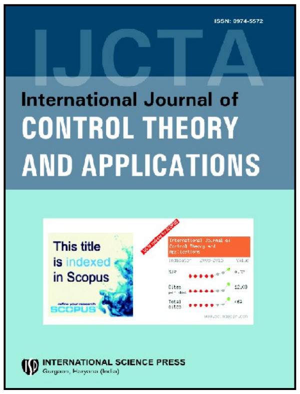

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Order Reduction of Continuous LTI Systems using Harmony Search Optimization with Retention of Dominant Poles

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Abstract: In this paper, a mixed approach is presented for the order reduction of a higher order continuous linear time invariant (LTI) systems in frequency domain. This approach utilizes the dominant pole retention method to synthesize the poles of reduced order model (ROM) whereas the Harmony search (HS) optimization technique is used to find the numerator polynomial of ROM. The resulting reduced order models are stable as the dominant poles are retained in reduced order model. On the other hand, Harmony search optimization is adapted through H_∞ norm minimization. Further, two numerical examples are considered in order to present the effectiveness of proposed approach over well-known existing methods available in literature.

Keywords: Dominant pole retention, Fitness function, Harmony Search, ISE.

1. INTRODUCTION

Model order reduction (MOR) is an active research area which deals with the generation of simpler model for a higher order system with preservation of input-output behavior. Conventional frequency domain MOR methods are continued fraction expansion (CFE) method [1-3], moment matching technique [4], Padè approximation [5], etc. but sometimes instability issue arises with these methods. Therefore, several stability preservation methods such as stability equation [6], Hurwitz polynomial approximation [7-8], Routh approximation [9] etc are developed. Recently, the combinatorial methods [10-12], have gained much attention over conventional approaches in which the denominator of reduced model is obtained by stability guaranteed method [6-9] and the numerator polynomial of ROM is determined by conventional approach [1-5]. Since last few years, several optimization techniques [13] shown in Figure. 1, are successfully implemented by several researchers in different areas of engineering. Hence, the optimization methods such as genetic algorithm (GA) [14-15], particle swarm optimization (PSO) [16-17], Big bang-big crunch (BB-BC) [18-20] cuckoo search (CS) [21], Differential evolution (DE) [22] and Harmony search (HS) [23] are used in MOR for numerator polynomial calculation for better approximation. In this article, the Harmony search algorithm and dominant pole retention [24] methods are combined to find out the numerator polynomial and denominator polynomial of reduced order model respectively.

The contents of paper are explained into five sections. The introduction part is given in section 1. Section 2 incorporates the problem formulation and the denominator and numerator polynomial reduction methods are explained in section 3. The validation of proposed method is given by numerical examples in section 4 and finally, the conclusion is given in section 5.

2. PROBLEM STATEMENT

The n^{th} order continuous LTI single input, single output system is considered with the transfer function as given below

$$T_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{\sum_{i=0}^{n-1} y_i s^i}{\sum_{i=0}^{n} x_i s^i} \quad (1)$$

where y_i's and x_i's are numerator and denominator polynomial coefficients of higher order system respectively.

Now, the problem is to find the r^{th} ($r < n$) order ROM with the generalized with the transfer function considered as

$$T_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{\sum_{i=0}^{r-1} b_i s^i}{\sum_{i=0}^{r} a_i s^i} \quad (2)$$

where and are denominator and numerator polynomial coefficients of the ROM respectively.

