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# Analysis, Adaptive Control and Synchronization of a Seven-Term Novel 3-D Chaotic System

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**ABSTRACT**

First, this paper introduces a seven-term novel 3-D chaotic system and discusses its qualitative properties. The proposed system is a seven-term novel polynomial chaotic system with three quadratic nonlinearities. The Lyapunov exponents of the novel chaotic system are obtained as $L_1 = 3.3226$, $L_2 = 0$ and $L_3 = -30.3406$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as $L_1 = 3.3226$ and Lyapunov dimension as $D_L = 2.1095$. Next, we derive new results for the adaptive control design of the novel chaotic system with unknown parameters. Next, we derive new results for the adaptive synchronization design of the identical novel chaotic systems with unknown parameters. The adaptive control and synchronization results have been established using adaptive control theory and Lyapunov stability theory. Numerical simulations with MATLAB have been shown to validate and illustrate all the new results derived in this paper.

***Keywords:*** Chaos, chaotic systems, novel chaotic system, adaptive control, adaptive synchroization.

## 1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions. The sensitivity of a nonlinear chaotic system in response to small changes in the initial conditions is commonly called as *butterfly effect* [1] and this is one of the characterizing features of a chaotic system.

The Lyapunov exponent of a dynamical system is a quantitative measure that characterizes the rate of separation of infinitesimally close trajectories of the system. Thus, a chaotic system is also defined as a dynamical system having at least one positive Lyapunov exponent.

In the last four decades, many chaotic systems have been found in the literature using modelling and other techniques. In 1963, Lorenz modelled a 3-D chaotic weather system and experimentally verified that a very small difference in the initial conditions resulted in very large changes in his deterministic weather model [2]. In the later years, many important chaotic systems were discovered such as Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], Sundarapandian-Pehlivan system [12], etc.

Chaos control and chaos synchronization are important research problems in the chaos literature, which have been studied extensively in the last four decades. Chaos theory finds applications in a variety of fields such as lasers [13-14], oscillators [15-16], chemical reactions [17-18], biology [19-20], neural networks [21-23], ecology [24-25], robotics [26-28], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [29-30]. Some popular methods for chaos control are active control [31-32], adaptive control [33-34], sliding mode control [35], etc.

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Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Because of the butterfly effect which results in the exponential divergence of two identical chaotic systems with nearly the same initial conditions, the synchronization of two identical chaotic systems or different chaotic systems is a challenging problem in the chaos literature. The synchronization of chaotic systems has applications in secure communications [36-39], cryptosystems [40-41], encryption [42-43] etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [44-45], many important methods have been derived for the chaos synchronization problem such as active control method [46-50], adaptive control method [51-56], sampled-data feedback control method [57-58], time-delay feedback approach [59], backstepping method [60-65], sliding mode control method [66-69], etc.

In this paper, we have proposed a seven-term novel 3-D chaotic system with three quadratic nonlinearities. We have obtained the Lyapunov exponents of the novel chaotic system as $L_1 = 3.3226$, $L_2 = 0$ and $L_3 = -30.3406$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is found as $L_1 = 3.3226$ and Lyapunov dimension as $D_L = 2.1095$. We have also derived new results for the adaptive control of the novel chaotic system and adaptive synchronization of identical novel chaotic systems with unknown parameters. The main adaptive results of this paper are proved using adaptive control theory and Lyapunov stability theory.

The rest of this paper is organized as follows. Section 2 describes the equations and phase portraits of the seven-term novel 3-D chaotic system with three quadratic nonlinearities. Section 3 describes the qualitative properties of the novel 3-D chaotic system including Lyapunov exponents and Lyapunov dimension of the system. Section 4 describes new results for the design of adaptive control to stabilize the novel chaotic system with unknown system parameters. Section 5 describes new results for the design of adaptive synchronization of the identical novel chaotic systems with unknown system parameters. The main results in Sections 3 and 4 have been established using adaptive control theory and Lyapunov stability theory. MATLAB simulations have been provided to illustrate all the main results obtained in this paper. Section 5 contains a summary of the main results derived in this paper.

## 2. A NOVEL SEVEN-TERM CHAOTIC SYSTEM

In this section, we describe the equations and properties of a novel seven-term 3-D chaotic system with three quadratic nonlinearities.

Our seven-term novel chaotic system is modelled by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= b x_1 + c x_1 x_3 \\ \dot{x}_3 &= -d x_3 - x_1 x_2 - x_1^2 \end{aligned} \tag{1}$$

where $x_1$, $x_2$, $x_3$ are the state variables and $a$, $b$, $c$, $d$ are constant, positive parameters of the system.

The system (1) exhibits a chaotic attractor for the values

$$a = 22, \ \ b = 400, \ \ c = 50, \ \ d = 0.5 \tag{2}$$

Figure 1 shows the strange chaotic attractor of the system (1). Figures 2-4 show the 2-D view of the chaotic attractor of the system (1) in $(x_1, x_2)$, $(x_2, x_3)$ and $(x_1, x_3)$ planes respectively.

