
\begin{align*}
& \sum_{h} x_{i}^{h}\left(p, r^{h}\right)-z_{i}-\sum_{j} y_{i}^{j}(p)=0 ; \quad i=0, \ldots, n  \tag{2}\\
& \quad \Rightarrow \sum_{i} \sum_{h} p_{i} x_{i}^{h}-\sum_{i} p_{i} z_{i}-\sum_{i} \sum_{j} p_{i} y_{i}^{j}=0 \\
& \quad \Rightarrow \sum_{h} r^{h}-\Pi-\sum_{j} \pi_{j}=0
\end{align*}
$$

where $\pi_{j}$ is the profit of firm $j$, and $\Pi$ is the profit of public enterprise.

We can see that the net profits of private firms and of the public enterprise are equal to the sum of lump-sum payments. Thus this model implies a total redistribution of private profits to the consumers and the public sector. If, however, private and public profits lead to an aggregate deficit, consumers are forced to finance it by lump-sum taxes.

### 2.2.2 The production technology

The public enterprise is assumed to produce efficiently according to a production function defined by equation (3). This implies that the public enterprise produces at point along its production-possibilities frontier rather than at a point below it. By not further restricting the production function, we allow for decreasing, constant, or increasing returns to scale.

$$
\begin{equation*}
g(z)=0 \tag{3}
\end{equation*}
$$

The technology of the private firms is not explicitly modelled. However, we assume that the public enterprise has a priori information on the net supply functions $y_{i}^{j}(p)$ of the private firms. In this model the private sector is exogenous and the public sector is expected to adjust to it, as is typical for a second-best approach.

### 2.2.3. Profits and deficits

We assume that the government decides that some goods, $z_{i}$, should be produced in the public sector and management of public enterprise has the right to set prices of particular goods (labeled $k \in K \subset I$ ). The public sector, moreover, is restricted by a revenue-cost constraint of the type:

$$
\begin{equation*}
\sum_{i=0}^{n} p_{i} z_{i}=\Pi \tag{4}
\end{equation*}
$$

where $\Pi=0$ implies break-even pricing; $\Pi<0$ determines a deficit; and $\Pi>0$ requires public sector profits.
The lowest $\Pi$ that can be found in practice will correspond to zero tariffs of the publicly supplied goods whereas the highest possible $\Pi$ corresponds to profit maximizing behavior of the public enterprise. The $\Pi$ may be an exogenously fixed value such as $\Pi=\Pi^{0}$. In its most general formulation, $\Pi=\Pi(p, z, \rho)$, the revenue-cost constraint depends on prices $p$, public sector netputs $z$, and exogenously given regulated variables $\rho$. The regulated variables, $\rho$, may be determined by a variety of ideological or economic motives regarding, for example, the desired size of the public sector, fears of losing votes because of higher tariffs, or opportunity costs of public sector deficits as compared with alternative use of resources in the private sector.

As is usual in Boiteux-type models, we will apply an exogenously fixed budget constraint, $\Pi=\Pi^{0}$, assumed to be given by the government. This could be a break-even constraint, for example, or a constraint based on previous year's budget and determined before prices are set. A higher value of $\Pi^{0}$ is typically associated with a higher price level and consequently lower demand for the good in question.

It should be noted that the controlled prices $\left\{p_{k}, k \in K \subset I\right\}$ are a subset of all prices whereas uncontrolled prices $\left\{p_{i}, i \notin K\right\}$ are exogenously given and public sector adjusts to the unregulated private
economy. The wage rate $p_{0}$, which serves as the numeraire, is unregulated. The production plans under the control of public enterprise $\left\{z_{i}, i \in I\right\}$, are a subset of all net production plans of the economy $\left\{z_{i}, y_{i}^{j}, i=0, \ldots, n ; j=0, \ldots, J\right\}$. The production plans whose prices are controlled by the public enterprise are a subset of $z_{i}$, denoted by $z_{k}, k \in K \subset I$. If, in addition to efficient allocation of resources, the public enterprise has distributional equity as an additional objective, then individual lump-sum incomes $\left\{r^{h}, h=1, . ., H\right\}$ are a third instrument under its control.

## 3. SOLVING THE MODEL

The welfare maximizing controlled prices and net production plan can be obtained from maximizing the following Lagrangian function:

$$
\begin{equation*}
\max L=W\left(v^{1}, . ., v^{H}\right)-\sum_{i=0}^{n} \alpha_{i}\left[\sum_{h} x_{i}^{h}\left(p, r^{h}\right)-z_{i}-\sum_{j} y_{i}^{j}(p)\right]-\beta g(z)-\gamma\left(\Pi^{0}-\sum_{i=0}^{n} p_{i} z_{i}\right) \tag{5}
\end{equation*}
$$

Differentiating with respect to prices $p_{k}$, quantities $z_{i}$, and the Lagrangian multipliers $\alpha_{i}, \beta$, and, $\gamma$ results in a system of five equations in five unknowns. The necessary conditions for optimal prices and quantities are as follows:

$$
\begin{gather*}
\sum_{h} \frac{\partial W}{\partial v^{h}} \frac{\partial v^{h}}{\partial p_{k}}-\sum_{i} \alpha_{i}\left(\sum_{h} \frac{\partial x_{i}^{h}}{\partial p_{k}}-\sum_{j} \frac{\partial y_{i}^{j}}{\partial p_{k}}\right)+\gamma z_{k}=0 ; \quad k \in K  \tag{6}\\
\alpha_{i}-\beta \frac{\partial g}{\partial z_{i}}+\not p_{i}=0 ; \quad i=0, \ldots, n \tag{7}
\end{gather*}
$$

Substituting the value of $\alpha_{i}$ from (6) into (7), we obtain:

$$
\begin{equation*}
\sum_{h} \frac{\partial W}{\partial v^{h}} \frac{\partial v^{h}}{\partial p_{k}}-\sum_{i}\left(\beta \frac{\partial g}{\partial z_{i}}-\gamma p_{i}\right)\left(\sum_{h} \frac{\partial x_{i}^{h}}{\partial p_{k}}-\sum_{j} \frac{\partial y_{i}^{j}}{\partial p_{k}}\right)+z_{k}=0 \tag{8}
\end{equation*}
$$

We divide these equations by $\beta_{0}:=\beta\left(\frac{\partial g}{\partial z_{0}}\right)>0,{ }^{1}$ and define $\lambda^{h}:=\left(\partial W / \partial v^{h}\right) / \beta_{0}$, $\gamma_{0}:=\gamma / \beta_{0}, c_{i}:=\left(\partial g / \partial z_{i}\right) /\left(\partial g / \partial z_{0}\right)$.

- $\quad \lambda^{h} \geq 0$ is the "normalized" marginal social welfare of individual utility. The welfare function is chosen so that $\lambda^{h}$ increases with decreasing individual utility.
- $\quad \gamma_{0}$ is a "normalized" measure of the welfare effects of the size of the public enterprise's deficit. If $\Pi^{0}$ exceeds the unconstrained welfare-optimal profit, then $0<\gamma_{0}<1$.
- $\quad c_{i}$ is a shadow price that measures the marginal labor cost of publicly produced goods $i$ (for $z_{i}>0$; otherwise it is a partial marginal rate of transformation). In this presentation of the model $c_{i}$ is used as marginal cost ${ }^{2}$.

