

RADIATION EFFECTS ON OSCILLATING VERTICAL PLATE WITH UNIFORM HEAT AND MASS FLUX

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ABSTRACT: Thermal radiation effects on flow past an impulsively started infinite vertical oscillating plate with uniform heat and mass flux is studied here. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The dimensionless governing equations are solved using the Laplace-transform technique. The velocity, temperature and concentration are studied for different physical parameters like radiation parameter, phase angle, Schmidt number and time. The variation of the skin-friction for different values of the parameters is also shown in a table.

Keywords: Oscillating vertical plate, Radiation, Heat and mass flux.

1. INTRODUCTION

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

The first exact solution of the Navier-stokes equation was given by Stokes (1851) which is concerned with the flow of a viscous incompressible fluid past an infinite horizontal plate oscillating in its own plane in an infinite mass of stationary fluid. Such a flow past an infinite vertical plate oscillating in its own plane was first studied by Soundalgekar (1979) in case of an isothermal plate. M. A. Mansour (1990) studied the interaction of free convection with thermal radiation of the oscillatory flow past a vertical plate. Zhang *et al.*, (2004) studied the free convection effects on a heated vertical plate subjected to a periodic oscillation. The effects of thermal radiation on flow past an oscillating plate with variable temperature were studied by Pathak *et al.*, (2006). Free convection effects on a vertical oscillating porous plate with constant heating was studied by Toki (2009). Chandrakala and Bhaskar (2009) studied the effects of thermal radiation on the flow past an infinite vertical oscillating isothermal plate in the presence of transversely applied magnetic field. Chandrakala (2011) studied the effects of thermal radiation on the flow past an infinite vertical oscillating plate with uniform heat flux.

The unsteady flow past a moving infinite vertical oscillating plate in the presence of radiation with uniform heat and mass flux has not received much attention from

contemporary researchers. It is now proposed to study the thermal radiation effects on the oscillatory past a vertical plate with uniform heat and mass flux.

2. MATHEMATICAL FORMULATION

Consider the flow of an incompressible viscous radiating fluid past an impulsively started infinite vertical plate with uniform heat and mass flux is considered. The x' -axis is taken along the plate in the vertical direction and the y' -axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration in a stationary condition. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity u_0 . At the same time, the heat is supplied from the plate to the fluid at a uniform rate and the concentration level near the plate is also raised at a uniform rate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \quad (2.1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \quad (2.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (2.3)$$

with the following initial and boundary conditions

$$\begin{aligned} t' \leq 0: \quad u' &= 0, & T' &= T'_\infty, & C' &= C'_\infty & \text{for all } y' \\ t' > 0: \quad u' &= u_0 \cos \omega t, & \frac{\partial T'}{\partial y} &= -\frac{q}{k}, & \frac{\partial C'}{\partial y} &= -\frac{j''}{D} & \text{at } y' = 0 \\ u' &= 0, & T' &\rightarrow T'_\infty, & C' &\rightarrow C'_\infty & \text{as } y' \rightarrow \infty \end{aligned} \quad (2.4)$$

The local radiant for the case of an optically thin gray gas is expressed by

$$q_r = -\frac{4\sigma}{3\kappa^*} \frac{\partial T'^4}{\partial y'} \quad (2.5)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is

accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3 T' - 3T'^4 \quad (2.6)$$

By using equations (2.5) and (2.6), equation (2.2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_\infty'^3}{3\kappa^*} \frac{\partial^2 T'}{\partial y'^2} \quad (2.7)$$

On introducing the following dimensionless quantities

$$u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T' - T'_\infty}{(q\nu/k u_0)}$$

$$Gr = \frac{g\beta q \nu^2}{k u_0^4}, \quad Pr = \frac{\mu C_p}{k}, \quad N = \frac{\kappa^* k}{4\sigma T_\infty'^3} \quad (2.8)$$

$$C = \frac{C' - C'_\infty}{j'' \nu / (D u_0)}, \quad Gc = \frac{g\beta^* \nu^2 j''}{D u_0^4}$$

in equations (2.1) to (2.7), leads to

$$\frac{\partial u}{\partial t} = Gr\theta + \frac{\partial^2 u}{\partial y^2} \quad (2.9)$$

$$3NPr \frac{\partial \theta}{\partial t} = (3N + Pr) \frac{\partial^2 \theta}{\partial y^2} \quad (2.10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad (2.11)$$

