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EFFECT OF MAGNETIC FIELD ON SCATTERING OF DUST PARTICLES WITH CHEMICAL REACTION IN THE ATMOSPHERE

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Abstract: The simultaneous effect of chemical reaction with and without magnetic field on scattering of dust particles in the atmosphere is studied. The scattering coefficient of dust particles is evaluated analytically using Taylor's method. The computed numerical results are depicted through graphs and conclusions are drawn.

Keywords: a*tmospheric fluid, electric number,Hartmann number.*

1. INTRODUCTION

Airborne dust are of special interest because they are the main reason for many lung diseases. Dust is a fine, dry powder consisting of tiny particles of earth or waste matter lying on the ground or carried in the air. Types of dust includes mineral dust, metallic dust, chemical dust, organic and vegetable dust and biohazards. Scattering is the redirection of electromagnetic energy by suspended particles in the atmosphere [1]. As scattering properties of dust particles play an important role, many researchers worked in this area. John Aitken [3] studied the methods for counting the dust particles in the air. Sunil et.al [5] dealt with the theoretical investigation of viscosity of saturating porous medium in ferro-magnetic fluid in the presence of dust particles.Meenapriya and Nirmala P.Ratchagar [4] investigated the electrohydrodynamic dispersion of atmospheric aerosols through a vertical channel with temperature distributions. Ulanowski et.al [7] concludes in his study that the alignment of dust can significantly alter dust optical depth and the presence of electric field modify dust transport. Haijiao Liu et.al [2] presented the results of a comprehensive study of dust flux and magnetic signatures of atmospheric dust fall originating from east Asia and China. The objective of this paper is to study the effects of scattering coefficient of dust particles in the atmosphere under the effect of electric field, both in the presence and absence of magnetic field.

2. MATHEMATICAL FORMULATION

We consider two-dimensional laminar incompressible viscous flow of dust particles as depicted in figure 1. It includes a horizontal atmospheric fluid layer bounded by electro conducting impermeable rigid plates embedded with electrodes located at y = 0 and y = h with an applied magnetic field B_0. On the boundaries, the electric potentials at y=0, $\phi = \frac{V}{h}x$ and at y=h, $\phi = \frac{V}{h}(x - x_0)$ are maintained, where V is potential.

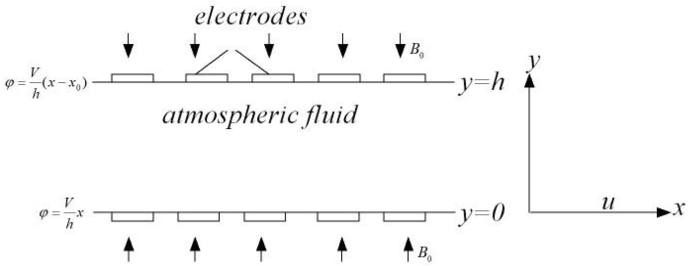

Figure 1. Physical configuration

The electric field $\vec{E}$, to be conservative, since we are considering very small electrical conductivity (σ) ,

$$\text{i.e. } \vec{E} = -\nabla\phi \tag{1}$$

The conservation of mass for an incompressible fluid

$$\nabla.\vec{q} = 0 \tag{2}$$

The conservation of momentum

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} + \rho_e\vec{E} + \mu(\vec{J}x\vec{B}) \tag{3}$$

where $\vec{q}$ the velocity, p the pressure, μ the viscosity, ρ_e the density of the charge distribution, $\vec{E}$ the electric field, $\vec{J}$ the current density and $\vec{B}$ the magnetic induction.

The species equation is

$$\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = D\nabla^2 C \tag{4}$$

The conservation of charges

$$\frac{\partial \rho_e}{\partial t} + (\vec{q}.\nabla)\rho_e + \nabla.\vec{J} = 0 \tag{5}$$

The Maxwell's equations

$$\nabla.\vec{E} = \frac{\rho_e}{\varepsilon_0} \tag{6}$$

$$\nabla x\vec{E} = \frac{-\partial\vec{B}}{\partial t} \tag{7}$$

$$\vec{J} = \sigma\vec{E} \tag{8}$$

where ε_0 the dielectric constant and σ the electrical conductivity. The following boundary conditions on velocity are used to solve the above equations

$$\left.\begin{aligned} u = 0 \text{ at } y = 0 \\ u = 0 \text{ at } y = h \end{aligned}\right\} \tag{9}$$