Using these new symbols the marginal conditions (8) can be rewritten as follows:

$$
\begin{equation*}
\sum_{h} \lambda^{h} \frac{\partial v^{h}}{\partial p_{k}}-\sum_{i}\left(c_{i}-\gamma_{0} p_{i}\right)\left[\sum_{h} \frac{\partial x_{i}^{h}}{\partial p_{k}}-\sum_{j} \frac{\partial y_{i}^{j}}{\partial p_{k}}\right]+\gamma_{0} z_{k}=0 \tag{9}
\end{equation*}
$$

We assume that optimal lump-sum incomes are exogenously given since the term for controlling the distribution of lump-sum incomes is not included in the equation (9). For our purpose it is sufficient to assume that the public enterprise controls the prices and production plans but accepts individual lumpsum incomes as given and optimally distributed at the outset. Equation (9) expresses a combination of distinct equity and efficiency effects of the particular public sector pricing and production decisions and second-best conditions considered in a Boiteux type model. In order to make the economic relationships clearer and interpretable, we wish to proceed to price-cost differences ( $p_{i}-c_{i}$ ) instead of using ( $\gamma_{0} p_{i}-c_{i}$ ), in line with Bös (1994), we restate (9) as follows:

$$
\begin{equation*}
\sum_{h} \lambda^{h} \frac{\partial v^{h}}{\partial p_{k}}-\left(1-\gamma_{0}\right) \sum_{i} \sum_{h} p_{i} \frac{\partial x_{i}^{h}}{\partial p_{k}}-\sum_{i}\left[c_{i}-p_{i}\left[\sum_{h} \frac{\partial x_{i}^{h}}{\partial p_{k}}-\sum_{j} \frac{\partial y_{i}^{j}}{\partial p_{k}}\right]=-\gamma_{0} z_{k}-\left(1-\gamma_{0}\right) \sum_{i} \sum_{j} p_{i} \frac{\partial y_{i}^{j}}{\partial p_{k}} ; k \in K .\right. \tag{10}
\end{equation*}
$$

This equation consists of five terms. We now explain the economic relationships of these terms from left to right with respect to their equity and efficiency effects.

### 3.1. Distributional Objectives

The first two terms of equation (10) represent the distributional objectives of the model. The first term, $\sum_{h} \lambda^{h}\left(\partial v^{h} / \partial p_{k}\right)$, represents the social valuation of the effect of the change in price $p_{k}$. The absolute value of this term is high for necessities and low for non-necessities. Bös makes it clear by applying Roy's identity:

$$
\begin{equation*}
\sum_{h} \lambda^{h} \frac{\partial v^{h}}{\partial p_{k}}=-\sum_{h} \lambda^{h} x_{k}^{h} \cdot \frac{\partial v^{h}}{\partial r^{h}} ; k \in K \tag{11}
\end{equation*}
$$

He defines a "distributional characteristic", $F_{k}$, of any good $k \in K$ as a distributionally weighted sum of individual consumption shares:

$$
\begin{equation*}
F_{k}:=\sum_{h} \lambda^{h} \frac{\partial v^{h}}{\partial r^{h}} \frac{x_{k}^{h}}{x_{k}} ; k \in K . \tag{12}
\end{equation*}
$$

According to the usual economic assumptions, the social valuation of changes in the individual lumpsum incomes, $\lambda^{h}\left(\partial v^{h} / \partial r^{h}\right)$, is a decreasing function of individual incomes, it brings about the distributional
weighting in (12). The term $\lambda^{h}\left(\partial v^{h} / \partial r^{h}\right)$ is a combination of the social and individual valuations of incomes and utility. $\lambda^{h}$ is the social valuation of individual utility, and $\left(\partial v^{h} / \partial r^{h}\right)$ is the individual marginal utility of lump-sum incomes. Social planner chooses the welfare function in such a way that at optimum $\lambda^{h}\left(\partial v^{h} / \partial r^{h}\right)$ is positive but decreasing with income.

The second distributional term, $\left(1-\gamma_{0}\right) \sum_{i} \sum_{h} p_{i} \frac{\partial x_{i}^{h}}{\partial p_{k}}$, refers to the price level. Its absolute value is the larger, the smaller $\gamma_{0}$. A smaller $\gamma_{0}$ usually results from a lower $\Pi^{0}$. A lower $\Pi^{0}$ is associated with a lower level of prices. Lower prices, in turn, imply higher demand and thus reinforce the distributional objectives. For example, the level of prices of a welfare-maximizing public enterprise will be lower than of an unregulated monopolist, for whom $\gamma_{0}$ would be equal to 1 .

Applying the Slutsky equation to the second term to separate the substitution and income effects results in the following equation:

$$
\begin{equation*}
\left(1-\gamma_{0}\right) \sum_{i} \sum_{h} p_{i} \frac{\partial x_{i}^{h}}{\partial p_{k}}=\left(1-\gamma_{0}\right)\left[\sum_{i} \sum_{h} p_{i} \frac{\partial \hat{x}_{i}^{h}}{\partial p_{k}}-\sum_{i} \sum_{h} p_{i} x_{k}^{h} \frac{\partial x_{i}^{h}}{\partial r^{h}}\right]=-\left(1-\gamma_{0}\right) \sum_{h} x_{k}^{h} \tag{13}
\end{equation*}
$$

where $\hat{x}_{i}^{h}$ denotes compensated demand.
We should note that for any individual $h$, the compensated expenditures for all goods do not react to price changes $\left(\sum_{i} p_{i}\left(\partial \hat{x}_{i}^{h} / \partial p_{k}\right)=0\right)$. Moreover, differentiating the individual budget constraint $\left(\sum_{i} p_{i} x_{i}^{h}=r^{h}\right)$ yields $\sum_{i} p_{i}\left(\partial x_{i}^{h} / \partial r^{h}\right)=1$.

Hence the first two terms in equation (10) can be rewritten as:

$$
\begin{equation*}
-F_{k} x_{k}+\left(1-\gamma_{0}\right) x_{k} \tag{14}
\end{equation*}
$$

where the first term refers to the price structure and the second one to the price level.

### 3.2. Allocation in the Public Sector

The third and fourth terms in equation (10) reflect the allocation in the public sector, which is core of the Boiteux model. It should be noted that the theoretical interest has shifted from marginal cost prices to second-best prices, which deviate from marginal cost. The $\left(p_{i}-c_{i}\right)$ in the third term represents, in a second-best pricing situation, by how much prices should deviate from marginal cost. Since the public sector allocation not only depends on the supply side but also on the price sensitivity of demand for publicly supplied goods, consumer demand for public supply can be represented as follows:

$$
\begin{equation*}
\frac{\partial z_{i}^{D}}{\partial p_{k}}:=\sum_{h} \frac{\partial x_{i}^{h}}{\partial p_{k}}-\sum_{j} \frac{\partial y_{i}^{j}}{\partial p_{k}} \tag{15}
\end{equation*}
$$

where $z_{i}^{D}$ is a Marshallian demand function and not a Hicksian compensated one.
With focus on second-best pricing, the Boiteux model also stresses the importance of the revenuecost constraint. In this model, this constraint is represented by the fourth term, $-\gamma_{0} z_{k}$.

