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Detection and Measurement of the Fast-Fluctuating Gaussian Random Process Dispersion Abrupt Change

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Abstract: We introduce the technically simple approach to the determination of the abrupt change of the unknown dispersion of the high-frequency fast-fluctuating Gaussian random process against white noise with an unknown spectral density. For this purpose, we determine new approximations of the decision statistics for various hypotheses, we carry out their maximization on unknown parameters, and we develop the block diagrams for the corresponding detector and measurer in the form of the comparatively simple single-channel units. For the analytical analysis of the performance of the synthesized algorithms, the asymptotically exact expressions for their characteristics, specifically - type I and type II error probabilities (when an abrupt change point is detected) and conditional biases and variances of the estimates (when measuring the parameters of the analyzed random process), are obtained by means of local Markov approximation method. We also illustrate a new procedure for determining the distribution law and the central moments of the estimate of the discontinuous parameter (abrupt change point), with an allowance of anomalous effects. The experimental testing of the presented theoretical results is implemented by the methods of statistical computer simulation.

Keywords: Random process, abrupt change, maximum likelihood method, parametrical prior uncertainty, discontinuous parameter, local Markov approximation method, detection and estimation characteristics, statistical simulation.

1. INTRODUCTION

The problem of the statistical analysis of the abrupt change (i.e., instantaneous jumping) of the mean power of the random process is of a great importance in the fields of technical and medical diagnostics, the control theory, in data processing, etc. [1-3]. In certain publications, the statement of this problem is accompanied by the assumption that the observable data realization has a normal distribution. As a rule, the additional restrictions are also imposed. Thus, in [1] it is presupposed that the processed samples are statistically independent, in [2, 3] the autoregressive models of the information process are mainly studied, etc. Besides, in many cases the synthesis of detection and estimation algorithms for the abrupt change is usually conducted in the conditions of complete prior certainty regarding spurious parameters of the analyzed random process and its random distortions. In the relative few papers on the statistical analysis of the abrupt change of Gaussian processes with unknown parameters, there are described comparatively complex iterative algorithms only operable in case of very high signal-to-noise ratios (SNRs) [1, etc.].

In the present study, we consider the problem of the analysis of the abrupt change in the power parameter of the random process, presupposing that its fluctuations are fast (strong condition) and that its spectral density is approximately uniform within the specified bandwidth (weak condition). We illustrate how to effectively overcome the parametrical prior uncertainty under arbitrary SNRs. We then suggest a technically simple approach to the determination of the moment of the stepwise change in the unknown dispersion of the band high-frequency Gaussian random process [4, 5]. That approach allows us to obtain the processing algorithms invariant to the average power value of the background noise with flat spectrum.

Analytically studied information process can be presented as follows

$$\xi(t) = \left[\sigma' + (\sigma'' - \sigma')\theta(t - \lambda_0)\right]\nu(t). \tag{1}$$

In Eq. (1) the designations are: $\theta(t) = 0$, if $t < 0$, and $\theta(t) = 1$, if $t \geq 0$ – Heaviside function, λ_0 – the moment of possible stepwise change, σ', σ'' – mean square deviations of the $\xi(t)$ under $t < \lambda_0$ and $t \geq \lambda_0$, accordingly, and $\nu(t)$ – stationary centered Gaussian random process possessing spectral density

$$G(\omega) = \frac{\pi}{\Omega}\begin{cases} 1, & \vartheta - \Omega/2 \leq |\omega| \leq \vartheta + \Omega/2, \\ 0, & |\omega| < \vartheta - \Omega/2 \text{ or } |\omega| > \vartheta + \Omega/2. \end{cases} \tag{2}$$

Here ϑ is the band center, and Ω is the bandwidth of the process $\nu(t)$.

We believe that the process (1) is observed against additive Gaussian white noise $n(t)$ with one-sided spectral density N_0. As a result, the mix

$$x(t) = \xi(t) + n(t),\ t \in [0, T], \tag{3}$$

can now be observed. The fluctuations of the process $\xi(t)$ are now considered as "fast", so the following condition is satisfied

$$\mu_{\min} = T_{\min}\Omega/2\pi >> 1, \tag{4}$$

where $T_{\min} = \min(\lambda_0, T - \lambda_0)$. With the observable realization (3), it is necessary to detect the stepwise change point and to estimate the parameters $\lambda_0 \in [\Lambda_1, \Lambda_2]$, σ', σ''.