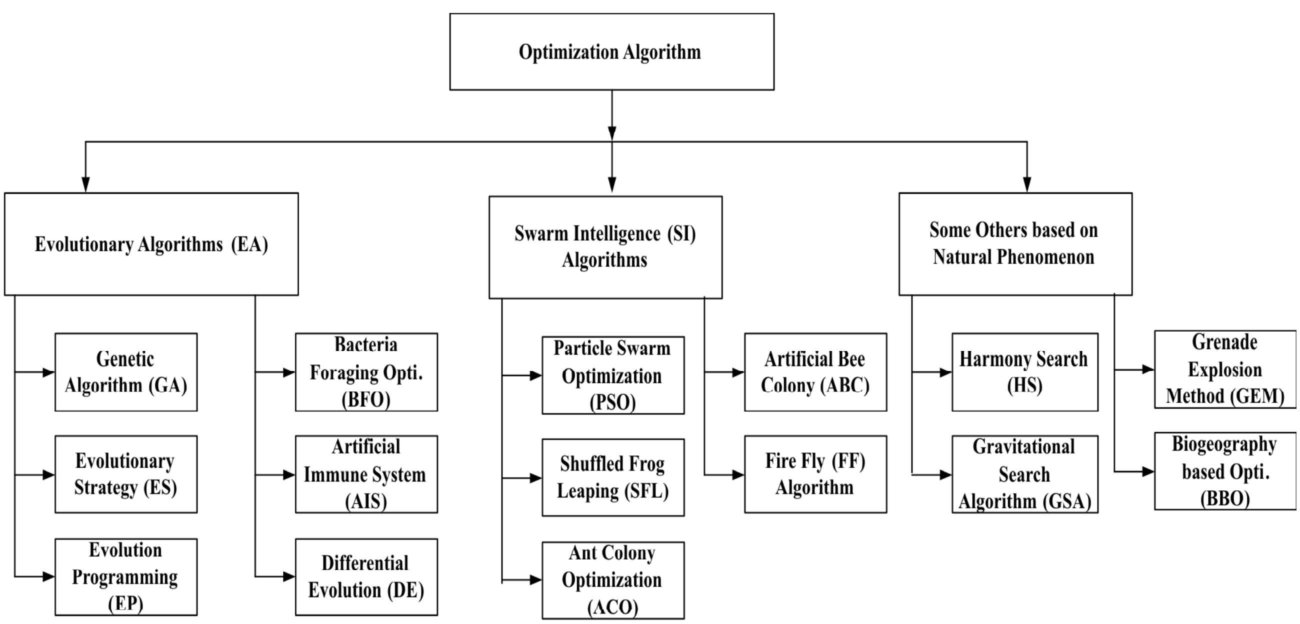

Figure 1: Classification of optimization algorithms

3. PROPOSED METHOD

In this section, the proposed method is discussed which utilizes the dominant pole retention and Harmony search optimization algorithm for reduction of higher order system into lower one.

3.1. Dominant pole retention [24]

In the proposed approach, the poles of ROM are obtained by the dominant pole retention method. In this method, the number of poles required for reduced model is the dominant poles of higher order model. Let the n^{th} order original higher order system have n poles located at $-p_1$, $-p_2$,……..,$-p_n$, then the 'r' dominant poles of r^{th} order ($r < n$) reduced order are $-p_1$, $-p_2$, ….,$-p_r$. Hence, the denominator of reduced order model is

$$D_r(s) = (s + p_1)(s + p_2)......(s + p_r) \tag{3}$$

3.2. Harmony Search Algorithm [25]

Once the denominator polynomial is obtained, the numerator polynomial coefficients are obtained by Harmony search (HS) optimization algorithm. Harmony search, one of the meta-heuristic optimization algorithms is proposed by Geem [25] in 2001 which is based on the perfect harmony search in musical process. The improvisation process of a skilled musician is interlaced to develop a qualitative optimization process called harmony search algorithm. A musician follows three choices for improvisation (1) Play a memorized music (harmony memory) (2) play a famous piece with adjustable pitch (pitch adjusting) or (3) compose something new (randomization).

Similarly HS follows the above musical rule:

1. Selection of a possible solution's value from Harmony Memory (HM);
2. Selection of adjacent solution from harmony memory;
3. Selection of a random solution value from the value of possible range.

Pitch adjusting rate (PAR) and harmony memory considering rate (HMCR) are two main parameters to implement the above rules on HS algorithm. The Flow chart of HS implementation is shown in Figure. 2 which is explained by the four steps given below:

Step 1: (Initialization): Initialize Harmony memory size (HMS) that is the number of possible solution vectors along with harmony memory consideration rate (HMCR), pitch adjustment rate (PAR), bandwidth (BW) and number of iterations. After that, define the fitness function which should be minimized. The initial solution vectors in the HS is the randomly generated n dimensional vectors with upper and lower bound value and given by

$$x_j^k = x_{jL} + rand(0,1) * (x_{jU} - x_{jL}) \tag{4}$$

where, x_{jU} and x_{jL} are lower and upper bound of solution vectors, $j = 1,2, ….., N$ and $k = 1, 2, ….., \text{HMS}$.