### 3.3. The Public and the Private Sector

The fifth term of equation (10) reflects the adjustment of the public sector pricing to the monopolistic structure of the unregulated private economy. If the private sector is perfectly competitive, this fifth term vanishes. The term does not vanish in case of monopolistic pricing. For monopolists, who change their levels of production from the efficient ones, marginal $\operatorname{cost} c_{i}^{j}\left(=-d y_{0}^{j} / d y_{i}^{j}\right.$ for $\left.y_{i}^{j}>0\right)$ can be interpreted as "producer price". We know that in the case of efficient production,

$$
\begin{equation*}
\sum_{i} \sum_{j} c_{i}^{j} \frac{\partial y_{i}^{j}}{\partial p_{k}}=0 \tag{16}
\end{equation*}
$$

Hence fifth term in equation (10) can be expressed as (Hagen, 1979):

$$
\begin{equation*}
\left(1-\gamma_{0}\right) \sum_{i} \sum_{j} p_{i} \frac{\partial y_{i}^{j}}{\partial p_{k}}=\left(1-\gamma_{0}\right) \sum_{i} \sum_{j}\left(p_{i}-c_{i}^{j}\right) \frac{\partial y_{i}^{j}}{\partial p_{k}} ; k \in K . \tag{17}
\end{equation*}
$$

Equation (17) clearly shows that monopolistic structure of the private economy influence the public sector pricing.

Taking into consideration of all above new definitions and transformations, equation (10) can be rewritten as:

$$
\begin{equation*}
F_{k} x_{k}-\left(1-\gamma_{0}\right) x_{k}+\sum_{i}\left(c_{i}-p_{i}\right) \frac{\partial z_{i}^{D}}{\partial p_{k}}=\gamma_{0} z_{k}-\left(1-\gamma_{0}\right) \sum_{i} \sum_{j}\left(c_{i}^{j}-p_{i}\right) \frac{\partial y_{i}^{j}}{\partial p_{k}} ; k \in K . \tag{18}
\end{equation*}
$$

Equation (18) contains the basic marginal conditions for optimal prices and quantities. This represents the general framework for a public enterprise's policy since it contains the interaction between public and private supply; includes distributional welfare judgments, and uses the usual non-compensated demand for public supply.

### 3.4. Compensating for Income Effects

Let us assume that distribution of lump-sum incomes can be controlled. Now, we maximize our Lagrangian function (5) not only with respect to prices and quantities, but also with respect to $r^{b}$.

The resulting marginal conditions

$$
\begin{equation*}
\frac{\partial W}{\partial v^{h}} \frac{\partial v^{h}}{\partial r^{h}}-\sum_{i} \alpha_{i} \frac{\partial x_{i}^{h}}{\partial r^{h}}=0 ; h=1, \ldots, H . \tag{19}
\end{equation*}
$$

can be transformed by inserting Roy's identity. Then we obtain,

$$
\begin{equation*}
\frac{\partial W}{\partial v^{h}} \frac{\partial v^{h}}{\partial p_{k}}=-\sum_{i} \alpha_{i} x_{k}^{h} \frac{\partial x_{i}^{h}}{\partial r^{h}} ; h=1, \ldots, H, k \in K . \tag{20}
\end{equation*}
$$

The incomes are redistributed in such a way that for each consumer the weighted sum of all income effects that result from changing price $p_{k}$ is just equalized with the policy maker's valuation of individual's utility change because of the change in price $p_{k}$. Hence at the optimum, the distributional valuations of the policy maker and all income effects cancel out. Distributional considerations are no longer needed so that the pricing structure now only concerns allocation. At the same time all income effects are eliminated, leaving only the substitution effect and thus compensated demand.

Using transformation analogous to those in the "non-compensated" case, we obtain the basic marginal conditions for the case of compensated demand:

$$
\begin{equation*}
\sum_{i}\left(c_{i}-p_{i}\right) \frac{\partial \hat{z}_{i}}{\partial p_{k}}=\gamma_{0} z_{k}-\left(1-\gamma_{0}\right) \sum_{i} \sum_{j}\left(c_{i}^{j}-p_{i}\right) \frac{\partial y_{i}^{j}}{\partial p_{k}} ; k \in K . \tag{21}
\end{equation*}
$$

This equation corresponds to the equation for Marshallian demand but omits the first two (distributional) terms in equation (18) since we are now dealing with compensated demand $\hat{z}_{i}$.

## 4. THE PRICING RULES

### 4.1. Marginal-Cost Pricing in the Public Sector

Marginal-cost pricing in a first-best environment requires the following assumptions (Jha, 2002): (i) the private sector is perfectly competitive; (ii) only prices of publicly produced goods are controlled and all other prices in the public sector are equal to marginal cost $c_{i}$; (iii) the distribution of lump-sum incomes is optimally chosen, hence we deal with compensated demand; and (iv) there is no revenue-cost constraint on the public sector.

In this case, equation (21) reduces to

$$
\begin{equation*}
\sum_{i \in K}\left(c_{i}-p_{i}\right) \frac{\partial \hat{z}_{i}}{\partial p_{k}}=0 ; k \in K . \tag{22}
\end{equation*}
$$

Assuming that $\partial \hat{z}_{i} / \partial p_{k}$ is a regular matrix, we obtain the well-known marginal-cost pricing rule $p_{i}=c_{i}(z), i \in K$. This rule is normatively valid for any kind of public enterprise: for competitive public enterprise as well as for monopolistic ones. In the case of decreasing-cost industries, the marginalcost pricing rule leads to welfare-optimal deficits which normatively have to be financed by the lump-sum taxes. These deficits are not indicative of mismanagement. ${ }^{3}$

Under the usual assumptions of the Boiteux-type model, the marginal-cost pricing rule results in a first-best allocation of resources. It arrives at a Pareto-optimal allocation of goods among consumers and
leads to the first-best utilization of the capacity of public enterprise. If both public and private enterprises follow marginal-cost pricing, the allocation between publicly and privately produced goods is also a firstbest allocation. In the decreasing-cost case, marginal-cost pricing leads to an extension of the public sector since prices will be lower than the average-cost and demand will therefore be relatively greater. It is difficult to draw a general conclusion regarding effects of marginal-cost pricing on income redistribution. Although, marginal-cost pricing does not have income redistribution as its main objective, it may have some distributional consequences. For example, if in the decreasing-cost case, the comparatively lower priced goods are mainly consumed by lower-income groups, the distributional effect may be positive. On the other hand, the positive effect may be offset by the fact that the deficits have to be financed by (possibly regressive) lump-sum taxes.