The initial and boundary conditions in non-dimensionless form are

$$u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } y, t' \leq 0$$

$$t' > 0: \quad u = u_0 \cos \omega t, \quad \frac{\partial \theta}{\partial y} = -1, \quad \frac{\partial C}{\partial y} = -1 \quad \text{at } y = 0 \quad (2.12)$$

$$u = 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

The equations (2.9), (2.10) and (2.11), subject to the boundary conditions (2.12), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\theta = 2\sqrt{t} \left[\frac{\exp(-\eta^2 a)}{\sqrt{\pi}} - \eta\sqrt{a} \operatorname{erfc}(\eta\sqrt{a}) \right] \quad (2.13)$$

$$C = 2\sqrt{t} \left[\frac{\exp(-\eta^2 Sc)}{\sqrt{\pi}} - \eta\sqrt{Sc} \operatorname{erfc}(\eta\sqrt{Sc}) \right] \quad (2.14)$$

$$\begin{aligned} u = & \frac{\exp(-i\omega t)}{4} \left[\exp(-2\eta\sqrt{-i\omega t} \operatorname{erfc}(\eta - \sqrt{-i\omega t})) \right. \\ & \left. + \exp(2\eta\sqrt{-i\omega t} \operatorname{erfc}(\eta + \sqrt{-i\omega t})) \right] \\ & + \frac{\exp(i\omega t)}{4} \left[\exp(-2\eta\sqrt{i\omega t} \operatorname{erfc}(\eta - \sqrt{i\omega t})) \right. \\ & \left. + \exp(2\eta\sqrt{i\omega t} \operatorname{erfc}(\eta + \sqrt{i\omega t})) \right] \\ & - \frac{Grt^{3/2}}{3\sqrt{a}(1-a)} \left[\frac{4}{\sqrt{\pi}} (1 + \eta^2) \exp(-\eta^2) - \eta(6 + 4\eta^2) \operatorname{erfc}(\eta) \right] \\ & - \frac{Gct^{3/2}}{3(1-Sc)} \left[\frac{4}{\sqrt{\pi}} (1 + \eta^2) \exp(-\eta^2) - \eta(6 + 4\eta^2) \operatorname{erfc}(\eta) \right] \\ & + \frac{Grt^{3/2}}{3\sqrt{a}(1-a)} \left[\frac{4}{\sqrt{\pi}} (1 + \eta^2 a) \exp(-\eta^2 a) - \eta\sqrt{a}(6 + 4\eta^2 a) \operatorname{erfc}(\eta\sqrt{a}) \right] \\ & + \frac{Gct^{3/2}}{3(1-Sc)} \left[\frac{4}{\sqrt{\pi}} (1 + \eta^2 Sc) \exp(-\eta^2 Sc^2) - \eta\sqrt{Sc}(6 + 4\eta^2 Sc) \operatorname{erfc}(\eta\sqrt{Sc}) \right] \quad (2.15) \end{aligned}$$

where $a = \frac{3NPr}{3N+4}$ and $\eta = \frac{y}{2\sqrt{t}}$.

3. DISCUSSION OF RESULTS

The numerical values of the velocity, temperature and wall concentration are computed for different parameters like radiation parameter, Grashof number, Schmidt number, time and $Pr = 0.71$. The purpose of the calculations given here is to assess the effects of the parameters N , M , t and ωt .

The velocity profiles for different values of the radiation parameter ($N = 2, 30$) are shown in Fig. 1. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

The velocity profiles for different values of the time ($t = 0.2, 0.4, 0.6$) are shown in Fig. 2. This shows that the velocity increases with increasing values of time t .