Step 2: (Improvising of a new solution): Improvisation is the process of New Harmony generation. This new solution is selected on the basis of HMCR. The term HMCR is the probability of selecting a component from the harmony memory. HMCR is selected from 0.7 to 0.9, since convergence speed of algorithm and selection of best harmonics are highly dependent upon its value. If HMCR value is very less, then only few best harmonics are selected and convergence speed is slow and if it is very high, then almost all vectors are selected and the best solution vector cannot be obtained. .In this process, a random parameter $r \in [0, 1]$, is generated, if this parameter is greater than HMCR, then new vector is generated by random selection otherwise, it is generated by consideration rate. Thereafter, vector generated by consideration rate is mutated by process of pitch adjustment with probability of PAR. This process is given as

$$x_j^{next} = x_j^{next} \pm rand(0,1) * BW \quad (5)$$

Step 3: (Replacement of solution vectors): The HM is updated by new solution if it the fitness value of the new solution is better than the that of the worst member in the HM, otherwise it is eliminated.

Step 4: The step 2 and step 3 are continuously repeated until maximum number of iteration or some termination criterion is met.

4. NUMERICAL EXAMPLES

Example 1: Consider the HOS with transfer function [18-21] as given below

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

The poles of the above system are as follows:

$$|p_1| = 1.00 \quad |p_2| = 2.00 \quad |p_3| = 3.00 \quad |p_4| = 4.00$$

Here, the objective is to reduce this system in a second order model. The poles of the ROM are obtained by the dominant pole retention approach as discussed in section 3.1 which state that the fast dynamics poles can be eliminated since these have less effect on system performance whereas the dominant poles should be retained in reduced model. Therefore, the poles of reduced order model are as follows:

$$|p_1| = 1.00 \quad |p_2| = 2.00$$

Now, the numerator polynomial of reduced order model is obtained by Harmony search optimization algorithm by minimization of $\|G - G_r\|_\infty$; where G_r is reduced order transfer function. The parameters of HS algorithm are as HMS = 100, HMCR = 0.9, PAR = 0.3 and BW = 0.2. Thereafter, steady state error is suppressed by DC matching and finally, the transfer function of the reduced order model is as follows

$$G_2(s) = \frac{0.7892s + 2}{s^2 + 3s + 2}$$

Figure 3 demonstrates the similarity between the step responses of reduced order models obtained by proposed method and existing methods [4, 19, 21]. Figure. 4 is plotted for the convergence of squared error with respect to time which shows that the proposed method converges very rapidly as compared to the methods [4, 19, 21]. A qualitative comparison of proposed method and method available in literature is shown in table 1 in terms of transient behavior whereas Table 2 shows the performance indices comparison for different error indices.

Table 1
Qualitative comparison of transient parameters for Example 1

Reduction Method	*Settling time*	*Rise Time*	*Overshoot (%)*
Original system	3.93	2.26	0
Proposed Method	4.1	2.35	0
Narwal and Prasad [21]	3.16	2.15	1.39
Boby and Pal [18]	4.55	2.53	0
Desai and Prasad [19]	3.15	2.12	1.25
Desai and Prasad [20]	3.62	2.28	0.274
Parmar *et al.* [4]	3.22	2.19	1.29
Pal [26]	5.57	2.2	2.69