### 4.2. Ramsey Pricing

When we restrict the public enterprise to meet a revenue-cost constraint, we are required to find the secondbest set of prices. In the context of optimal taxation, Ramsey (1927) derived a formal mathematical solution to the optimal pricing in the industries in which marginal-cost prices do not cover total costs. Ramsey pricing allows the firm to use monopoly power to meet its revenue requirement. Optimal Ramsey pricing requires the following assumptions (Jha, 2002): ${ }^{4}$ (i) the private sector is perfectly competitive; (ii) only prices of publicly produced goods are controlled and all other prices in the public sector are equal to respective marginal cost $c_{i}$; (iii) the distribution of lump-sum incomes is optimally chosen, hence we deal with compensated demand; and (iv) the public enterprise is restricted by an exogenously fixed deficit or profit $\Pi^{0}$.

In this case, equation (21) reduces to

$$
\begin{equation*}
\sum_{i \in K}\left(c_{i}-p_{i}\right) \frac{\partial \hat{z}_{i}}{\partial p_{k}}=\gamma_{0} z_{k} ; k \in K . \tag{23}
\end{equation*}
$$

where $0<\gamma_{0}<1$ when, as in the most relevant cases, $\Pi^{0}$ exceeds the unconstrained welfare-optimal profit. If on the other hand, $\Pi^{0}$ falls below the unconstrained welfare optimizing profit, $\gamma_{0}<0$.

Ramsey pricing is characterized by a trade-off between the level of prices and the structure of prices. The level of prices is mainly influenced by $\Pi^{0}$. A low $\Pi^{0}$ implies a low price and, if demand reacts normally, a larger public-sector. Low $\Pi^{0}$ may imply lower prices of publicly provided goods in order to help the low-income group. The structure of prices is determined by the price elasticities of demand. For example, prices of goods that are price-inelastic can be increased by a higher percentage to meet $\Pi^{0}$ than prices of goods that are price-elastic. The trade-off exists because a low $\Pi^{0}$ favors low-income group, but a high $\gamma_{0}$ which meets the revenue-cost constraint by increasing prices of price-inelastic goods, will have the opposite effect on income distribution if these goods are bought primarily by low-income groups. If $\gamma_{0}=1$, Ramsey pricing converges to monopoly pricing for compensated demand. An organization that chooses Ramsey pricing behaves as if it was an unconstrained monopolist inflating all compensated price by a factor of $1 / \gamma_{0}$. The level of prices when $0<\gamma_{0}<1$, would be lower than the case of a profitmaximizing monopolist.

The Ramsey pricing condition can be expressed in terms of elasticities. Since substitution effects are symmetric i.e., $\frac{\partial \hat{z}_{i}}{\partial p_{k}}=\frac{\partial \hat{z}_{k}}{\partial p_{i}}$, equation (23) can be rewritten as:.5

$$
\begin{equation*}
\sum_{i \in K} \frac{\left(p_{i}-c_{i}\right)}{p_{i}} \frac{\partial \hat{z}_{k}}{\partial p_{i}} \frac{p_{i}}{\hat{z}_{k}}=-\gamma_{0} ; k \in K . \tag{24}
\end{equation*}
$$

where $\left(p_{i}-c_{i}\right) / p_{i}$ is the Lerner index $L_{i}$, and $\left(\partial \hat{z}_{k} / \partial p_{i}\right)\left(p_{i} / \hat{z}_{k}\right)$ is the compensated price elasticity of demand $\eta_{k i}$.

If we totally neglect all cross-price elasticities of demand ( $\left.\partial \hat{z}_{i} / \partial p_{k}=0 ; i, k \in K, i \neq k\right)$, the Ramsey price structure reduces to the well-known "inverse elasticity rule",

$$
\begin{equation*}
\frac{p_{k}-c_{k}}{p_{k}}=-\frac{\gamma_{0}}{\eta_{k k}} ; k \in K \tag{25}
\end{equation*}
$$

where $\eta_{k k}$ is the own compensated price elasticity of demand, $\left(\partial \hat{z}_{k} / \partial p_{k}\right)\left(p_{k} / \hat{z}_{k}\right)$.
Equation (25) clearly reveals that, for price above marginal cost $\gamma_{0}>0$, and for price below marginal $\operatorname{cost} \gamma_{0}<0 .{ }^{6}$ In other words, for positive Lerner index $\gamma_{0}>0$ and for negative Lerner index $\gamma_{0}<0$. Inverse elasticity rule asserts that the Lerner index, that is, the optimal percentage deviation of the price of any goods from its marginal cost should be inversely proportional to its own price elasticity of demand. It also implies that the optimal percentage deviation of price from marginal cost will be larger, the smaller the absolute value of the goods' price elasticity. If we assume that goods mainly bought by lower-income consumers are comparatively price inelastic, then lower-income consumers are burdened in the case of positive price-cost margins, favored in the case of negative ones. When $\Pi^{0}=0$, in a decreasing-cost case, at least one Ramsey price will exceed marginal cost; if cross-elasticities of demand are zero, all Ramsey prices will be above their respective marginal-cost prices. The effect on allocation in this case will be that the size of public sector will be smaller than under marginal-cost pricing without a revenue-cost constraint.

## 5. APPLICATION TO PUBLICLY SUPPLIED BUS TRANSPORT SERVICES USING A CASE STUDY OF STATE TRANSPORT UNDERTAKINGS IN INDIA

In most of the countries, bus transport services are directly or indirectly controlled by the government. In India, bus transport industry is dominated by the publicly owned State Transport Undertakings (STUs) since private sector is highly fragmented (Singh and Raghav, 2014). The STUs have a special responsibility to provide road-based passenger mobility in India, as they are the biggest undertakings in the hands of the respective state governments. Although, government by statutory provision calls STUs as autonomous corporations and lays before them the objective of operating the transport services on business principles under the section 22 of RTC Act 1950, but the STUs do not have freedom to formulate the pricing policy of their services. Currently, bus transport pricing in India is regulated under section 67(1) of Motor Vehicle Act 1988 under which the State Government can issue directives to the State Transport Authority regarding fixing of fares. The procedure of fare hike is usually very cumbersome and government takes considerable
time to approve it. In fact, it is very common that the decisions of respective state governments are more often than not influenced by the factors other than the economic rationality.

In this section, we try to examine whether STUs in India charge optimal prices for their services; if not, how much dead weight loss they create? This is evaluated by estimating the monetary value of social welfare and profitability ${ }^{7}$, and $\gamma_{0}$ at different level of prices. Estimation of social welfare, profitability, and $\gamma_{0}$ requires information about marginal and average cost curves as well as demand curve. Using annual data for a sample of nine STUs that operated from 1983-84 to 1996-97, Singh (2002) estimated a translog cost function jointly with factor share equations subject to required coefficient restrictions by using the method of 'Zellner's iterative' technique. We used this cost function to estimate the marginal and average cost curves for the two largest STUs, Maharashtra State Road Transport Corporation (MSRTC) and Andhra Pradesh State Road Transport Corporation (APSRTC), to illustrate the application of Ramsey pricing in publicly supplied bus transport services.