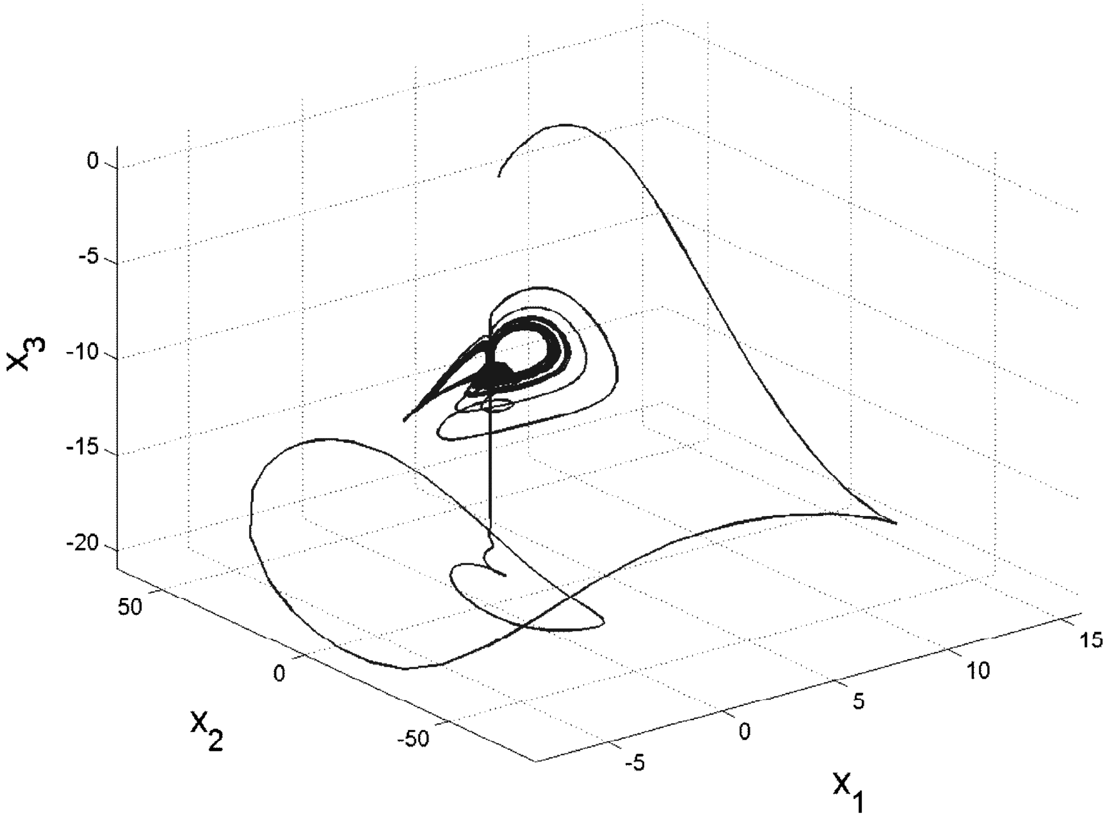


Figure 1: Strange Attractor of the Novel Chaotic System

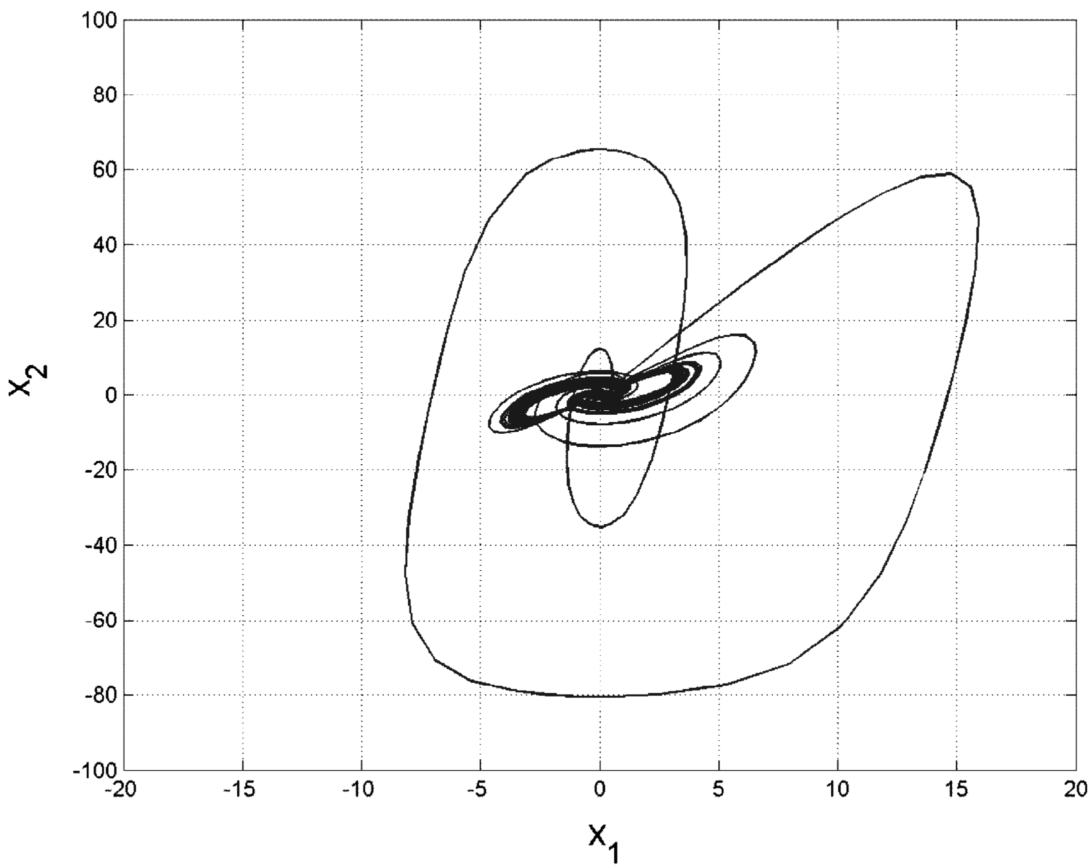


Figure 2: 2-D View of the Novel Chaotic System in $(x_1, x_2)$ Plane

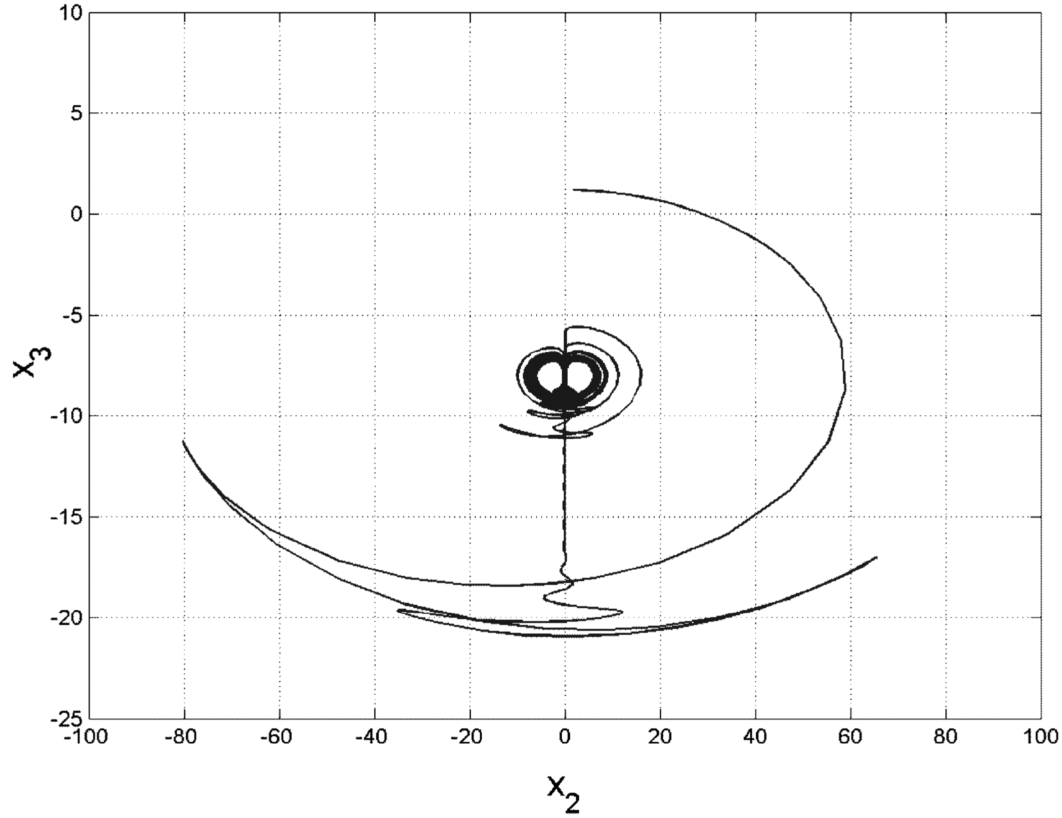


**Figure 3: 2-D View of the Novel Chaotic System in ($x_2$, $x_3$) Plane**

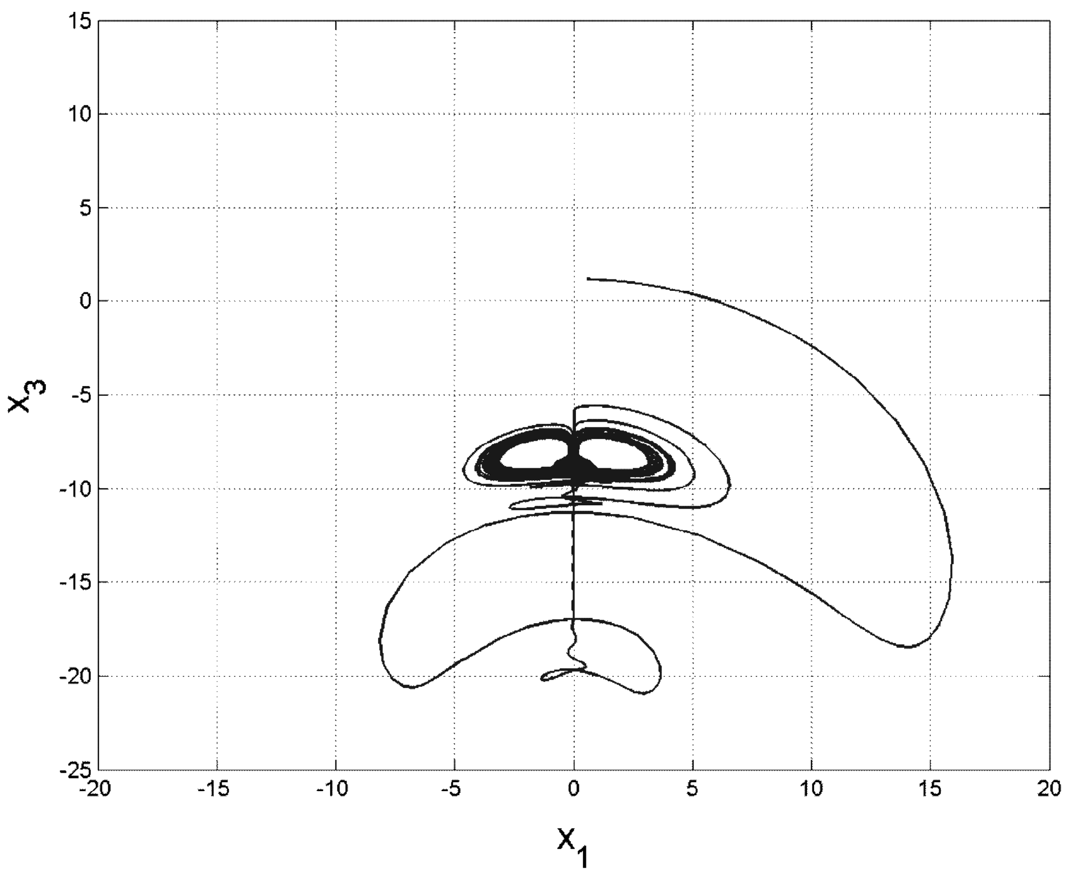


**Figure 4: 2-D View of the Novel Chaotic System in ($x_1$, $x_3$) Plane**

## 3. PROPERTIES OF THE NOVEL CHAOTIC SYSTEM

### (A) Symmetry

The novel 3-D chaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \to (-x_1, -x_2, x_3) \tag{3}$$

Since the transformation (3) persists for all values of the system parameters, the novel chaotic system (1) has rotation symmetry about the $x_3$–axis and that any non-trivial trajectory must have a twin trajectory.

### (B) Invariance

The $x_3$–axis ($x_1 = 0$, $x_2 = 0$) is invariant for the system (1). Hence, all orbits of the system (1) starting on the $x_3$– axis stay in the $x_3$– axis for all values of time.

### (C) Dissipativity

We write the system (1) in vector notation as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} \tag{4}$$

where

$$\begin{aligned} f_1(x) &= a(x_2 - x_1) + x_2 x_3 \\ f_2(x) &= b x_1 + c x_1 x_3 \\ f_3(x) &= -d x_3 - x_1 x_2 - x_1^2 \end{aligned} \tag{5}$$

The divergence of the vector field $f$ on $R^3$ is obtained as

$$\text{div}\, f = \frac{\partial f_1(x)}{\partial x_1} + \frac{\partial f_2(x)}{\partial x_2} + \frac{\partial f_3(x)}{\partial x_3} = -(a+d) < 0 \tag{6}$$

since $a$ and $d$ are positive constants.

Let $\Omega$ be any region in $R^3$ having a smooth boundary.

Let $\Omega(t) = \Phi_t(\Omega)$, where $\Phi_t$ is the flow of $f$. Let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, it follows that

$$\frac{dV(t)}{dt} = \iiint_{\Omega(t)} (\text{div}\, f)\; dx_1\; dx_2\; dx_3 \tag{7}$$

By substituting the value of **div** $f$ from (6) into (7), we obtain

$$\frac{dV(t)}{dt} = -(a+d) \iiint_{\Omega(t)} dx_1\; dx_2\; dx_3 = -(a+d)V(t) \tag{8}$$

Integrating the linear differential equation (8), we get the solution as

$$V(t) = V(0)\exp(-(a+d)t) = V(0)\exp(-\mu t) \tag{9}$$

where $\mu = a + d > 0$.

From Eq. (9), it follows that the volume $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Thus, the novel chaotic system (1) is dissipative. Hence, the asymptotic motion of the system (1) settles exponentially onto a set of measure zero, *i.e.* a strange attractor.

### (D) Equilibrium Points

The equilibrium points of the novel chaotic system (1) are obtained by solving the nonlinear equations