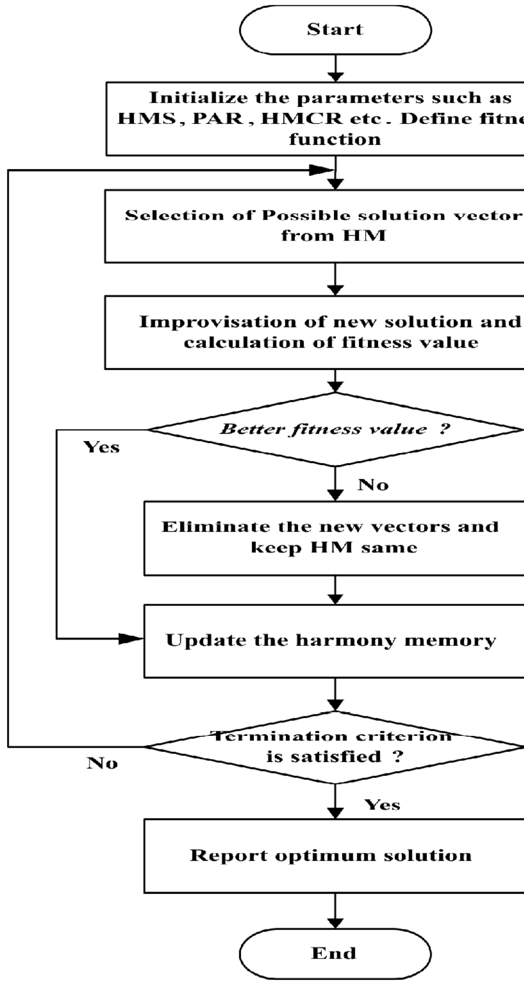

Figure 2: Flow chart of harmony search algorithms

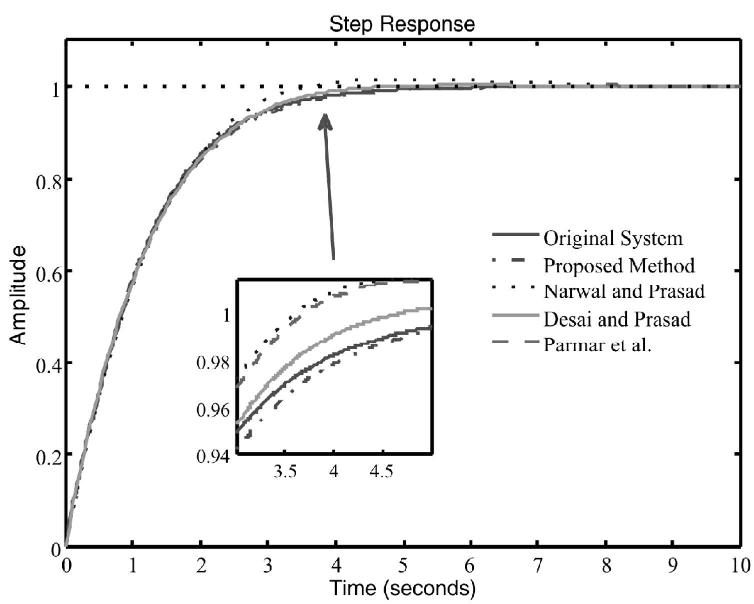

Figure 3: Step responses of ROM for Example 1

Example 2: The state space model of tape-drive function [27] with fifth order dynamics with single input and three outputs is shown as below:

$$A=\begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -1.35 & 0.1 & 4.1 & 0.75 \\ 0 & 0 & 0 & 5 & 0 \\ 0.35 & 0.4 & -1.4 & -5.4 & 0 \\ 0 & -0.03 & 0 & 0 & -10 \end{bmatrix}; \qquad B=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \quad C=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ -0.2 & -0.2 & 0.2 & 0.2 & 0 \end{bmatrix}$$

Here, the aim is to reduce the above 5th order system into 2nd order lower model of a close approximation with the original system. The poles of the original system are as follows:

$$|p_1| = 0.2055, \quad |p_2| = 0.6778, \quad |p_3| = 1.8992, \quad |p_4| = 3.9702, \quad |p_5| = 9.9973$$

According to section 3.1, the poles near to origin are slow decaying poles whereas the poles far away the origin will decay very fast. Hence, the poles of the reduced order model are p_1 and p_2.

Once the denominator polynomial is obtained the numerator polynomial of reduced model is obtained by the Harmony search by minimizing the fitness function. The harmony search algorithm is applied to find the 2nd order reduced model. Hence, the state space matrix of reduced model is given by:

$$A_r=\begin{bmatrix} -0.8833 & -0.2786 \\ 0.5 & 0 \end{bmatrix}; \qquad B_r=\begin{bmatrix} 0.5 \\ 0 \end{bmatrix}; \qquad C_r=\begin{bmatrix} -0.01896 & 0.1393 \\ -0.02391 & 0.3482 \\ -0.02357 & -0.08357 \end{bmatrix}$$

The step response of tape drive original system and 2nd order ROM obtained by the proposed algorithm is shown in Figure. 5. It is observed that the reduced model obtained by proposed method gives close approximation to original system.