2. DETECTION OF THE RANDOM PROCESS DISPERSION STEPWISE CHANGE

For the solution of the problem of detection of the process $\xi(t)$ dispersion stepwise change, we separate three possible cases (three hypotheses): 1) $\sigma' = \sigma''$, i.e. jumping is absent (H_0 hypothesis); 2) $\sigma' > \sigma''$ (H_1

hypothesis); 3) $\sigma'' > \sigma'$ (H_2 hypothesis). Then, taking into account the condition (4), we present the initial process $\xi(t)$ (1) when implementing H_0, H_1, H_2 hypotheses in the form of

$$H_0:\ \xi(t)=\nu_1(t),\ H_1:\ \xi(t)=\nu_1(t)+[1-\theta(t-\lambda_0)]\nu_2(t),\ H_2:\ \xi(t)=\nu_1(t)+\theta(t-\lambda_0)\nu_2(t).$$

Here $\nu_i(t)$, $i=1,2$ are the stationary centered statistically independent Gaussian random processes with dispersions $\sigma_i^2 = d_{0i}\Omega/2\pi$ and spectral densities $G_i(\omega)=\sigma_i^2 G(\omega)$, and d_{0i} is the intensity of the process $\nu_i(t)$. Parameters σ', σ'' and σ_1, σ_2 are bound by the relationships $\sigma'=\sigma''=\sigma_1$ under H_0 hypothesis, by $\sigma'=\sqrt{\sigma_1^2+\sigma_2^2}$, $\sigma''=\sigma_1$ under H_1 hypothesis, and by $\sigma'=\sigma_1$, $\sigma''=\sqrt{\sigma_1^2+\sigma_2^2}$ under H_2 hypothesis.

The problem of the specified hypotheses testing is solved by means of the maximum likelihood method. For this purpose, with the results of the previous studies [4-6] in mind, the expressions for decision statistics (logarithms of the functionals of likelihood ratio) for hypotheses H_0, H_1, H_2 against alternative H: $x(t)=n(t)$ are written down as

$$H_0: L_0(d_1)=\frac{d_1}{N_0(N_0+d_1)}\int_0^T y^2(t)\ dt-\frac{\Omega T}{2\pi}\ln\left(1+\frac{d_1}{N_0}\right),$$

$$H_1: L_1(\lambda,d_1,d_2)=\frac{d_2}{(N_0+d_1)(N_0+d_1+d_2)}\int_0^\lambda y^2(t)\ dt+\frac{d_1}{N_0(N_0+d_1)}\int_0^T y^2(t)\ dt-$$
$$-\frac{\Omega T}{2\pi}\ln\left(1+\frac{d_1}{N_0}\right)-\frac{\Omega\lambda}{2\pi}\left[\ln\left(1+\frac{d_1+d_2}{N_0}\right)-\ln\left(1+\frac{d_1}{N_0}\right)\right], \quad (5)$$

$$H_2: L_2(\lambda,d_1,d_2)=\frac{d_2}{(N_0+d_1)(N_0+d_1+d_2)}\int_\lambda^T y^2(t)\ dt+\frac{d_1}{N_0(N_0+d_1)}\int_0^T y^2(t)\ dt-$$
$$-\frac{\Omega T}{2\pi}\ln\left(1+\frac{d_1}{N_0}\right)-\frac{\Omega(T-\lambda)}{2\pi}\left[\ln\left(1+\frac{d_1+d_2}{N_0}\right)-\ln\left(1+\frac{d_1}{N_0}\right)\right].$$

Here $y(t)=\int_{-\infty}^{\infty}x(t')h(t-t')dt'$ is the output signal of the filter with the transfer function $H(\omega)$ satisfying the condition $|H(\omega)|^2=\Omega G(\omega)/\pi$ (2), and λ, d_1, d_2 are current values of the parameters λ_0, d_{01}, d_{02}, accordingly. The choice is made in favor of the hypothesis for which the value of an absolute maximum of decision statistics is the greatest.