$$\begin{aligned} f_1(x) &= a(x_2 - x_1) + x_2 x_3 = 0 \\ f_2(x) &= bx_1 + cx_1 x_3 = 0 \\ f_3(x) &= -dx_3 - x_1 x_2 - x_1^2 = 0 \end{aligned} \tag{10}$$

We take the parameter values as in the chaotic case, *viz.*

$$a = 22, \; b = 400, \; c = 50, \; d = 0.5 \tag{11}$$

Solving the equations (10) using the values (11), we obtain three equilibrium points of the novel chaotic system (1) as

$$\begin{aligned} &E_0 : (0,0,0) \\ &E_1 : (1.2472, 1.9599, -8) \\ &E_2 : (-1.2472, -1.9599, -8) \end{aligned} \tag{12}$$

The Jacobian matrix of the novel chaotic system (1) is obtained as

$$J(x) = \begin{bmatrix} -a & a + x_3 & x_2 \\ b + cx_3 & 0 & cx_1 \\ -x_2 - 2x_1 & -x_1 & -d \end{bmatrix} \tag{13}$$

The Jacobian matrix at the equilibrium $E_0$ is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} -22 & 22 & 0 \\ 400 & 0 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}, \tag{14}$$

which has the eigenvalues

$$\lambda_1 = -0.5, \;\; \lambda_2 = -105.451, \;\; \lambda_3 = 83.451 \tag{15}$$

This shows that the equilibrium $E_0$ is a saddle-point, which is unstable.

The Jacobian matrix at the equilibrium $E_1$ is obtained as

$$J_1 = J(E_1) = \begin{bmatrix} -22 & 14 & 1.9599 \\ 0 & 0 & 62.36 \\ -4.4543 & -1.2472 & -0.5 \end{bmatrix}, \tag{16}$$

which has the eigenvalues

$$\lambda_1 = -26.7022, \; \lambda_2 = 2.1011 + 14.3283i, \; \lambda_3 = 2.1011 - 14.3283i \tag{17}$$

This shows that the equilibrium $E_1$ is a saddle-focus, which is unstable.

The Jacobian matrix at the equilibrium $E_2$ is obtained as

$$J_2 = J(E_2) = \begin{bmatrix} -22 & 14 & -1.9599 \\ 0 & 0 & -62.36 \\ 4.4543 & 1.2472 & -0.5 \end{bmatrix}, \tag{18}$$

which has the eigenvalues

$$\lambda_1 = -26.7022,\ \lambda_2 = 2.1011 + 14.3283i,\ \lambda_3 = 2.1011 - 14.3283i \tag{19}$$

This shows that the equilibrium $E_2$ is a saddle-focus, which is unstable.

Hence, $E_0$, $E_1$, $E_2$ are all unstable equilibrium points, where $E_0$ is a saddle point and $E_1$, $E_2$ are saddle-focus points.

### (E) Lyapunov Exponents

We take the parameter values of the system (1) as

$$a = 22,\ b = 400,\ c = 50,\ d = 0.5 \tag{20}$$

We take the initial state as

$$x_1(0) = 0.6,\ \ x_2(0) = 1.8,\ \ x_3(0) = 1.2 \tag{21}$$

The Lyapunov exponents of the system (1) are numerically obtained with MATLAB as

$$L_1 = 3.3226,\ \ L_2 = 0,\ \ L_3 = -30.3406 \tag{22}$$

Eq. (22) shows that the system (1) is chaotic, since it has a positive Lyapunov exponent. Since the sum of the Lyapunov exponents is negative, the system (1) is a dissipative chaotic system.

Also, the maximal Lyapunov exponent (MLE) of the system (1) is obtained as $L_1$ = 3.3226.

The dynamics of the Lyapunov exponents is depicted in Figure 5.