Table 1 shows the STUs included in the sample by Singh (2002) to estimate the translog cost function, together with some indicators concerning their size. The sample STUs are publicly owned and having similar organizational structure. Table 2 presents average fare (in paise per pass.-km at constant 1996-97 prices) charged by the sample STUs over the sample period. This table shows that there is a wide variation in fare structure across STUs whereas such variation is relatively low within a STU. Figures presented in the table reveal that average fare, in terms of real monetary unit, of most STUs has declined from 1983-84 to 1996-97. During 1996-97 in comparison to 1983-84, only three STUs, UPSRTC, KSRTC, and STPJB, increased their average fare rates. The proportionate change in average fare is found to be relatively higher in case of MSRTC and STPJB.

Table 1
Some indicators concerning the size of the STUs (mean of the sample period)

| STUs | Pass.-kms <br> $\left(\times 10^{6}\right)$ | Bus-kms <br> $\left(\times 10^{6}\right)$ | Route-kms <br> $\left(\times 10^{3}\right)$ |
| :--- | :---: | :---: | :---: |
| Maharashtra State Road Transport Corporation (MSRTC) | 49435 | 1243 | 1040 |
| Andhra Pradesh State Road Transport Corporation (APSRTC) | 52725 | 1310 | 735 |
| Karnataka State Road Transport Corporation (KnSRTC) | 33047 | 809 | 670 |
| Gujarat State Road Transport Corporation (GSRTC) | 31444 | 788 | 879 |
| Uttar Pradesh State Road Transport Corporation (UPSRTC) | 20641 | 567 | 456 |
| Kerala State Road Transport Corporation (KSRTC) | 12999 | 284 | 181 |
| Rajasthan State Road Transport Corporation (RSRTC) | 12668 | 323 | 355 |
| Madhya Pradesh State Road Transport Corporation (MPSRTC) | 7945 | 210 | 258 |
| State Transport Punjab (STPJB) | 7853 | 199 | 166 |

### 5.1. Optimal Pricing in Two Largest STUs, MSRTC and APSRTC

### 5.1.1. Optimal Pricing in MSRTC

Simulation based on equation (25) requires estimation of own compensated price elasticity of demand. The compensated or Hicksian demand is derived by minimizing the consumer's expenditure for achieving

Table 2
Average fare charged by the STUs (in paise/pass.-km); 1983-84 to 1996-97.

|  | MSRTC | APSRTC | KnSRTC | GSRTC | UPSRTC | KSRTC | RSRTC | MPSRTC | STPJB |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1983-84$ | 31.64 | 25.10 | 22.87 | 22.90 | 23.62 | 22.44 | 27.77 | 26.50 | 15.56 |
| $1984-85$ | 29.62 | 23.72 | 22.27 | 21.55 | 25.04 | 21.22 | 26.12 | 24.89 | 14.62 |
| $1985-86$ | 28.42 | 25.51 | 24.60 | 20.86 | 22.33 | 24.62 | 28.50 | 29.15 | 14.00 |
| $1986-87$ | 27.28 | 23.72 | 26.95 | 19.61 | 19.08 | 23.35 | 27.93 | 29.63 | 14.95 |
| $1987-88$ | 26.63 | 23.34 | 25.06 | 23.66 | 24.30 | 21.11 | 25.81 | 26.81 | 16.43 |
| $1988-89$ | 24.76 | 22.61 | 23.11 | 22.18 | 23.39 | 22.51 | 25.94 | 24.44 | 15.29 |
| $1989-90$ | 25.32 | 21.87 | 21.99 | 20.72 | 21.51 | 21.77 | 25.20 | 23.73 | 14.43 |
| $1990-91$ | 30.62 | 23.97 | 24.74 | 21.28 | 22.55 | 21.69 | 24.48 | 22.96 | 16.62 |
| $1991-92$ | 29.65 | 22.16 | 22.16 | 22.08 | 20.91 | 21.23 | 26.11 | 25.41 | 16.71 |
| $1992-93$ | 28.51 | 21.42 | 20.83 | 19.51 | 23.66 | 21.46 | 23.94 | 24.76 | 17.20 |
| $1993-94$ | 30.92 | 23.54 | 22.05 | 28.24 | 22.89 | 21.12 | 26.18 | 22.89 | 25.47 |
| $1994-95$ | 27.89 | 21.55 | 19.82 | 24.18 | 23.07 | 22.16 | 24.68 | 22.62 | 25.94 |
| $1995-96$ | 25.95 | 23.13 | 18.38 | 18.97 | 21.88 | 23.15 | 23.51 | 25.60 | 23.27 |
| $1996-97$ | 24.58 | 24.02 | 22.16 | 17.71 | 23.92 | 24.75 | 23.46 | 23.75 | 25.00 |

a given level of utility whereas the ordinary or Marshallian demand is derived by maximizing a representative consumer's utility function subject to a budget constraint. Since the utility level is held constant in the case of compensated demand, the compensated price elasticity measures only the substitution effect of a price change. In contrast, the ordinary price elasticity measures both the substitution and income effects of a price change. In practice, however, the compensated demand function is not estimable because it is a function of utility, which is not directly observable. Hence, virtually all passenger travel demand studies estimate an ordinary demand function and report the associated elasticities. One should note that if we assume expenditure share of public bus transport services is relatively small and that public bus transport demand is not highly income elastic then both the compensated and uncompensated price elasticities of demand will almost be identical.

We estimated a demand function for passenger transport services provided by MSRTC using required annual data from 1983-84 to 1996-97. The purpose of this empirical estimation is to determine whether there is indeed a negative relationship between the (revenue) passenger-kilometer and fare rate (paise/ pass.-km), and if so, whether it is quantitatively significant. We assume that demand for passenger transport at time $t, D_{t}$, consists of a systematic component - which depends on the price charged by MSRTC, $\mathrm{P}_{\mathrm{t}}$ as well as on other, non-price factors, X , affecting $\mathrm{D}_{\mathrm{t}}$ and a stochastic component, $e_{t}$, representing a surrogate for all variables that cannot be separately included in the model but which collectively affect $\mathrm{D}_{\mathrm{t}}$.

We assume the systematic relationship between the quantity demanded and the variables to be linear and express it as a standard linear model as follows:

$$
\begin{equation*}
D_{t}=\beta_{1}+\beta_{2} P_{t}+\sum \beta_{j} X_{j t}+\varepsilon_{t} \tag{26}
\end{equation*}
$$

where the variables are defined as above, and $\mathrm{X}_{\mathrm{jt}}$ represents factors other than average fare rate that affect the quantity demanded.

In the context of the empirical demand estimation, the model may be expressed as follows:

$$
\begin{equation*}
\ln (\text { MPKm })=\beta_{1}+\beta_{2} \ln (\text { AvgFare })+\beta_{3} \ln (P C I)+\beta_{4} \ln (\text { SDP })+\beta_{5}(\text { Time })+\varepsilon \tag{27}
\end{equation*}
$$

where the quantity demanded is measured as pass.-km in million, price is expressed as average fare rate (paise/pass.-km) at constant 1996-97 prices, PCI is per capita income measured in Rs, at constant 1980-81 prices, and SDP is state domestic product measured in Rs. crore at constant 1980-81 prices.