The boundary conditions on electric potential are,

$$\left.\begin{aligned} \phi = \frac{V}{h} x \text{ at } \quad & y = 0 \\ \phi = \frac{V}{h}(x - x_0) \text{ at } \quad & y = h \end{aligned}\right\} \tag{10}$$

The above approximation equation (3) can be written in Cartesian form as below,

$$0 = -\frac{\partial p}{\partial x} + \mu\nabla^2 u + We\rho_e E_x - M^2 u, \nabla^2 = \frac{\partial^2}{\partial y^2}$$

where $M^2 = \frac{\sigma B_0^2 h^2}{\mu}$ (M is the Hartmann number), B_0 the uniform applied magnetic field, σ the electrical conductivity and μ the coefficient of viscosity. The electrical conductivity is assumed to vary linearly with temperature in the poorly conducting fluid and increases with temperature in the form

$$\sigma = \sigma_0 [1+\alpha_h(T_b-T_0)] \tag{11}$$

where α_h is the volumetric coefficient of expansion.

After making dimensionless, using

$$y^* = \frac{y}{h}; \quad u^* = \frac{u}{\frac{\nu}{h}}; \quad E_x^* = \frac{E_x}{\frac{\nu}{h}}; \quad \rho_e^* = \frac{\rho_e}{\frac{\varepsilon_0 \nu}{h^2}}; \quad P^* = \frac{P}{\rho\left(\frac{\nu}{h}\right)^2}; x^* = \frac{x}{h};$$

The electric potential V is obtained through electrodes and the above equations (3) to (11) becomes,

$$\frac{d^2u}{dy^2} - M^2 u = W_e \rho_e E_x + P \tag{12}$$

where $W_e = \frac{\varepsilon_0 V^2}{\mu}$, $P = \frac{-\partial p}{\partial x}$, Equation (5) becomes, $\nabla.\vec{J}=0$

From equation (1) we get,

$$\sigma(\nabla^2\phi)+\nabla\phi.\nabla\sigma=0 \tag{13}$$

After dimensionless the boundary conditions on velocity and electric potential are

$$\left.\begin{array}{l} u=0 \text{ at } y=0 \\ u=0 \text{ at } y=1 \end{array}\right\} \tag{14}$$

$$\left.\begin{array}{ll} \phi=x \text{ at} & y=0 \\ \phi=x-x_0 \text{ at} & y=1 \end{array}\right\} \tag{15}$$

The electrical conductivity σ <<1 which is negligibly small, and it depends on the conduction temperature T_b as given in equation (11),

$$\text{Also, } \frac{d^2T_b}{dy^2}=0 \tag{16}$$

using the boundary conditions

$$\left.\begin{array}{ll} T_b=T_0 \text{ at} & y=0 \\ T_b=T_1 \text{ at} & y=h \end{array}\right\} \tag{17}$$

$$\text{is } T_b-T_0=\Delta T_y \tag{18}$$

equation (11) becomes

$$\sigma=\sigma_0[1+\alpha_h\Delta T_y]=\sigma_0(1+\alpha y)=\sigma_0 e^{\alpha y}$$

$$\sigma\approx e^{\alpha y} \tag{19}$$

where $\alpha=\alpha_h\nabla T$.

Then (13) using (19) we get

$$\frac{d^2\phi}{dy^2}+\alpha\frac{d\phi}{dy}=0 \tag{20}$$

The solution for ϕ is given by

$$\phi=x-\frac{x_0}{1-e^{-\alpha}}[1-e^{-\alpha y}] \tag{21}$$

The equations (6), (7) and (8) can be written using the non-dimensional quantities and equation (21),

$$\rho_e = \nabla.\vec{E} = -\nabla^2\phi = -\frac{x_0\alpha^2 e^{-\alpha y}}{1-e^{-\alpha}}\text{ ; } E_x = -1$$

Then $\rho_e E_x = \frac{x_0^2\alpha^2 e^{-\alpha y}}{1-e^{-\alpha}}$ (22)

3. SCATTERING OF AEROSOLS

We consider two cases. First case is scattering in the presence of magnetic field and second without magnetic field

Case 1: M ≠ 0 (with magnetic field)