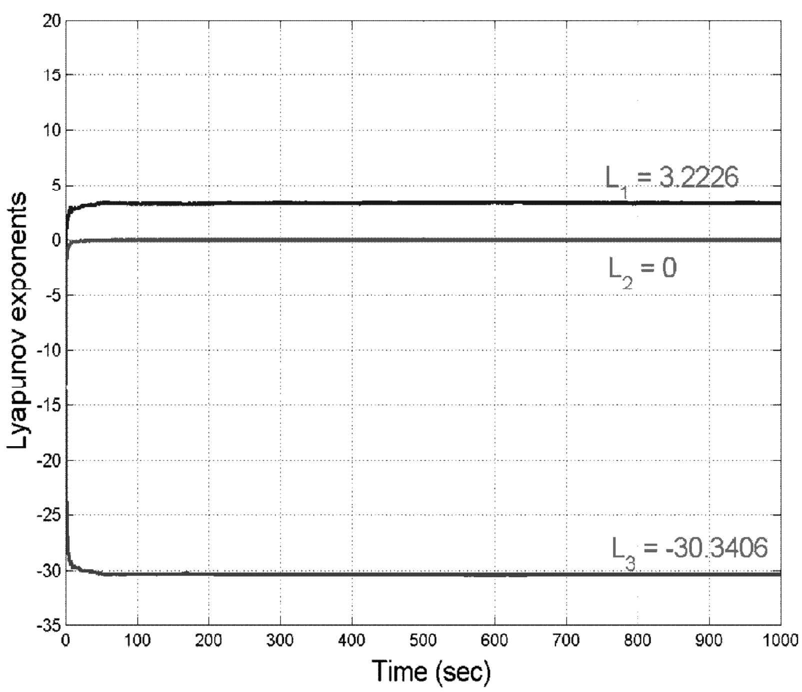


**Figure 5: Dynamics of the Lyapunov Exponents**

### (F) Lyapunov Dimension

The Lyapunov dimension of the chaotic system (1) is determined as

$$D_L = j + \frac{\sum_{i=1}^{j} L_i}{|L_{j+1}|} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.1095, \tag{23}$$

which is fractional. Thus, the seven-term 3-D system (1) is a dissipative chaotic system with fractional Lyapunov dimension.

## 4. ADAPTIVE CONTROL OF THE NOVEL CHAOTIC SYSTEM

In this section, we derive new results for the adaptive controller to stabilize the unstable novel chaotic system with unknown parameters for all initial conditions.

Thus, we consider the controlled novel 3-D chaotic system

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 + u_1 \\ \dot{x}_2 &= b x_1 + c x_1 x_3 + u_2 \\ \dot{x}_3 &= -d x_3 - x_1 x_2 - x_1^2 + u_3 \end{aligned} \tag{24}$$

where $x_1, x_2, x_3$ are state variables, $a, b, c, d$ are constant, unknown, parameters of the system and $u_1, u_2, u_3$ are adaptive controls to be designed.

We aim to solve the adaptive control problem by considering the adaptive feedback control law

$$\begin{aligned} u_1(t) &= -A(t)(x_2 - x_1) - x_2 x_3 - k_1 x_1 \\ u_2(t) &= -B(t) x_1 - C(t) x_1 x_3 - k_2 x_2 \\ u_3(t) &= D(t) x_3 + x_1 x_2 + x_1^2 - k_3 x_3 \end{aligned} \tag{25}$$

where $A(t), B(t), C(t), D(t)$ are estimates for the unknown parameters $a, b, c, d$ respectively, and $k_1, k_2, k_3$ are positive gain constants.

The closed-loop system is obtained by substituting (25) into (24) as

$$\begin{aligned} \dot{x}_1 &= (a - A(t))(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= (b - B(t)) x_1 + (c - C(t)) x_1 x_3 - k_2 x_2 \\ \dot{x}_3 &= -(d - D(t)) x_3 - k_3 x_3 \end{aligned} \tag{26}$$

To simplify (26), we define the parameter estimation error as

$$\begin{aligned} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \\ e_c(t) &= c - C(t) \\ e_d(t) &= d - D(t) \end{aligned} \tag{27}$$

Substituting (27) into (26), we obtain

$$\begin{aligned} \dot{x}_1 &= e_a(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= e_b x_1 + e_c x_1 x_3 - k_2 x_2 \\ \dot{x}_3 &= -e_d x_3 - k_3 x_3 \end{aligned} \tag{28}$$

Differentiating the parameter estimation error (27) with respect to $t$, we get

$$\begin{aligned}\dot{e}_a(t) &= -\dot{A}(t) \\ \dot{e}_b(t) &= -\dot{B}(t) \\ \dot{e}_c(t) &= -\dot{C}(t) \\ \dot{e}_d(t) &= -\dot{D}(t)\end{aligned} \tag{29}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(x_1, x_2, x_3, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{30}$$

which is positive definite on $R^7$.

Differentiating $V$ along the trajectories of (28) and (29), we obtain

$$\begin{aligned}\dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{A}\right] + e_b\left[x_1x_2 - \dot{B}\right] \\ + e_c\left[x_1x_2x_3 - \dot{C}\right] + e_d\left[-x_3^2 - \dot{D}\right]\end{aligned} \tag{31}$$

In view of (31), we define an update law for the parameter estimates as

$$\begin{aligned}\dot{A} &= x_1(x_2 - x_1) + k_4e_a \\ \dot{B} &= x_1x_2 + k_5e_b \\ \dot{C} &= x_1x_2x_3 + k_6e_c \\ \dot{D} &= -x_3^2 + k_7e_d\end{aligned} \tag{32}$$

where $k_4$, $k_5$, $k_6$, $k_7$ are positive gain constants.

**Theorem 1**. The novel chaotic system (24) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (25) and the parameter update law (32), where $k_i$, $(i = 1, 2, \ldots, 7)$ are positive constants. All the parameter estimation errors $e_a$, $e_b$, $e_c$, $e_d$ globally and exponentially converge to zero with time.

*Proof*. The result is proved using Lyapunov stability theory [70]. We consider the quadratic Lyapunov function $V$ defined by (30), which is a positive definite function on $R^7$.

Substituting the parameter update law (32) into (31), we obtain $\dot{V}$ as

$$\dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 - k_4e_a^2 - k_5e_b^2 - k_6e_c^2 - k_7e_d^2 \tag{33}$$

which is a negative definite function on $R^7$.

Thus, by Lyapunov stability theory [70], all the states $x_1$, $x_2$, $x_3$ and the parameter estimation errors $e_a$, $e_b$, $e_c$, $e_d$ globally and exponentially converge to zero with time.

## NUMERICAL RESULTS

For the novel chaotic system (24), the parameter values are taken as in the chaotic case, *viz*.

$$a = 22, \quad b = 400, \quad c = 50, \quad d = 0.5 \tag{34}$$

We take the feedback gains as $k_i = 5$ for $i = 1, 2, \ldots, 7$.

The initial values of the chaotic system (24) are taken as

$$x_1(0) = 3.4,\ x_2(0) = 2.7,\ x_3(0) = -4.2 \tag{35}$$

The initial values of the parameter estimates are taken as

$$A(0) = 6,\ B(0) = 17,\ C(0) = 24,\ \ D(0) = 10 \tag{36}$$

Figure 6 depicts the time-history of the controlled novel chaotic system.

Figure 7 depicts the time-history of the parameter estimation errors.