In Fig. 3, the velocity profiles are shown for different values of ωt . It is observed that velocity increases with decreasing phase angle ωt .

The temperature profiles are calculated for different values of thermal radiation parameter ($N = 2, 5, 30$) are shown in Fig. 4. It is observed that temperature increases with decreasing radiation parameter.

The temperature profiles are calculated for different values of the Prandtl number $Pr = 0.71$ and $Pr = 7.0$ are shown in Fig. 5. The effect of Prandtl number is important in temperature profiles. It is observed that temperature increases in the presence of air than in water.

Figure 6 represents, the effect of concentration profiles for different Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$) and time $t = (0.2, 0.4)$. The effect of Schmidt number is important in concentration field. It is observed that the wall concentration increases with decreasing values of the Schmidt parameter. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

From the velocity field, we now study the skin-friction. It is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0} \quad (3.1)$$

Hence, from equations (2.15) and (3.1),

$$\begin{aligned} \tau = \frac{1}{\sqrt{\pi t}} \left[\frac{\exp(-i\omega t)}{2} \left(1 + \sqrt{-i\omega\pi t} \operatorname{erf}(\sqrt{-i\omega t}) \right) \right. \\ \left. + \frac{\exp(i\omega t)}{2} \left(1 + \sqrt{i\omega\pi t} \operatorname{erf}(\sqrt{i\omega t}) \right) - \frac{Gr t^{3/2} \sqrt{\pi}}{\sqrt{a}(1+\sqrt{a})} - \frac{Gct^{3/2} \sqrt{\pi}}{(1+\sqrt{Sc})} \right] \quad (3.2) \end{aligned}$$

The numerical values of τ are presented in the following Table 1. It is observed from this table, that an increase in the Schmidt number parameter or radiation parameter leads to rise in the value of the skin-friction. As time advances the value of skin-friction decreases. It is also observed that the skin-friction decreases with increasing values of the thermal Grashof number or mass Grashof number.