As discussed above, $\mathrm{b}_{2}$ is expected to have a negative sign, meaning that if MSRTC increases its price, the demand for its services will decrease. We also hypothesize that MSRTC sales are positively related with economic activity in the region, so that $b_{4}$ is expected to have a positive sign. $b_{3}$ is expected to have $a$ negative sign since as the per capita income in the state increases, use of personalized vehicle will increase and demand for bus services will go down. The model also includes Time variable to verify whether MSRTC sales show a trend - an average increase or decrease over time - not explained by the other independent variables. Moreover, an attractive feature of the $\log$ model is that the estimated slope coefficient $b_{2}$ will directly measure the price elasticity of demand, which will be used to estimate the optimal fare rate for MSRTC.

Table 3 presents results of estimated demand function. ${ }^{8}$ The estimated price elasticity, $\beta_{2} \cong \eta_{\mathrm{kk}}$, is expected to always be negative because of the inverse relationship between quantity and price implied by the 'law of demand' and represented by the downward sloping demand curve (see also Figure 1). However, traditionally the negative sign is omitted and the price elasticity is expressed as an absolute value. The price elasticities of demand are to be interpreted as follows:
If $\quad \beta_{2}=0$, the demand is perfectly inelastic;
$\beta_{2}=1$, the demand has unitary elasticity;
$\beta_{2}=\infty$, the demand is perfectly elastic;
$0<\beta_{2}<1$, the demand is inelastic;
$1<\beta_{2}<\infty$, the demand is elastic.
This has implications for policy decisions related to raising the revenue. If the demand is inelastic, an increase in price leads to an increase in total revenue, and a decrease in price leads to a fall in total revenue; if the demand is elastic, an increase in price leads to a decrease in total revenue, and a decrease in price leads to an increase in total revenue; and if the demand has unitary elasticity, total revenue is not affected by changes in price. The result of Table 3 shows that the prices of bus transport services of MSRTC do influence the quantity demanded. The estimated price elasticity of demand is 0.77 which indicates that an increase in average fare rate will increase the traffic revenue of MSRTC while reducing sales by a relatively smaller magnitude.

An application of equation (25) requires estimation of marginal cost at different levels of output. We estimated marginal cost of MSRTC at different level of pass.-km based on results presented in Singh (2002) and depicted the same in Figure 2. Table 4 presents an indicator of social welfare and profitability
of MSRTC during 1996-97 and 1983-84 at different level of prices. Result shows that during 1996-97, MSRTC was able to recover its operating cost if its fare rate would have been around 22 paise per pass. -km . One should note that operating cost does not include depreciation on buses, tax, and interest payment which account for around 31 percent of the total operating cost during 1996-97 (see Table 5). So, to recover the total cost, MSRTC's profit over operating cost should be at least 31 percent of the operating cost. During the same year when price is equal to 26 paise per pass.-km (which is more than the actual charged price of 24.58 paise per pass.km), MSRTC was able to recover its total cost. Economically efficient pricing rule tells us that prices should be equal to marginal cost. In this case, marginal cost is 27 paise per pass.-km which is more than average operating cost as well as average total cost. Hence, marginal cost pricing of MSRTC's bus transport services will not only be economically efficient but also profitable to the organization. This is because MSRTC operates on decreasing returns to scale (Singh, 2002).

Last column of Table 4 presents an indicator of social welfare at different level of prices. The proportionality factor $g_{0}$, calculated from the Lagrangian parameter of the budget constraint, is a measure of the welfare effect of the size of the deviation from marginal cost. It depends on all the variables and functions of the model. $\mathrm{g}_{0}=0$ if marginal cost pricing is applied (since marginal cost pricing is welfare maximizing). Any deviation from zero is taken as socially sub-optimal pricing. In the case of increasingcost production, as is the case with MSRTC, marginal cost pricing will be able to recover the operating cost.

Figure 3 measures the deadweight loss ${ }^{9}$ (DWL) due to sub optimal pricing in MSRTC. We have chosen the year 1996-97 for illustrative purpose. During this year, MSRTC charged, on an average, 24.58 paise per pass.-km for its transport services whereas optimal price was 27 paise per pass.-km when marginal cost equals price. MSRTC produced around 63 billion pass.-km during 1996-97 whereas under marginal cost pricing it would have produced only around 59 billion pass.-km. How to measure the total loss in efficiency due to sub optimal pricing? We know how to measure the gain to the consumers from having to pay $\mathrm{p}_{\mathrm{a}}$ (actual price) rather than $\mathrm{p}_{\mathrm{c}}$ (marginal cost price) when $\mathrm{p}_{\mathrm{c}}>\mathrm{p}_{\mathrm{a}}-$ we just look at the change in consumers' surplus. Similarly, for MSRTC we know how to measure the loss in profits from charging $\mathrm{p}_{\mathrm{a}}$ rather than $\mathrm{p}_{c}$ - we use the change in producer's surplus. So, deadweight loss due to sub optimal pricing will be the sum of change in consumers' as well as producer's surplus. Therefore, dead weight loss due to MSRTC's charged price $p_{a}\left(<p_{c}\right)$ during 1996-97 would be (see Figure 3):
DWL $=$ DEJD - ABJA $+($ DKGED - FHKGF $)-($ AKCBA - BHKCB $)$
$=$ BADEB + DKGED - FBCGF - AKCBA
$=\mathrm{BAKGEB}-\mathrm{AKCBA}-\mathrm{FBCGF}$
= BCGEB - FBCGF
$=\mathrm{BEFB}$
$=\frac{1}{2} \cdot \mathrm{BI} \cdot \mathrm{FE}$
$=(0.5) .(4) .(4)$ (in paise in billion)
$=$ Rs. 80 million.
A similar exercise can be done to find out the dead weight loss due to sub optimal pricing in MSRTC during 1983-84 as well as for other sample periods. One should note that during 1983-84, price charged by

MSRTC was very close to the marginal cost of production. Since MSRTC is operating on decreasing returns to scale, by charging slightly more than marginal cost it not only made profit over operating cost but also over total cost during 1983-84. Furthermore, optimal fare rate for MSRTC has declined from the year 1983-84 to 1996-97 due to many factors including technological progress. It is evident from this analysis that for the firm such as MSRTC, which operates on decreasing returns to scale, marginal cost pricing will not only be efficient but also be sufficient to recover the operating as well as total cost of production.

Table 3
Estimated demand function for MSRTC (dep. var.: natural log of MPKm; ${ }^{10}$

| Parameter |  |
| :--- | :---: |
| Constant | -6.40 |
| AvgFare | $(1.38)$ |
| PCI | -0.77 |
|  | $(7.69)$ |
| SDP | -11.45 |
|  | $(6.05)$ |
| Time | 11.24 |
|  | $(5.66)$ |
| Number of observations | -0.20 |
| R-square | $(3.88)$ |
| Log-Likelihood | 14 |



Figure 1: Demand for MSRTC's bus transport services during 1996-97.