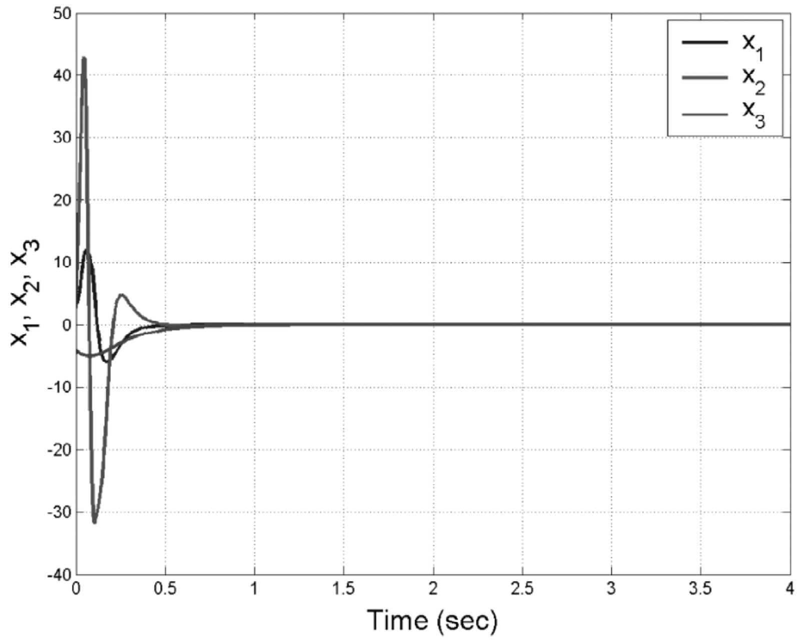


**Figure 6: Time-History of the Controlled Novel Chaotic System**

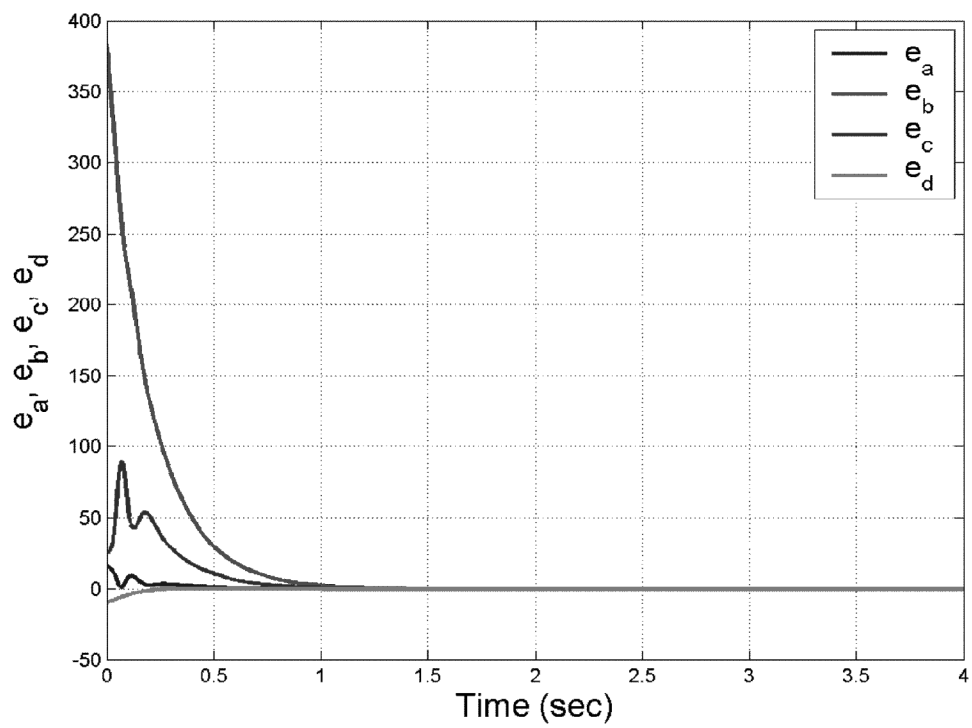


**Figure 7: Time History of the Parameter Estimation Errors $e_a$, $e_b$, $e_c$, $e_d$**

## 5. ADAPTIVE SYNCHRONIZATION OF THE IDENTICAL NOVEL CHAOTIC SYSTEMS

In this section, we derive new results for the adaptive synchronization design of the identical novel chaotic systems with unknown parameters.

As the master system, we take the novel 3-D chaotic system

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= b x_1 + c x_1 x_3 \\ \dot{x}_3 &= -d x_3 - x_1 x_2 - x_1^2 \end{aligned} \tag{37}$$

where $x_1, x_2, x_3$ are state variables and $a, b, c, d$ are constant, unknown, parameters of the system.

As the slave system, we take the controlled novel 3-D chaotic system

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= b y_1 + c y_1 y_3 + u_2 \\ \dot{y}_3 &= -d y_3 - y_1 y_2 - y_1^2 + u_3 \end{aligned} \tag{38}$$

where $y_1, y_2, y_3$ are state variables and $u_1, u_2, u_3$ are adaptive controllers to be designed.

The synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{39}$$

The error dynamics is obtained using (37) and (38) as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 &= b e_1 + c(y_1 y_3 - x_1 x_3) + u_2 \\ \dot{e}_3 &= -d e_3 - y_1 y_2 + x_1 x_2 - y_1^2 + x_1^2 + u_3 \end{aligned} \tag{40}$$

We consider the adaptive control law defined by

$$\begin{aligned} u_1 &= -A(t)(e_2 - e_1) - y_2 y_3 + x_2 x_3 - k_1 e_1 \\ u_2 &= -B(t) e_1 - C(t)(y_1 y_3 - x_1 x_3) - k_2 e_2 \\ u_3 &= D(t) e_3 + y_1 y_2 - x_1 x_2 + y_1^2 - x_1^2 - k_3 e_3 \end{aligned} \tag{41}$$

where $k_1, k_2, k_3$ are positive gain constants.

Substituting (41) into (40), we get the closed-loop error dynamics as

$$\begin{aligned} \dot{e}_1 &= (a - A(t))(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= (b - B(t)) e_1 + (c - C(t))(y_1 y_3 - x_1 x_3) - k_2 e_2 \\ \dot{e}_3 &= -(d - D(t)) e_3 - k_3 e_3 \end{aligned} \tag{42}$$

To simplify the error dynamics (42), we define the parameter estimation error as

$$\begin{aligned} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \\ e_c(t) &= c - C(t) \\ e_d(t) &= d - D(t) \end{aligned} \tag{43}$$

Using (43), we can simplify the error dynamics (42) as

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e_b e_1 + e_c(y_1 y_3 - x_1 x_3) - k_2 e_2 \\ \dot{e}_3 &= -e_d e_3 - k_3 e_3 \end{aligned} \tag{44}$$

Differentiating the parameter estimation error (43) with respect to $t$, we get

$$\begin{aligned} \dot{e}_a(t) &= -\dot{A}(t) \\ \dot{e}_b(t) &= -\dot{B}(t) \\ \dot{e}_c(t) &= -\dot{C}(t) \\ \dot{e}_d(t) &= -\dot{D}(t) \end{aligned} \tag{45}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{46}$$

which is positive definite on $R^7$.

Differentiating $V$ along the trajectories of (44) and (45), we obtain

$$\begin{aligned} \dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 &+ e_a\left[e_1(e_2 - e_1) - \dot{A}\right] + e_b\left[e_1 e_2 - \dot{B}\right] \\ &+ e_c\left[e_2(y_1 y_3 - x_1 x_3) - \dot{C}\right] + e_d\left[-e_3^2 - \dot{D}\right] \end{aligned} \tag{47}$$

In view of (47), we define an update law for the parameter estimates as

$$\begin{aligned} \dot{A} &= e_1(e_2 - e_1) + k_4 e_a \\ \dot{B} &= e_1 e_2 + k_5 e_b \\ \dot{C} &= e_2(y_1 y_3 - x_1 x_3) + k_6 e_c \\ \dot{D} &= -e_3^2 + k_7 e_d \end{aligned} \tag{48}$$

where $k_4, k_5, k_6, k_7$ are positive gain constants.

**Theorem 2**. The identical novel chaotic systems (37) and (38) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (41) and the parameter update law (48), where $k_i$, (i = 1, 2,..., 7) are positive constants. All the parameter estimation errors $e_a$, $e_b$, $e_c$, $e_d$ globally and exponentially converge to zero with time.