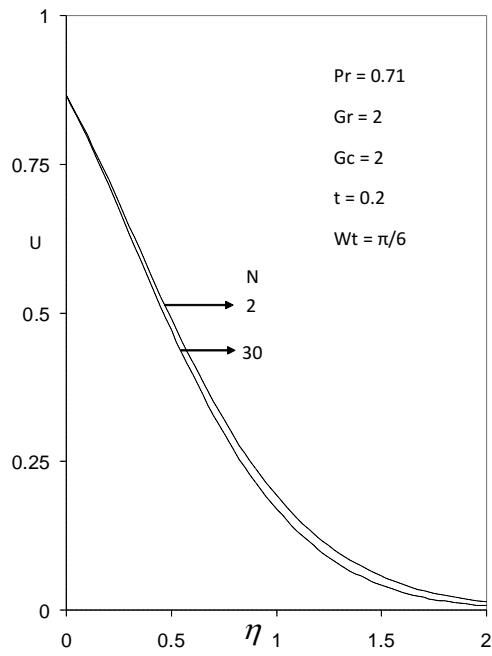


Figure 1: Velocity Profiles for Different N

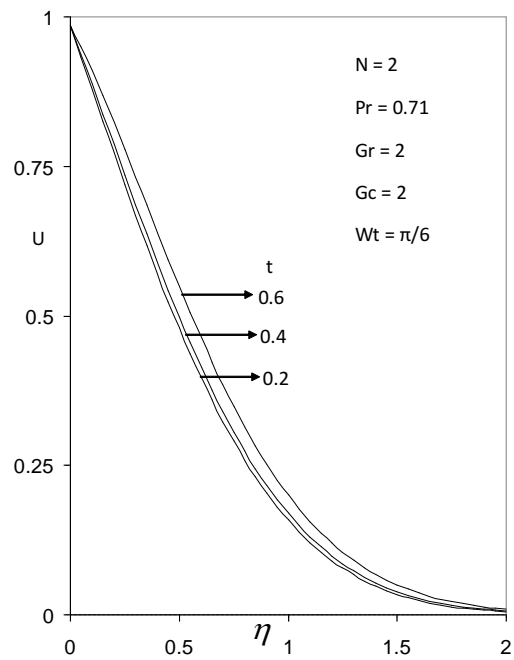


Figure 2: Velocity Profiles for Different t

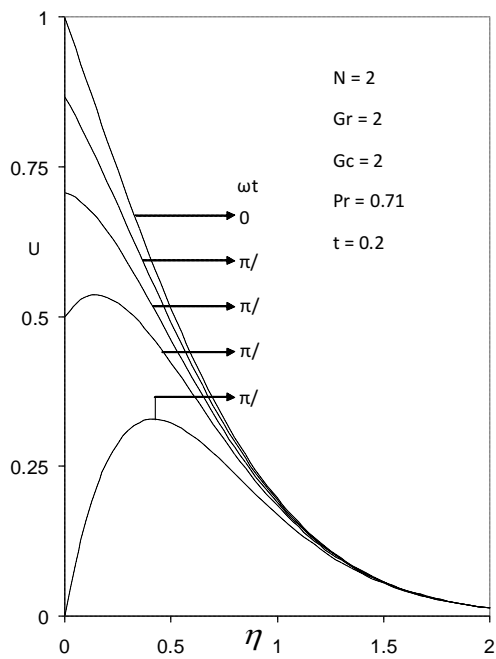


Figure 3: Velocity Profiles for Different wt

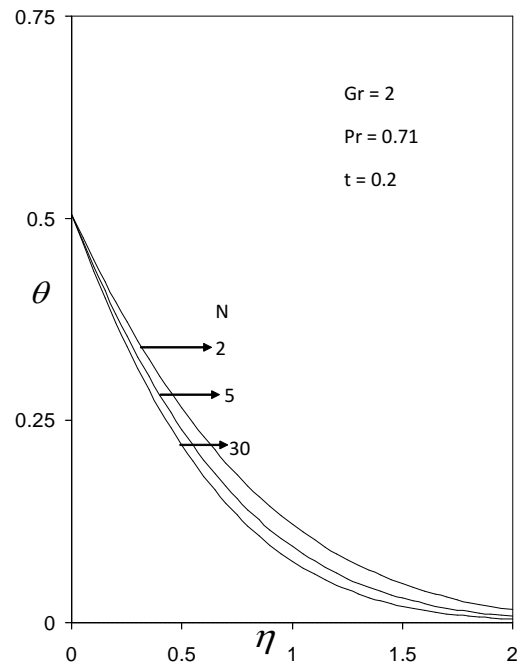


Figure 4: Temperature Profiles for Different N

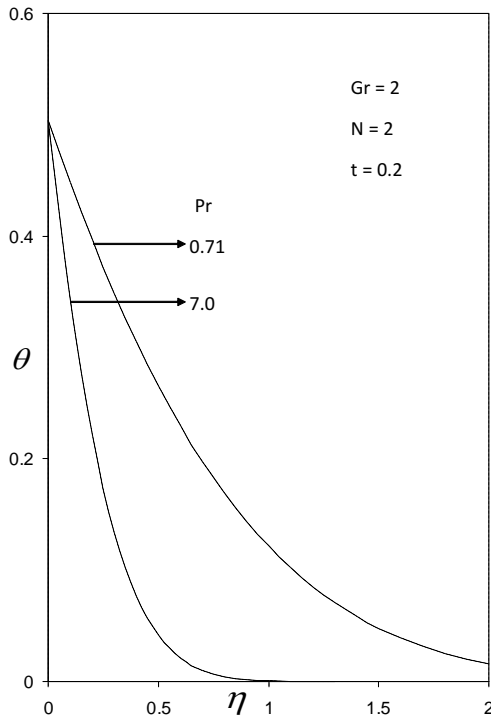


Figure 5: Temperature Profiles for Different Pr

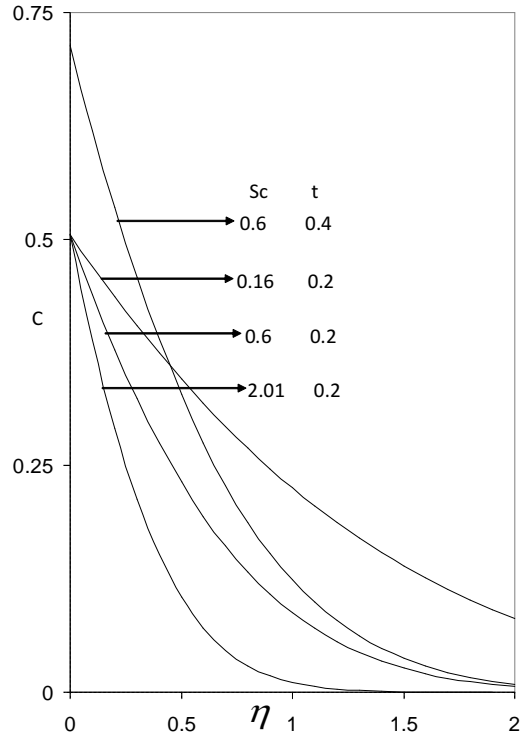


Figure 6: Concentration Profiles for Different Sc and t

Table 1
Values of the Non-Dimensional Skin-Friction

N	Gr	Gc	Sc	t	$\omega t = 0$	$\omega t = \pi/6$	$\omega t = \pi/4$	$\omega t = \pi/3$
0.2	2	5	0.6	0.2	1.1912	-0.4950	-2.1043	-3.9863
2	2	5	0.6	0.2	1.2197	-0.4665	-2.0759	-3.9578
2	2	5	0.6	0.2	1.2160	-0.4702	-2.0796	-3.9615
2	2	5	0.6	0.4	.06556	-0.5367	-1.6747	-3.0054
2	2	5	0.6	0.2	1.1949	-0.4914	-2.1007	-3.9827
2	2	2	0.6	0.2	1.2349	-0.4514	-2.0607	-3.9427
5	2	5	0.6	0.2	1.2227	-0.4636	-2.0729	-3.9549
10	2	5	0.6	0.2	1.2237	-0.4625	-2.0719	-3.9538

4. CONCLUSIONS

An exact analysis is performed to study the thermal radiation effects on flow past an impulsively started infinite oscillating vertical plate with uniform heat and mass flux.

The dimensionless governing equations are solved by the usual Laplace-transform technique. The velocity, temperature and wall concentration are studied for different physical parameters are studied graphically. The conclusions of the study are as follows.

- (i) The presence of radiation causes a fall in the velocity and temperature.