The solution of equation (12) satisfying the condition (14) is

$$u = A\cosh My + B\sinh My + \frac{W_e a_0 e^{-\alpha y}}{(\alpha^2 - M^2)} - \frac{P}{M^2} \quad (23)$$

where $a_0 = \frac{W_e x_0 \alpha^2}{e^{-\alpha}-1}$; $A = \frac{P}{M^2} - \frac{W_e a_0 e^{-\alpha y}}{(\alpha^2 - M^2)}$;

$$B = \frac{1}{\text{Sinh}M}\left(\frac{P}{M^2}(1-\text{Cosh}M) - \frac{W_e a_0 e^{-\alpha y}}{(\alpha^2 - M^2)}(e^{-\alpha} - \text{Cosh}M)\right)$$

The average velocity is given by,

$$\bar{u} = \int_0^1 u\,dy = \frac{A\,\text{Sinh}M}{M} - \frac{B\,\text{Cosh}M - 1}{M} + \frac{W_e a_0}{\alpha(\alpha^2 - M^2)}(e^{-\alpha} - 1) - \frac{P}{M^2} \quad (24)$$

The concentration of aerosol C with chemical reaction K in the atmosphere can be written as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - KC \quad (25)$$

The longitudinal diffusion is very much less than the transverse diffusion which implies $\frac{\partial^2 C}{\partial x^2} << \frac{\partial^2 C}{\partial y^2}$

Equation (25) becomes

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial y^2} - KC \quad (26)$$

The dimensionless boundary conditions on concentrations are

$$\left.\begin{aligned} \frac{\partial C}{\partial y} = 0 \text{ at} \qquad & y = 0 \\ \text{and } C = 1 \text{ at} \qquad & y = 1 \end{aligned}\right\} \tag{27}$$

Now using the dimensionaless variables,

$$y^* = \frac{y}{h}; C^* = \frac{C}{C_0}; t^* = \frac{t}{\bar{t}}; \xi = \frac{x - \bar{u}t}{L}; \beta^2 = \frac{h^2}{D}K$$

where the characteristic length L is along the flow direction and β is the dimensionless reaction rate parameter. Equation (26) can be written as

$$\frac{1}{\bar{t}} \cdot \frac{\partial C^*}{\partial t^*} + \frac{(u - \bar{u})}{L} \cdot \frac{\partial C^*}{\partial \xi} = \frac{D}{h^2} \cdot \frac{\partial^2 C^*}{\partial y^{*2}} - \frac{\beta^2 D}{h^2} C$$

For simplicity, neglecting the asterisks (*) we get

$$\frac{1}{\bar{t}} \cdot \frac{\partial C}{\partial t} + \frac{V}{L} \cdot \frac{\partial C}{\partial \xi} = \frac{D}{h^2} \cdot \frac{\partial^2 C}{\partial y^2} - \frac{\beta^2 D}{h^2} C \tag{28}$$

where $V = u - \bar{u} = A\cosh My + B\sinh My + \dfrac{W_e a_0 e^{-\alpha y}}{(\alpha^2 - M^2)} + f$ (29)

where $f = -\dfrac{A\mathrm{Sinh}M}{M} - \dfrac{B\mathrm{Cosh}M - 1}{M} + \dfrac{W_e a_0}{\alpha(\alpha^2 - M^2)}(e^{-\alpha} - 1)$

By approximating equation (28) we obtain C as the variation of y,

$$\frac{\partial^2 C}{\partial y^2} - \beta^2 C = QV \tag{30}$$

where $Q = \dfrac{h^2}{DL} \dfrac{\partial C}{\partial \xi}$ and β the reaction rate parameter.

Using the equation (29) and satisfying the boundary condition (27), the solution of equation (30) we get

$$C = Q\,(C1\ \mathrm{Cosh}\beta y + C2\mathrm{Sinh}\beta y + \frac{A\mathrm{Cosh}My}{M^2 - \beta^2} + \frac{B\mathrm{Sinh}My}{M^2 - \beta^2} + \frac{W_e a_0}{(\alpha^2 - M^2)(\alpha^2 - \beta^2)} e^{-\alpha y} - \frac{f}{\beta^2}) \tag{31}$$

where

$C1 = \dfrac{1}{\mathrm{Cosh}\beta}(1 - \dfrac{B}{M^2 - \beta^2}(\dfrac{-M\mathrm{Sinh}\beta}{\beta} + \mathrm{Sinh}M) - \dfrac{W_e a_0}{(\alpha^2 - M^2)(\alpha^2 - \beta^2)}(\dfrac{\alpha\mathrm{Sinh}\beta}{\beta} + e^{-\alpha} + \dfrac{f}{\beta^2});$

$$C2 = \frac{1}{\beta}\left(-\frac{BM}{M^2-\beta^2}+\frac{\alpha W_e a_0}{(\alpha^2-M^2)(\alpha^2-\beta^2)}\right);$$