*Proof*. The result is proved using Lyapunov stability theory [70]. We consider the quadratic Lyapunov function $V$ defined by (46), which is a positive definite function on $R^7$.

Substituting the parameter update law (48) into (47), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 - k_7 e_d^2 \tag{49}$$

which is a negative definite function on $R^7$.

Thus, by Lyapunov stability theory [70], all the synchronization errors $e_1$, $e_2$, $e_3$ and the parameter estimation errors $e_a$, $e_b$, $e_c$, $e_d$ globally and exponentially converge to zero with $t$.

## NUMERICAL RESULTS

For the novel chaotic systems, the parameter values are taken as in the chaotic case, *viz.*

$$a = 22, \quad b = 400, \quad c = 50, \quad d = 0.5 \tag{50}$$

We take the feedback gains as $k_i = 5$ for $i = 1, 2, ..., 7$.

The initial values of the master system (37) are taken as

$$x_1(0) = 4.7, \; x_2(0) = 1.5, \; x_3(0) = -2.8 \tag{51}$$

The initial values of the slate system (38) are taken as

$$y_1(0) = 3.2, \; y_2(0) = 6.8, \; y_3(0) = 1.4$$

The initial values of the parameter estimates are taken as

$$A(0) = 4, \; B(0) = 12, \; C(0) = 16, \; D(0) = 21 \tag{52}$$

Figure 8 depicts the complete synchronization of the identical novel chaotic systems.

Figure 9 depicts the time-history of the synchronization errors. Figure 10 depicts the time-history of the parameter estimation errors.