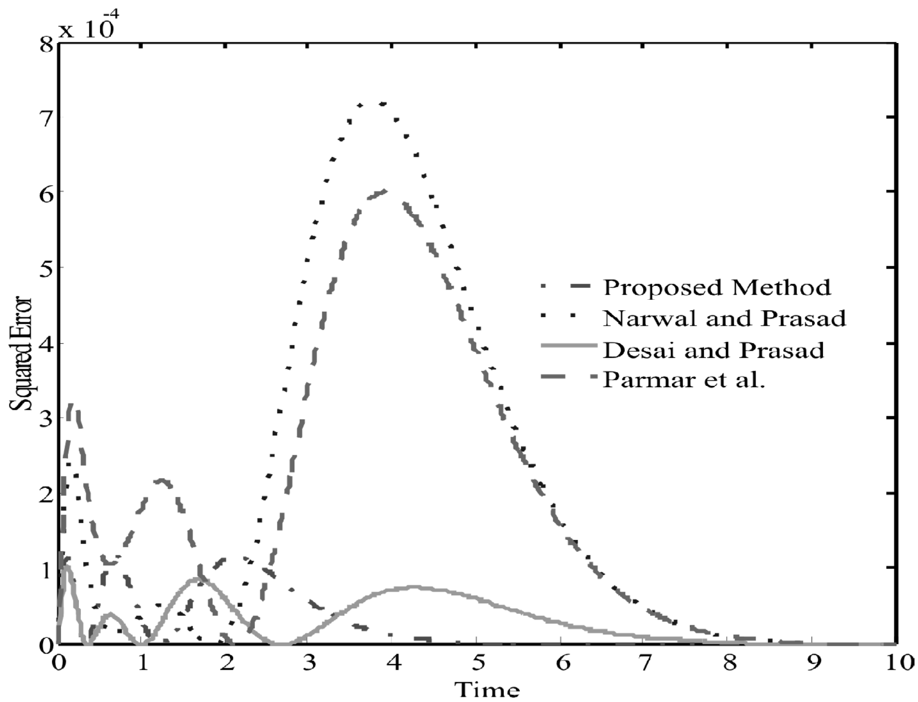

Figure 4: Convergence of squared error obtained by different method for Example 1

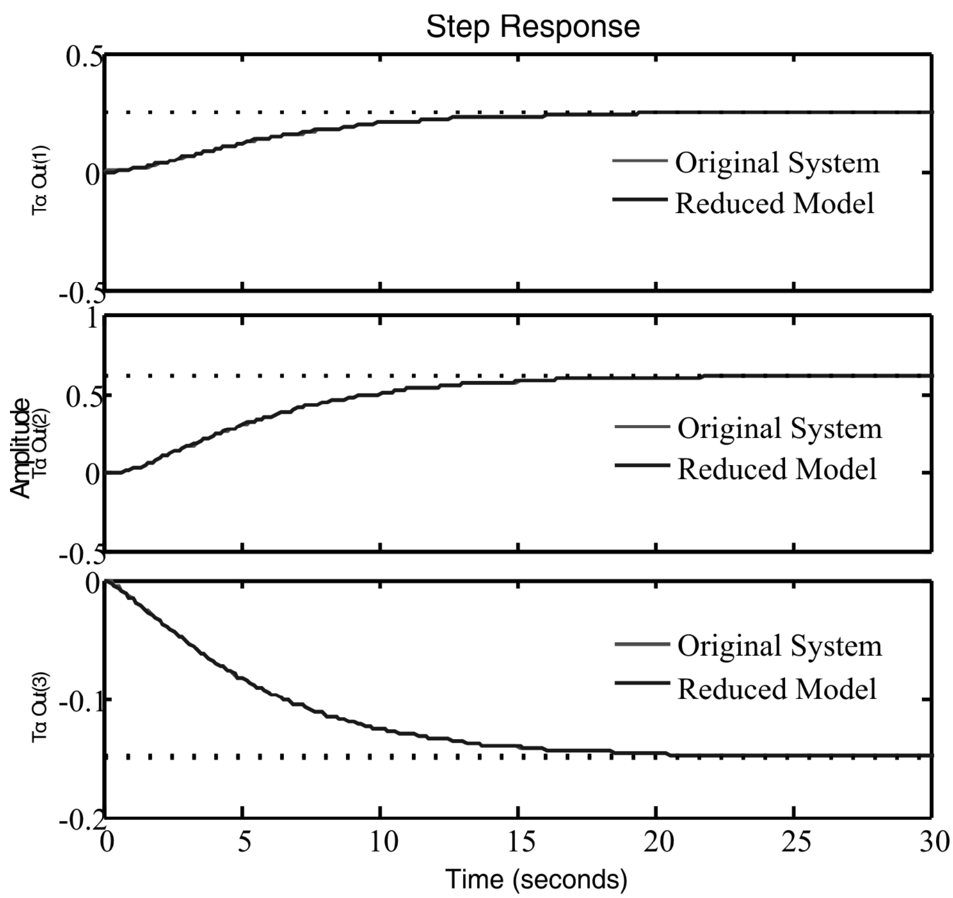

Figure 5: Step responses of ROM for Example 2

Table 2
Performance index comparison of various reduced order approach for Example 1