Where C is the concentration of aerosols in the presence of chemical reaction. The fluid is transfered across the section of layer per unit breadth, then the volumetric rate of the fluid is given by,

$$M = h\int_0^1 CVdy \tag{32}$$

Using equations (31) and (29), performing the integration and after simplification we get,

$$M = \frac{h^3}{DL} G \frac{\partial C}{\partial \xi} \tag{33}$$

Also we make the assumption that the variation of C with ξ is small compared with those in the longitudinal direction and if C_m is the mean concentration over a section, then $\frac{\partial C}{\partial \xi}$ is indistinguishable from $\frac{\partial C_m}{\partial \xi}$ (Taylor [6]) so (33) can be written as

$$M = \frac{h^3}{DL} G \frac{\partial C_m}{\partial \xi} \tag{34}$$

The fact that no material is lost in the process which is expressed by the continuity equation for C_m namely

$$\frac{\partial M}{\partial \xi} = -\frac{2}{L}\frac{\partial C_m}{\partial t} \tag{35}$$

where $\frac{\partial}{\partial t}$ refers to differentiation with respect to time at point ,and ξ is constant.

Using (35),equation (34) becomes,

$$\frac{\partial C_m}{\partial t} = D^* \frac{\partial^2 C_m}{\partial \xi^2} \tag{36}$$

where $D^* = -\frac{h^3}{2D} G$ (36a)

Case 2: M= 0 (without magnetic field)

When M = 0, Following the same procedure explained in case1,from equation (12)

$$\frac{\partial C_m}{\partial t} = D1 * \frac{\partial^2 C_m}{\partial \xi^2} \tag{37}$$

where, $D1^* = \frac{h^3}{2D} G_0$ (37a)

which is the equation of the longitudinal dispersion. From equations (36a) and (37a), the lengthy expression of scattering coefficients D^* and $D1^*$ are computed and the results obtained from the study are discussed in the next section.

4. RESULTS AND DISCUSSIONS

Tthe analytical results reported in the previous section is performed using MATHEMATICA 8.0 and a representative set of results are depicted graphically. The scattering coeficient given in equation (36a) and (37a) is calculated for many values of electric number and reaction rate for both in the presence and absence of Hartmann number which are graphically represented in figures 2 to 5. Figures 2 and 3 represent the dispersion coefficient D^* with reaction rate β for some values of electric number We and scattering coefficient D^* with electric number We for different values of reaction rate β . It is observed that D^* decreases with an increase in reaction rate β in the presence of Hartmann number M and D^* increases with increase in electric number We. Similarly in the absence of Hartmann number the scattering coefficient against reaction rate β and versus electric number We are represented through figures 4 and 5. It is observed that the scattering coefficient D^* increases with an increase in electric number We and decreases with increases in reaction rate β and Hartmann number M.

5. CONCLUSIONS

From the figures, it is concluded that for both with and without magnetic field the presence of electric field increases the scattering of dust particles and the reaction rate reduces the transport of particles. The mathematical model presented here is the representative of distribution of ultrafine dust particles in the atmosphere and it suggests that the scattering of particles would depend upon the chemical reaction, electric field, magnetic field and other parameters in the atmosphere.