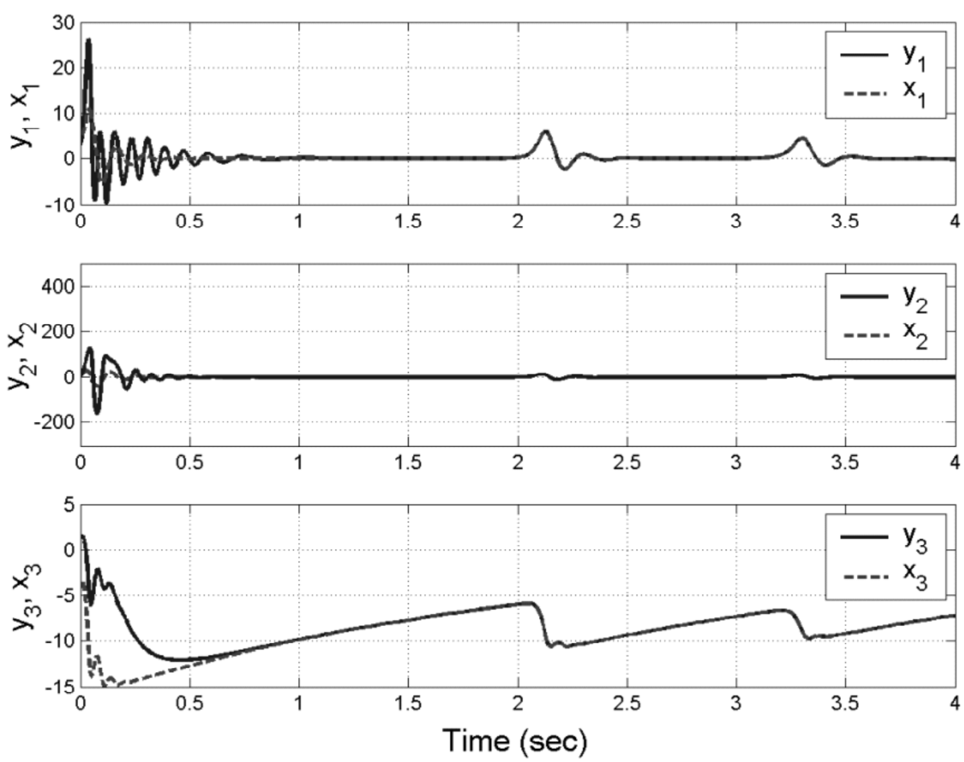


**Figure 8: Complete Synchronization of the Novel Chaotic Systems**

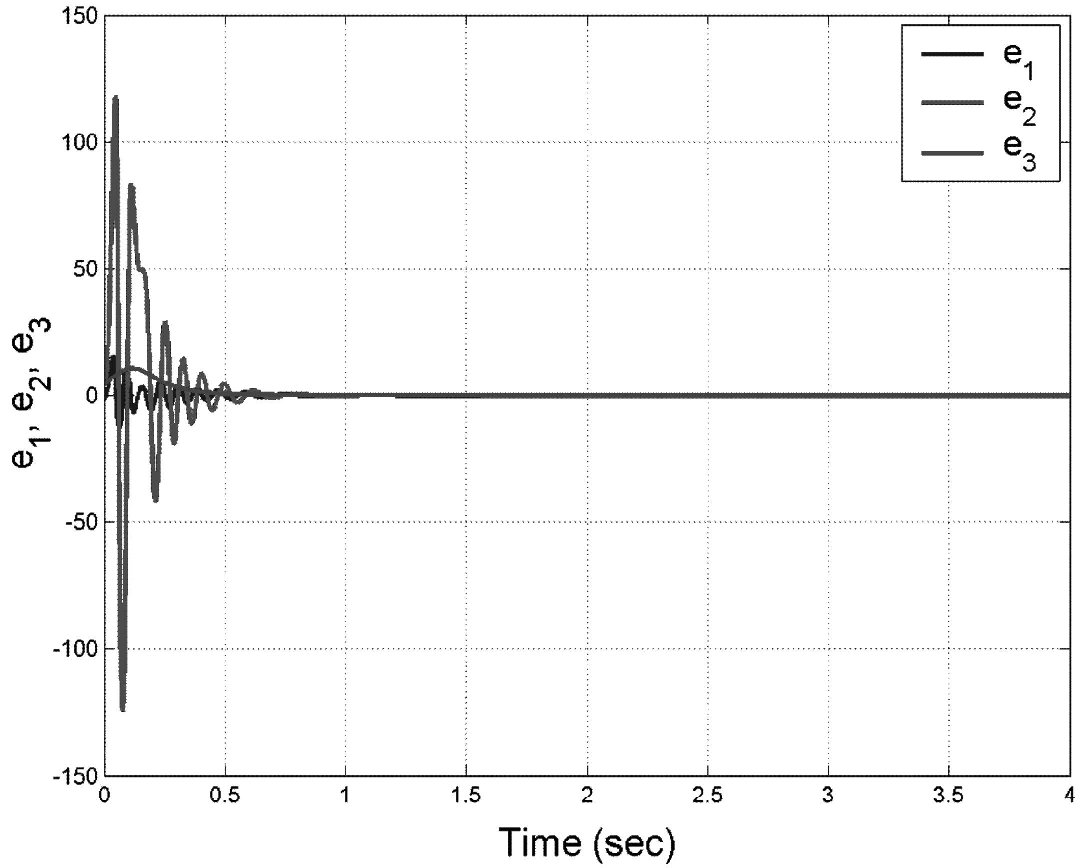


**Figure 9: Time History of the Chaos Synchronization Errors $e_1$, $e_2$, $e_3$**

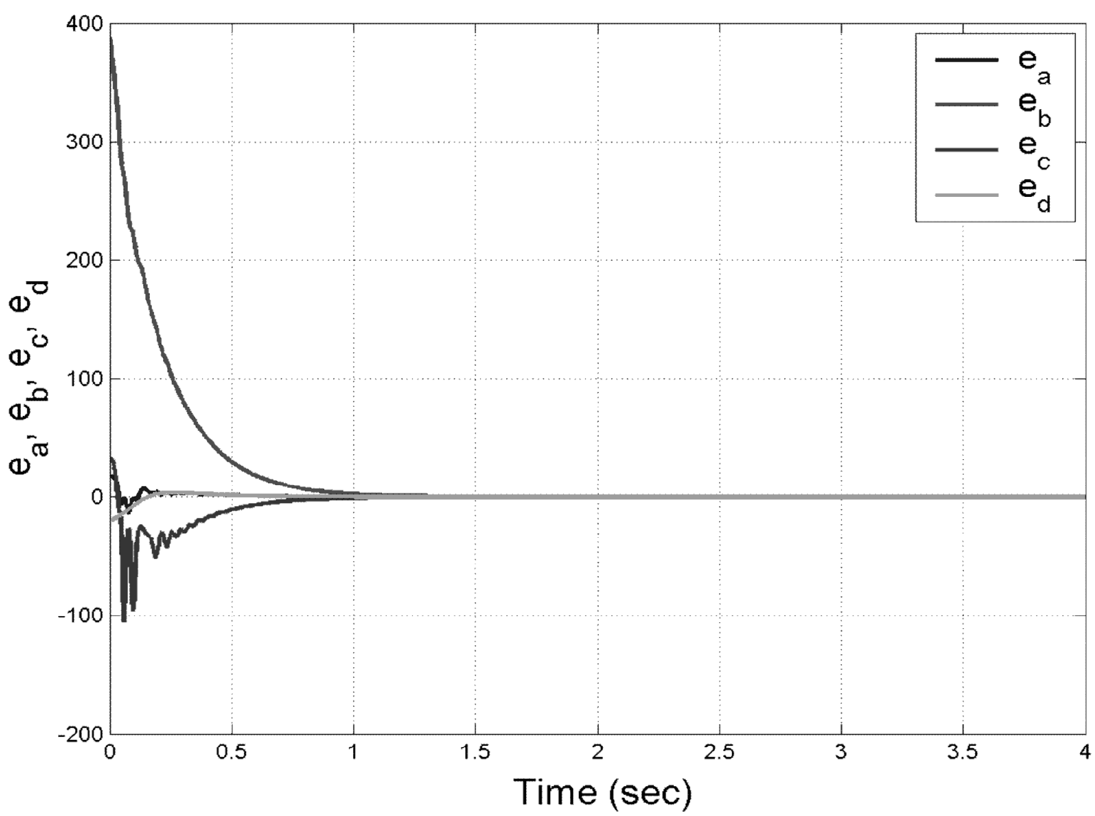


**Figure 10: Time History of the Parameter Estimation Errors $e_a$, $e_b$, $e_c$, $e_d$**

## 6. CONCLUSIONS

In this paper, we have derived a seven-term novel 3-D chaotic system with three quadratic nonlinearities and detailed its qualitative properties. We determined the Lyapunov exponents of the novel chaotic system as $L_1 = 3.3226$, $L_2 = 0$ and $L_3 = -30.3406$. The maximal Lyapunov exponent (MLE) for the novel chaotic system was found as $L_1 = 3.3226$ and Lyapunov dimension as $D_L = 2.1095$. Next, we have derived adaptive control and synchronization results for the novel chaotic system with unknown parameters, which have been established using adaptive control theory and Lyapunov stability theory. Numerical simulations with MATLAB were exhibited to validate and illustrate the novel chaotic system and the adaptive results derived in this paper.

## REFERENCES


[1] K. T. Alligood, T. Sauer and J.A. Yorke, *Chaos: An Introduction to Dynamical Systems*, Springer-Verlag: New York, USA, 1997.

[2] E. N. Lorenz, "Deterministic nonperiodic flow," *Journal of Atmospheric Sciences*, **20**, 130-141, 1963.

[3] O.E. Rössler, "An equation for continuous chaos," *Physics Letters*, **57A**, 397-398, 1976.

[4] T. Shimizu and N. Morioka, "On the bifurcation of a symmetric limit cycle to an assymetric one in a simple model," *Physics Letters A*, **76**, 201-204, 1980.

[5] R. Shaw, "Strange attractors, chaotic behavior and information flow," *Z. Naturforsch*, **36A**, 80-112, 1981.

[6] G. Chen and T. Ueta, "Yet another chaotic oscillator," *International Journal of Bifurcation and Chaos*, **9**, 1465-1466, 1999.

[7] J. Lü and G. Chen, "A new chaotic attractor coined," *International Journal of Bifurcation and Chaos*, **12**, 659-661, 2002.

[8] H.K. Chen and C.I. Lee, "Anti-control of chaos in rigid body motion," *Chaos, Solitons and Fractals*, **21**, 957-965, 2004.

[9] G. Cai and Z. Tan, "Chaos synchronization of a new chaotic system via nonlinear control," *Journal of Uncertain Systems*, **1**, 235-240, 2007.

[10] G. Tigan and D. Opris, "Analyis of a 3D chaotic system," *Chaos, Solitons and Fractals*, **36**, 1315-1319, 2008.

[11] D. Li, "A three-scroll chaotic attractor," *Physics Letters A*, **372**, 387-393, 2008.

[12] V. Sundarapandian and I. Pehlivan, "Analysis, control, synchronization and circuit design of a novel chaotic system," *Mathematics and Computer Modelling*, **55**, 1904-1915, 2012.

[13] S. Xiang, W. Pan, L. Yan, B. Luo, N. Jiang and K. Wen, "Using polarization properties to enhance performance of chaos synchronization communication between vertical-cavity surface-emitting lasers", *Optics and Laser Technology*, **42**, 674-684, 2010.

[14] R.S. Fyath and A.A. Al-mfrji, "Investigation of chaos synchronization in photonic crystal lasers", *Optics and Laser Technology*, **44**, 1406-1419, 2012.

[15] M. Lakshmanan and K. Murali, *Chaos in Nonlinear Oscillators: Controlling and Synchronization*, World Scientific: Singapore, 1996.

[16] S.K. Han, C. Kerrer and Y. Kuramoto, "Dephasing and hursting in coupled neural oscillators," *Physical Review Letters*, **75**, 3190-3193, 1995.

[17] J.S. Lee and K.S. Chang, "Applications of chaos and fractals in process systems engineering", *Journal of Process Control*, **6**, 71-87, 1996.

[18] M. Villegas, F. Augustin, A. Gilg, A. Hmaidi and U. Wever, "Application of the polynomial chaos expansion to the simulation of chemical reactors with uncertainities", *Mathematics and Computers in Simulation*, **82**, 805-817, 2012.

[19] J. J. Jiang, Y. Zhang and C. McGilligan, "Chaos in voice, from modeling to measurement", *Journal of Voice*, 20, 2-17, 2006.

[20] E. Carlen, R. Chatelin, P. Degond and B. Wennberg, "Kinetic hierarchy and propagation of chaos in biological swarm models", *Physica D: Nonlinear Phenomena*, **260**, 90-111.

[21] K. Aihira, T. Takabe and M. Toyoda, "Chaotic neural networks", *Physics Letters A*, **144**, 333-340, 1990.

[22] I. Tsuda, "Dynamic link of memory – chaotic memory map in nonequilibrium neural networks", *Neural Networks*, **5**, 313-326, 1992.

[23] Q. Ke and B.J. Oommen, "Logistic neural networks: their chaotic and pattern recognition properties", *Neurocomputing*, **125**, 184-194, 2014.

[24] B. Blasius, A. Huppert and L. Stone, "Complex dynamics and phase synchronization in spatially extended ecological system," *Nature,* **399**, 354-359, 1999.

[25] I. Suárez, "Mastering chaos in ecology", *Ecological Modelling*, 117, 305-314, 1999.

[26] S. Lankalapalli and A. Ghosal, "Chaos in robot control equations," *Interntional Journal of Bifurcation and Chaos*, **7**, 707-720, 1997.