Reduction Method	*Reduced Model*	*ISE*	*ITSE*	*IAE*	*ITAE*
Proposed Method	$\frac{0.7892s+2}{s^2+3s+2}$	2.478 x 10^{-4}	4.953 x 10^{-4}	3.240 x 10^{-2}	7.539 x 10^{-2}
Narwal and Prasad [21]	$\frac{0.7597s+0.6997}{s^2+1.45771s+0.6997}$	1.991 x 10^{-3}	7.995 x 10^{-3}	1.105 x 10^{-1}	4.593 x 10^{-1}
Desai and Prasad [19]	$\frac{0.8058s+0.7944}{s^2+1.65s+0.7944}$	2.84 x 10^{-4}	1.009 x 10^{-3}	4.44 x 10^{-2}	1.7377 x 10^{-1}
Boby and Pal [18]	$\frac{0.9315s+1.6092}{s^2+2.75612s+1.6092}$	1.7274 x 10^{-3}	3.8134 x 10^{-3}	9.3144 x 10^{-2}	2.5526 x 10^{-1}
Desai and Prasad [20]	$\frac{0.8s+0.686}{s^2+1.47s+0.686}$	1.9455 x 10^{-3}	7.6188 x 10^{-3}	1.1122 x 10^{-1}	4.5368 x 10^{-1}
Parmar *et al.* [4]	$\frac{0.7442s+0.6997}{s^2+1.458s+0.6997}$	1.8487 x 10^{-3}	6.9771 x 10^{-3}	1.1194 x 10^{-1}	4.3906 x 10^{-1}
Pal [26]	$\frac{16s+24}{30s^2+42s+24}$	1.1884 x 10^{-2}	2.4226 x 10^{-2}	2.5372 x 10^{-1}	7.1708 x 10^{-1}

5. CONCLUSION

Harmony search and dominant pole retention based mixed method is presented to develop the reduced order model for linear time invariant continuous time systems. The stability guaranteed dominant pole retention method is used to find the poles of reduced model and after that, the numerator polynomial is obtained by the meta-heuristic harmony search optimization method by minimization of fitness function. Numerical examples of SISO and SITO system is considered to validate the superiority of proposed method over existing methods and the comparative analysis is included in terms of step response, performance indices, transient parameters and convergence of squared error. Further, the proposed method can easily be extended for discrete time systems.

REFERENCES

[1] C F. Chen and L. S. Shich, A novel approach to Linear model simplification, *International journal of Control*, 1968, vol. 8, no. 6, pp. 561-570.

[2] S. C. Chuang, Application of continued-fraction method for modeling transfer function to give more accurate transient response, *Electronics Letters*, 1970, vol. 6, no. 26, pp. 861-863.

[3] Y. Shamash, Continued fraction methods for reduction of constant linear multivariable systems, *Int. J. Syst. Sci.*, 1976, vol. 7, no.7, pp. 743-758.

[4] V. Zakian, Simplification of linear time-invariant systems by moment approximants, Int. J. Contr., vol. 18, no. 3, pp. 455-460.

[5] Y. Shamash, Stable reduced-order models using Padé type approximations, *IEEE Transactions on Automatic control*, 1973, vol. 19, no.5, pp. 615-616, 1974.

[6] T.C. Chen, C.Y. Chang, and K.W. Han, Stable reduced-order Padè approximants using stability equation method, *Electronics Letters*, 1980, vol. 16, no. 9, pp. 345–346.

[7] R. K. Appiah, Linear model reduction using Hurwitz polynomial approximation, *International Journal of Control*, 1978, vol. 28, no. 3, pp. 477–488.

[8] R. K. Appiah, Padé methods of Hurwitz polynomial with application to linear system reduction, *International Journal of Control*, 1979, vol. 29, no. 1, pp. 39–48.