- (ii) Velocity increases with decreasing phase angle ωt .
- (iii) As time increases, it is found that there is a rise in velocity.

Nomenclature

- A – constant
- C' – Species concentration in the fluid
- C'_w – concentration of the plate
- C'_∞ – concentration in the fluid far away from the plate
- C – dimensionless concentration
- C_p – specific heat at constant pressure
- D – mass diffusion coefficient
- g – acceleration due to gravity
- Gr – thermal Grashof number
- Gc – mass Grashof number
- f – mass flux per unit area at the plate
- k – thermal conductivity of the fluid
- Pr – Prandtl number
- p – pressure
- q_r – radiative heat flux in the y -direction
- N – radiation parameter
- Sc – Schmidt number
- T' – temperature of the fluid near the plate
- T'_∞ – temperature of the plate
- T'_w – temperature of the fluid far away from the plate
- t' – time

- t' – dimensionless time
 u' – velocity of the fluid in the x -direction
 u_0 – velocity of the plate
 u – dimensionless velocity
 y – coordinate axis normal to the plate
 y' – dimensionless coordinate axis normal to the plate
 k^* – mean absorption coefficient
 β – volumetric coefficient of thermal expansion
 β^* – volumetric coefficient of expansion with concentration
 η – similarity parameter
 μ – coefficient of viscosity
 ν – kinematic viscosity
 ρ – density
 $\bar{\tau}$ – dimensionless average skin-friction
 θ – dimensionless temperature
 ωt – phase angle
 $erfc$ – complementary error function

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