[27] Y. Nakamura and A. Sekiguchi, "The chaotic mobile robot," *IEEE Transactions on Robotics and Automation*, **17**, 898-904, 2001.

[28] M. Islam and K. Murase, "Chaotic dynamis of a behavior-based miniature mobile robot: effects of environment and control structure", *Neural Networks*, **18**, 123-144, 2005.

[29] E. Ott, C. Grebogi and J.A. Yorke, "Controlling chaos," *Physical Review Letters*, **64**, 1196-1199, 1990.

[30] J. Wang, T. Zhang and Y. Che, "Chaos control and synchronization of two neurons exposed to ELF external electric field," *Chaos, Solitons and Fractals*, **34**, 839-850, 2007.

[31] V. Sundarapandian, "Output regulation of the Tigan system," *International Journal on Computer Science and Engineering*, **3**, 2127-2135, 2011.

[32] V. Sundarapandian, "Output regulation of the Sprott-G chaotic system by state feedback control," *International Journal of Instrumentation and Control Systems*, **1**, 20-30, 2011.

[33] V. Sundarapandian, "Adaptive control and synchronization of uncertain Liu-Chen-Liu system," *International Journal of Computer Information Systems*, **3**, 1-6, 2011.

[34] V. Sundarapandian, "Adaptive control and synchronization of the Shaw chaotic system," *International Journal in Foundations of Computer Science and Technology*, **1**, 1-11, 2011.

[35] V. Sundarapandian, "Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system," *International Journal of Control Theory and Applications*, **5**, 15-20, 2012.

[36] M. Feki, "An adaptive chaos synchronization scheme applied to secure communication," *Chaos, Solitons and Fractals*, **18**, 141-148, 2003.

[37] L. Kocarev and U. Parlitz, "General approach for chaos synchronization with applications to communications," *Physical Review Letters,* **74**, 5028-5030, 1995.

[38] K. Murali and M. Lakshmanan, "Secure communication using a compound signal using sampled-data feedback," *Applied Math. Mech.,* **11**, 1309-1315, 2003.

[39] J. Yang and F. Zhu, "Synchronization for chaotic systems and chaos-based secure communications via both reduced-order and step-by-step sliding mode observers," *Communications in Nonlinear Science and Numerical Simulation*, **18**, 926-937, 2013.

[40] L. Kocarev, "Chaos-based cryptography: a brief overview," *IEEE Circuits and Systems*, **1**, 6-21, 2001.

[41] J.M. Amigo, J. Szczepanski and L. Kocarev, "A chaos-based approach to the design of cryptographically secure substitutions," *Physics Letters A*, **343**, 55-60, 2008.

[42] H. Gao, Y. Zhang, S. Liang and D. Li, "A new chaotic algorithm for image encryption," *Chaos, Solitons and Fractals*, **29**, 393-399, 2006.

[43] Y. Wang, K.W. Wang, X. Liao and G. Chen, "A new chaos-based fast image encryption," *Applied Soft Computing*, **11**, 514-522, 2011.

[44] L. M. Pecora and T. I. Carroll, "Synchronization in chaotic systems," Phys. Rev. Lett., 64, 821-824, 1990.

[45] L. M. Pecora and T. L. Carroll, "Synchronizing in chaotic circuits," IEEE Trans. Circ. Sys., 38, 453-456, 1991.

[46] L. Huang, R. Feng and M. Wang, "Synchronization of chaotic systems via nonlinear control," *Physics Letters A*, **320**, 271-275, 2004.

[47] H. K. Chen, "Global chaos synchronization of new chaotic systems via nonlinear control," *Chaos, Solitons and Fractals*, **23**, 1245-1251, 2005.

[48] V. Sundarapandian and R. Karthikeyan, "Global chaos synchronization of hyperchaotic Liu and hyperchaotic Lorenz systems by active nonlinear control*", International Journal of Control Theory and Applications*, **3**, 79-91, 2010.

[49] V. Sundarapandian, "Active controller design for the hybrid synchronization of hyperchaotic Zheng and hyperchaotic Yu systems," *International Journal of Soft Computing, Artificial Intelligence and Applications*, **2**, 27-37, 2013.

[50] V. Sundarapandian, "Active controller design for the hybrid synchronization of hyperchaotic Xu and hyperchaotic Li systems", *International Journal of Information Technology, Modeling and Computing*, **1**, 21-35, 2013.

[51] B. Samuel, "Adaptive synchronization between two different chaotic dynamical systems," *Adaptive Commun. Nonlinear Sci. Num. Simul.*, **12**, 976-985, 2007.

[52] J. H. Park, S. M. Lee and O. M. Kwon, "Adaptive synchronization of Genesio-Tesi system via a novel feedback control," *Physics Letters A*, **371**, 263-270, 2007.

[53] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control," *International Journal of System Signal Control and Engineering Applications*, **4**, 18-25, 2011.

[54] V. Sundarapandian and I. Pehlivan, "Analysis, control, synchronization and circuit design of a novel chaotic system," *Mathematical and Computer Modelling*, **55**, 1904-1915, 2012.

[55] V. Sundarapandian, "Analysis, control and synchronization of Shimizu-Morioka chaotic system," *International Journal in Foundations of Computer Science and Technology*, **2**, 29-42, 2012.

[56] V. Sundarapandian, "Adaptive control and synchronization design for the Lu-Xiao chaotic system," *Springer-Verlag Lecture Notes in Electrical Engineering*, **131**, 319-327, 2013.

[57] T. Yang and L.O. Chua, "Control of chaos using sampled-data feedback control," *International Journal of Bifurcation and Chaos*, **9**, 215-219, 1999.

[58] N. Li, Y. Zhang, J. Hu and Z. Nie, "Synchronization for general complex dynamical networks with sampled-data", *Neurocomputing*, **74**, 805-811, 2011.

[59] J.H. Park and O.M. Kwon, "A novel criterion for delayed feedback control of time-delay chaotic systems," *Chaos, Solitons and Fractals*, **17**, 709-716, 2003.

[60] X. Wu and J. Lü, "Parameter identification and backstepping control of uncertain Lü system," *Chaos, Solitons and Fractals*, **18**, 721-729, 2003.

[61] Y.G. Yu and S.C. Zhang, "Adaptive backstepping synchronization of uncertain chaotic systems," *Chaos, Solitons and Fractals*, **27**, 1369-1375, 2006.

[62] R. Suresh and V. Sundarapandian, "Global chaos synchronization of WINDMI and Coullet chaotic systems by backstepping control," *Far East Journal of Mathematical Sciences*, **67**, 265-287, 2012.

[63] R. Suresh and V. Sundarapandian, "Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping controller with recursive feedback," *Far East Journal of Mathematical Sciences*, **73**, 73-95, 2013.

[64] R. Suresh and V. Sundarapandian, "Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems via backstepping control with novel feedback," *Archives of Control Sciences*, **22**, 255-278, 2012.

[65] R. Suresh and V. Sundarapandian, "Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback," *Malaysian Journal of Mathematical Sciences*, **7**, 219-246, 2013.

[66] V. Sundarapandian, "Global chaos synchronization of Lorenz-Stenflo and Qi chaotic systems by sliding mode control," *International Journal of Control Theory and Applications*, **4**, 161-172, 2011.

[67] V. Sundarapandian and S. Sivaperumal, "Sliding controller design of hybrid synchronization of four-wing chaotic systems", *International Journal of Soft Computing*, **6**, 224-231, 2011.

[68] V. Sundarapandian and S. Sivaperumal, "Anti-synchronization of four-wing chaotic systems via sliding mode control", *International Journal of Automation and Computing*, **9**, 274-279, 2012.

[69] V. Sundarapandian, "Sliding controller design for the global chaos synchronization of hyperchaotic Yujun systems," *International Journal of Advanced Information Technology*, **2**, 13-22, 2012.

[70] W. Hahn, *The Stability of Motion*, Springer-Verlag, New York, 1967.