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Figure 2: Marginal cost of MSRTC's bus transport services during 1996-97.


Figure 3: Deadweight loss due to sub optimal pricing in MSRTC during 1996-97.

Table 4
Measuring the social welfare and profitability of MSRTC at different level of prices; all monetary units are at constant 1996-97 prices

| Price (paise) <br> PKm)1996-97 <br> (=24.58) | Demand (= Supply) <br> (PKm in million) | Profit over operating cost (Rs. in million) | Profit as a percentage of operating cost | Average operating cost (paise/ PKm) | Marginal cost (paise/PKm) | $\gamma_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 74779 | -1281 | -8 | 21.71 | 32.91 | -0.50 |
| 21 | 72018 | -219 | -1 | 21.30 | 31.84 | -0.40 |
| 22 | 69481 | 738 | 5 | 20.94 | 30.86 | -0.31 |
| 23 | 67140 | 1607 | 12 | 20.61 | 29.97 | -0.23 |
| 24 | 64973 | 2399 | 18 | 20.31 | 29.16 | -0.17 |
| 25 | 62959 | 3125 | 25 | 20.04 | 28.41 | -0.11 |
| 26 | 61084 | 3793 | 31 | 19.79 | 27.72 | -0.05 |
| 27 | 59332 | 4411 | 38 | 19.57 | 27.08 | 0.00 |
| 28 | 57692 | 4984 | 45 | 19.36 | 26.48 | 0.04 |
| 29 | 56152 | 5518 | 51 | 19.17 | 25.93 | 0.08 |
| 30 | 54703 | 6017 | 58 | 19.00 | 25.41 | 0.12 |
| 31 | 53338 | 6484 | 65 | 18.84 | 24.93 | 0.15 |
| 32 | 52048 | 6923 | 71 | 18.70 | 24.48 | 0.18 |
| 33 | 50827 | 7337 | 78 | 18.56 | 24.05 | 0.21 |
| 34 | 49671 | 7727 | 84 | 18.44 | 23.65 | 0.23 |
| 35 | 48573 | 8098 | 91 | 18.33 | 23.27 | 0.26 |

1983-84 (= 31.64 )

| 20 | 45812 | -3226 | -26 | 27.04 | 38.94 | -0.73 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 44120 | -2475 | -21 | 26.61 | 37.75 | -0.61 |
| 22 | 42566 | -1797 | -16 | 26.22 | 36.66 | -0.51 |
| 23 | 41132 | -1183 | -11 | 25.88 | 35.67 | -0.42 |
| 24 | 39804 | -622 | -6 | 25.56 | 34.76 | -0.35 |
| 25 | 38571 | -109 | -1 | 25.28 | 33.93 | -0.28 |
| 26 | 37422 | 363 | 4 | 25.03 | 33.15 | -0.21 |
| 27 | 36349 | 800 | 9 | 24.80 | 32.44 | -0.16 |
| 28 | 35344 | 1204 | 14 | 24.59 | 31.77 | -0.10 |
| 29 | 34400 | 1581 | 19 | 24.40 | 31.15 | -0.06 |
| 30 | 33513 | 1933 | 24 | 24.23 | 30.57 | -0.01 |
| 31 | 32676 | 2262 | 29 | 24.08 | 30.03 | 0.02 |
| 32 | 31886 | 2571 | 34 | 23.94 | 29.52 | 0.06 |
| 33 | 31138 | 2862 | 39 | 23.81 | 29.04 | 0.09 |
| 34 | 30430 | 3137 | 44 | 23.69 | 28.58 | 0.12 |
| 35 | 29757 | 3396 | 48 | 23.59 | 28.16 | 0.15 |

Table 5
Taxes, interest payment, and depreciation on buses as a percentage of total operating cost during 1983-84 and 1996-97

| STUs | $1983-84$ | $1996-97$ |
| :--- | :---: | :---: |
| MSRTC | 23 | 31 |
| APSRTC | 32 | 30 |
| KnSRTC | 34 | 27 |
| GSRTC | 60 | 27 |
| UPSRTC | 50 | 14 |
| KSRTC | 24 | 26 |
| RSRTC | 47 | 33 |
| MPSRTC | 41 | 36 |
| STPJB | 29 | 40 |

### 5.1.2. Optimal Pricing in APSRTC

This involves an exercise similar to that is done in sub-section 5.1.1. We estimated ordinary demand function for passenger transport services provided by APSRTC using required annual data from 1983-84 to 199697. The estimated model is similar to the model reported in the previous sub-section. Table 6 presents results of estimated demand function. The result shows that the price of bus transport services of APSRTC significantly influences the quantity demanded. The estimated price elasticity of demand is 0.60 which indicates that an increase in average fare rate will increased the traffic revenue of APSRTC while reducing the sales by a relatively smaller percentage.

Table 6
Estimated demand function for APSRTC (dep. var.: natural log of MPKm) ${ }^{11}$

| Parameter |  |
| :--- | :---: |
| Constant | -14.41 |
|  | $(1.92)$ |
| AvgFare | -0.60 |
|  | $(1.69)$ |
| PCI | -14.83 |
|  | $(4.91)$ |
| SDP | 14.95 |
|  | $(4.19)$ |
| Time | -0.23 |
|  | $(3.22)$ |
| Number of observations | 14 |
| R-square | 0.985 |
| Log-Likelihood | 25.75 |



Figure 4: Deadweight loss due to sub optimal pricing in APSRTC during 1996-97
An application of equation (25) requires estimation of marginal cost at different level of output. We estimated marginal cost of APSRTC at different level of pass.-km based on results reported by Singh (2002) and depicted the same in Figure 4. Table 7 presents an indicator of social welfare and profitability of APSRTC during 1996-97 and 1983-84 at different level of prices. Result shows that during 1996-97, APSRTC charged around 24 paise per pass.-km which was slightly higher than the optimum fare rate of 22.5 paise per pass.-km. It may be justifiable because although marginal cost pricing was able to recover total operating cost, it was unable to recover the total cost. When taxes, interest payment, and depreciation on buses account for around 30 percent of total operating cost, a fare rate of 24 paise per pass. km will result in a profit (over total cost) of around 6 percent of total operating cost (see, Table 7). Moreover, we calculated the deadweight loss due to higher price than the optimal one. Area BEFB in Figure 4 represents deadweight loss due to actual price charged by APSRTC. The result reveals that deadweight loss is negligible in comparison to total operating cost. We calculated deadweight loss due to APSRTC's sub-optimal pricing during 1996-97 as follows:

> DWL = Area BEFB

$$
\begin{aligned}
& =\frac{1}{2} \cdot \mathrm{BI} \cdot \mathrm{FE}=(0.5) \cdot(2 \cdot 92) \cdot(2 \cdot 24) \quad \text { (in paise in billion) } \\
& =\text { Rs. } 32.7 \text { million. }
\end{aligned}
$$