[9] M. Hutton, B. Friedland, Routh approximations for reducing order of linear, time-invariant systems, *IEEE Transactions on Automatic control*, 1975, vol. AC-20, no. 3, pp. 329–337.

[10] J. Pal, System Reduction by a Mixed Method, *IEEE Transactions on Automatic Control*, 1980, vol. AC-25, no. 5, pp. 973-976.

[11] R. Prasad, S. P. Sharma, A. K. Mittal, Linear model reduction using the advantages of Mihailov criterion and factor division, *Journal of Institution of Engineers IE (I),* 2003, Journal-EL 84, pp. 7-10.

[12] G. Parmar, R. Prasad, S. Mukherjee, A Mixed Method for Large-Scale Systems Modelling using Eigen Spectrum Analysis and Cauer Second Form, *IETE Journal of Research*, 2007, vol. 53, no. 2, pp. 93-102.

[13] R. V. Rao, G. G. Waghmare, A new optimization algorithm for solving complex constrained design optimization problems. *Engineering Optimization*, 2016, pp. 1-24, doi: 10.1080/0305215X.2016.1164855

[14] Satakshi, S. Mukherjee, R.C. Mittal, Order reduction of linear discrete systems using a genetic algorithm, *Applied Mathematical Modelling*, 2005, vol. 29, no. 6, pp. 565-578.

[15] G. Parmar, R. Prasad, & S. Mukherjee, Order reduction of linear dynamic systems using stability equation method and GA, *IJECEECE, World Academy of Science, Engineering and Technology*, 2007, vol. 1, no. 2, pp. 236-242.

[16] S.N. Sivanandam, S.N. Deepa, A comparative study using genetic algorithm and particle swarm optimization for lower order system modelling, *Int.J.Comput. Internet Manage*. 2009, vol. 17, no. 3, pp. 1–10.

[17] Z. Gallehdari, M. Karrari, and O.P. Malik, Model Order Reduction using PSO Algorithm And It's Application to Power Systems, *IEEE International Conference on Electric Power and Energy Conversion Systems, EPECS '09* , 2009, Sharjah.

[18] B. Philip and J. Pal, An evolutionary computation based approach for reduced order modelling of linear systems, *IEEE International Conference on Computational Intelligence and Computing Research (ICCIC)*, 2010, Coimbatore, India.

[19] S.R. Desai, R. Prasad, A novel order diminution of LTI systems using Big Bang Big Crunch optimization and Routh Approximation, *Applied Mathematical Modelling*, 2013, vol. 37, no. 16-17, pp. 8016-8028.

[20] S.R. Desai, R. Prasad, A new approach to order reduction using stability equation and big bang big crunch optimization, *Systems Science & Control Engineering*, 2013, vol. 1, no. 1, pp. 20–27.

[21] Amit Narwal & B. Rajendra Prasad, A Novel Order Reduction Approach for LTI Systems Using Cuckoo Search Optimization and Stability Equation, *IETE Journal of Research*, 2016, vol. 62, no. 2, pp. 1-10.

[22] J. S. Yadav, N. P. Patidar, J. Singhai and C. Ardil, A Combined Conventional and Differential Evolution Method for Model Order Reduction, *IJECEECE, World Academy of Science, Engineering and Technology*, 2011, vol. 5, no. 9, pp. 1284-1291.

[23] H. Nasiri Soloklo, M. Maghfoori arsangi, Chebyshev rational functions approximation for model order reduction using harmony search, *Scientia Iranica*, 2013, vol. 20, no. 3, pp. 771-777.

[24] Y. Shamash, Linear system reduction using Padè approximation to allow retention of dominant modes, *Int. J. Control*, 1975, vol. 21, no. 2, pp. 257-272.

[25] Z. W. Geem, J. H. Kim, G. V. Loganathan, A new heuristic optimization algorithm: harmony search, *Trans. Soc. Model Simul. Int.*, 2001, vol. 76, no. 2, pp. 60-68.

[26] J. Pal, Stable reduced order Pade approximants using the Routh Hurwitz array, *Electronic Letters*, 1979, vol. 15, no. 8, pp. 225-226.

[27] G. F. Franklin, J. D. Powell and A. Emami-Naeini, Feedback Control of Dynamic Systems *(Addison-Wesley, 1994).*