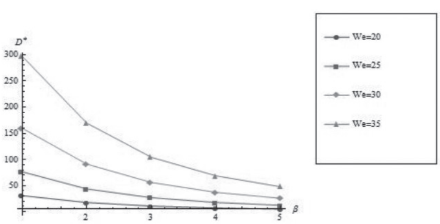

Figure 2. Scattering Coefficient D* Versus Reaction Rate β For Different Values Of Electric Number We With M=0.2

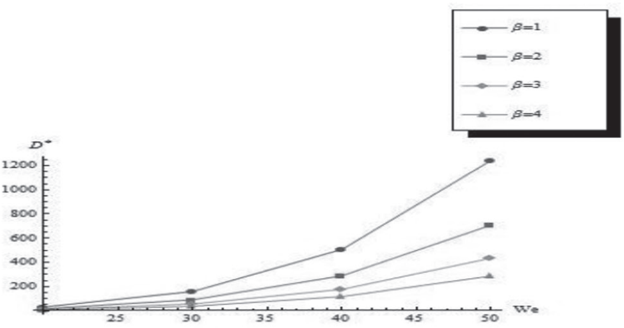

Figure 3. Scattering coefficient D* versus electric number fordifferent value of β with M=0.2

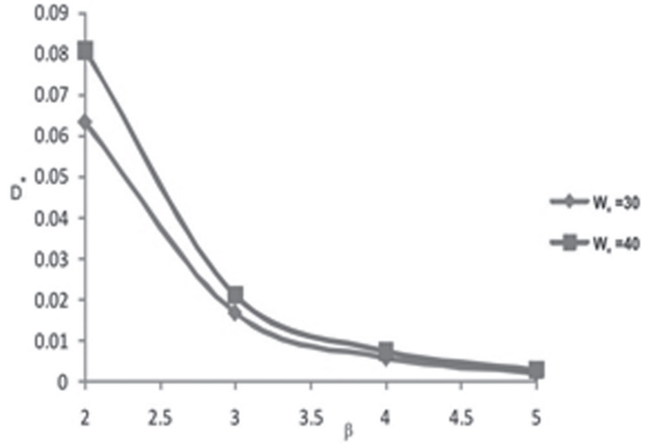

Figure 4. Scattering coefficient D1* versus reaction rate β for different values of electric number We with M=0

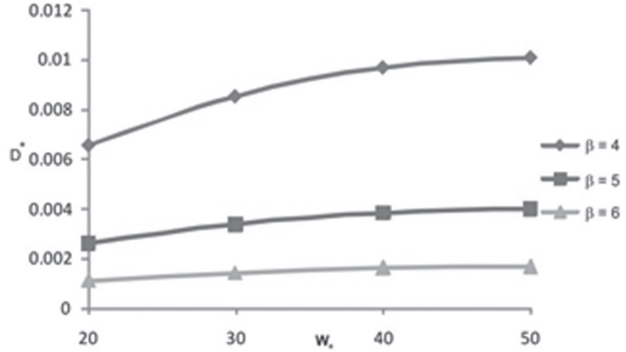

Figure 5. Scattering coefficient D1* versus electric number We for different values reaction rate β with M=0

REFERENCES

[1] Golokhvast K.S., Manakov Yu.A., Bykov A.A., Chayka V.V., Nikiforov.P.A., Rogulin.S., Romanova T.Yu., Karabstov A.A., Semenikhin V.A.,(2005),Some characteristics of dust particles in atmosphere of Kemerovo city according to pollution data of snow cover, IOP Conf. Series: Earth and environmental science, 87.

[2] Haijiao Liu., Yan Yan., Hong Chang., Hongyun Chen., Lianji Liang., Xingxing Liu., Xiaoke Qiang., Youbin Sun.,(2018), Magnetic signatures of natural and anthropogenic sources of urban dust particles, Atmospheric chemistry and physics, 452.

[3] John Aitken (1889), On the number of dust particles in the atmosphere, Earth and Envi. sci. transactions of the Royal society of Edinburgh. 35, 1-19.

[4] Meenapriya P. ,Nirmala P. Ratchagar (2012). EHD dispersion of atmospheric aerosols through a vertical channel with temperature distributions. World Journal of Engineering. 9(4) pp. 293-306.

[5] Sunil, Divya sharma, and Sharma.R.C., (2005), The effect of a magnetic field dependent viscosity on the thermal convection in a ferromagnetic fluid in a porous medium. Journal of eophysics and engineering , 277-286.

[6] Taylor, G.I. Dispersion of soluble matter in solvent flowing slowly through a tube. Proc. Roy. Soc. London. A. 219 (1953), 186.

[7] Ulanowski Z., Bailey J., Lucas P.W., Hough J.H., Hirst E.,(2007), Alignment of atmospheric mineral dust due to electric field. Atmospheric chemistry and physics, 7, 6161-6173.

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