## 6. CONCLUDING REMARKS

The driving force behind the argument in favor of economically efficient pricing rule (marginal cost pricing) for public enterprises is the assertion that government ought to maximize welfare rather than profits (Singh, 2012). A public-sector pricing model in the Boiteux tradition also provides optimal pricing rule for

Table 7
Measuring the social welfare and profitability of APSRTC at different level of prices; all monetary units are at constant 1996-97 prices.

| Price (paise) <br> PKm)1996-97 <br> (=24.02) | Demand ( $=$ Supply) <br> (PKm in million) | Profit over operating cost (Rs. in million) | Profit as a percentage of operating cost | Average operating cost (paise) PKm) | Marginal cost (paise/PKm) | $\gamma_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | 90544 | -3889 | -22 | 19.30 | 26.85 | -0.47 |
| 16 | 87116 | -2626 | -16 | 19.01 | 26.04 | -0.38 |
| 17 | 84014 | -1486 | -9 | 18.77 | 25.31 | -0.29 |
| 18 | 81191 | -449 | -3 | 18.55 | 24.66 | -0.22 |
| 19 | 78608 | 502 | 3 | 18.36 | 24.07 | -0.16 |
| 20 | 76233 | 1378 | 10 | 18.19 | 23.52 | -0.11 |
| 21 | 74041 | 2190 | 16 | 18.04 | 23.03 | -0.06 |
| 22 | 72010 | 2947 | 23 | 17.91 | 22.57 | -0.02 |
| 23 | 70121 | 3655 | 29 | 17.79 | 22.15 | 0.02 |
| 24 | 68359 | 4320 | 36 | 17.68 | 21.76 | 0.06 |
| 25 | 66710 | 4947 | 42 | 17.58 | 21.40 | 0.09 |
| 26 | 65164 | 5541 | 49 | 17.50 | 21.06 | 0.11 |
| 27 | 63710 | 6104 | 55 | 17.42 | 20.74 | 0.14 |
| 28 | 62339 | 6639 | 61 | 17.35 | 20.44 | 0.16 |
| 29 | 61044 | 7150 | 68 | 17.29 | 20.16 | 0.18 |
| 30 | 59819 | 7638 | 74 | 17.23 | 19.90 | 0.20 |
| 1983-84 (= 25.10) |  |  |  |  |  |  |
| 15 | 38507 | -1176 | -17 | 18.05 | 23.44 | -0.34 |
| 16 | 37049 | -687 | -10 | 17.85 | 22.79 | -0.25 |
| 17 | 35730 | -244 | -4 | 17.68 | 22.20 | -0.18 |
| 18 | 34529 | 161 | 3 | 17.53 | 21.67 | -0.12 |
| 19 | 33431 | 533 | 9 | 17.41 | 21.20 | -0.07 |
| 20 | 32421 | 877 | 16 | 17.30 | 20.76 | -0.02 |
| 21 | 31488 | 1197 | 22 | 17.20 | 20.35 | 0.02 |
| 22 | 30625 | 1496 | 29 | 17.12 | 19.98 | 0.05 |
| 23 | 29821 | 1777 | 35 | 17.04 | 19.64 | 0.09 |
| 24 | 29072 | 2041 | 41 | 16.98 | 19.32 | 0.12 |
| 25 | 28371 | 2291 | 48 | 16.93 | 19.02 | 0.14 |
| 26 | 27713 | 2528 | 54 | 16.88 | 18.74 | 0.17 |
| 27 | 27095 | 2753 | 60 | 16.84 | 18.48 | 0.19 |
| 28 | 26512 | 2968 | 67 | 16.81 | 18.24 | 0.21 |
| 29 | 25961 | 3173 | 73 | 16.78 | 18.01 | 0.23 |
| 30 | 25440 | 3370 | 79 | 16.75 | 17.79 | 0.24 |

public enterprises that have a mandated budget constraint. The price and output combinations that it computes minimize the deadweight loss due to unavoidable deviation of price from marginal cost. Since this pricing rule takes into account price elasticity of demand, it is superior to the average-cost pricing rule that most of the publicly owned bus transport companies tend to adopt. Average-cost pricing does not explicitly include demand-side information.

In this paper, we discussed the theory of public sector pricing and its application to publicly supplied bus transport services using a case study of STUs in India. We did a simulation study to measure the level of social welfare and profitability of the two largest STUs in India, MSRTC and APSRTC, at different level of prices charged for passenger transport services offered by them. Since the price elasticity of demand is an important ingredient of the optimal pricing rule, we first estimated, by regression analysis, the derived demand for passenger transport services offered by them, using a constant elasticity demand function. Marginal cost calculation is based on estimated cost function, which is presented in Singh (2002). Finally, we compared prices with the marginal cost and calculated a measure of the level of social welfare and profitability of MSRTC and APSRTC at different level of prices.

The analysis shows that both the STUs deviate from economically efficient fare rates. By comparing deadweight losses, we found that in every year of the study period there would have been a welfare gain if optimal prices had been charged rather than actual ones. During the year 1996-97, marginal cost pricing rule for MSRTC would have resulted in a significant amount of financial profit since it not only faced huge diseconomies of scale but also inelastic demand. In fact, MSRTC actually charged less than its marginal cost and could not recover the total cost of production. During the same year, marginal cost pricing for APSRTC would have resulted in recovery of its operating cost but not the total cost of production. APSRTC actually charged $6 \%$ more than its marginal cost of production; it may be justifiable because marginal cost pricing was unable to recover the total cost of production. Moreover, we found that deadweight loss created by MSRTC due to sub-optimal pricing was significantly higher than that of APSRTC. This is because fare of MSRTC deviated far more from its marginal cost of production than the corresponding deviation in APSRTC. Unlike MSRTC, dead weight loss created by APSRTC was negligible in comparison to its operating cost.

## NOTES

1. Differentiate the Lagrangian function $L$ with respect to initial endowments of labor $z_{0}$ and $y_{0}$ respectively. $\alpha_{0}>0$ and $\beta_{0}>0$ follow with economic plausibility, see Drèze and Marchand (1976, p. 67).
2. This is valid if the public enterprise operates at minimum cost which implies that marginal rates of input substitution equal input prices, see Bös (1994, p. 62-66).
3. Provided, of course, that the cost we are talking of is the minimum cost curve.
4. Ramsey pricing is named after Frank P. Ramsey; it is sometimes called Ramsey-Boiteux pricing to recognize its further development by Marcel Boiteux (1956).
5. At the optimum, $z_{k}\left(p, r^{h}\left(p, u^{h^{*}}\right)\right)=\hat{z}_{k}\left(p, r^{h}\right)$.
6. But, of course, this does not hold generally if all cross-price elasticities are taken into account as in equation (23).
7. The level of profitability is measured as traffic revenue minus operating cost. This definition is followed throughout this section.

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8. We fit different models for demand function estimation e.g., demand as a function of price only and demand as function of price and per capita income etc. Result of only the "best" fit model is presented here.
9. This is a proximate deadweight loss since we use the Marshallian rather than a Hicksian demand function.
10. T -values in parentheses and all variables except Time are in natural log.
11. T-values in parentheses and all variables except Time are in natural log.

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