Under unknown parameters λ_0, d_{01}, d_{02}, the maximization of the functionals (5) on variables d_1, d_2 can be performed analytically. As a result, it is found that

$$H_0:\ L_{0\max}=\max_{d_1} L_0(d_1)=\frac{\Omega T}{2\pi}\left[\frac{2\pi}{N_0\Omega T}\int_0^T y^2(t)dt-\ln\left(\frac{2\pi}{N_0\Omega T}\int_0^T y^2(t)dt\right)-1\right],$$

$$H_1:\ L_{1\max}=\max_{d_1,d_2} L_1(\lambda,d_1,d_2)=L_{0\max}+\frac{\Omega T}{2\pi}\ln\left[\frac{\frac{1}{T}\int_0^T y^2(t)\,dt}{\frac{1}{T-\lambda}\int_\lambda^T y^2(t)\,dt}\right]-\frac{\Omega\lambda}{2\pi}\ln\left[\frac{\frac{1}{\lambda}\int_0^\lambda y^2(t)\,dt}{\frac{1}{T-\lambda}\int_\lambda^T y^2(t)\,dt}\right], \quad (6)$$

$$H_2:\ L_{2\max}=\max_{d_1,d_2} L_2(\lambda,d_1,d_2)=L_{0\max}+\frac{\Omega T}{2\pi}\ln\left[\frac{\frac{1}{T}\int_0^T y^2(t)\,dt}{\frac{1}{T-\lambda}\int_\lambda^T y^2(t)\,dt}\right]-\frac{\Omega\lambda}{2\pi}\ln\left[\frac{\frac{1}{\lambda}\int_0^\lambda y^2(t)\,dt}{\frac{1}{T-\lambda}\int_\lambda^T y^2(t)\,dt}\right].$$

From Eqs. (6) it follows that the maximum likelihood detection algorithm of the stepwise change in the dispersion of Gaussian random process takes a form of

$$\max_{\lambda\in[\Lambda_1,\Lambda_2]} M(\lambda) \underset{H_0}{\overset{H_1 \text{ or } H_2}{\gtrless}} 0,\ M(\lambda)=\ln[M_3/M_2(\lambda)]-(\lambda/T)\ln[M_1(\lambda)/M_2(\lambda)]. \quad (7)$$

and it is an invariant to the spectral density of the white noise and to the direction of variation, be it increasing or decreasing, of the dispersion value of the random process. In Eq. (7) it is designated as:

$$M_1(\lambda)=\frac{1}{\lambda}\int_0^\lambda y^2(t)\,dt,\ M_2(\lambda)=\frac{1}{T-\lambda}\int_\lambda^T y^2(t)\,dt,\ M_3=\frac{1}{T}\int_0^T y^2(t)\,dt. \quad (8)$$

It should be noted that instead of algorithm (7) it is possible to use the generalized detection algorithm [6, 7], based on the comparison of the greatest maximum of the functional $M(\lambda)$ with a particular (nonzero, generally) threshold c, calculated according to the specified optimality criterion.

Detection quality is characterized by type I (false alarm) and type II (missing) error probabilities, designated as α and β, respectively [6, 7]. In order to determine α and β, we present the functionals $M_1(\lambda)$, $M_2(\lambda)$, M_3 as the sums of signal and noise components [7]

$$M_i(\lambda)=S_i(l)+N_i(l),\ i=1,2,\ M_3=S_3+N_3, \quad (9)$$

Here $S_i(l)=\langle M_i(\lambda)\rangle$, $S_3=\langle M_3\rangle$ are signal, $N_i(l)=M_i(\lambda)-\langle M_i(\lambda)\rangle$, $N_3=M_3-\langle M_3\rangle$ are noise components, $l=\lambda/T$ is current value of the normalized parameter $l_0=\lambda_0/T$, and the averaging $\langle\cdot\rangle$ is performed in terms of the all possible realizations $x(t)$ with fixed values for the all unknown parameters λ_0, σ', σ''. While executing the ratio (4), we get

$$S_1(l)=E_N\left[1+q'+(q''-q')\ \max(0,l-l_0)/l\right],\ S_3=E_N\left[1+q'+(q''-q')\ (1-l_0)\right],$$

$$S_2(l)=E_N\left[1+q'+(q''-q')\ (1-\max(l_0,l))/(1-l)\right], \quad (10)$$

$$\langle N_1(l_1)N_1(l_2)\rangle=\left(E_N^2/\mu l_1 l_2\right)\left[(1+q')^2\min(l_1,l_2)+(q''-q')\ (2+q'+q'')\max\left(0,\min(l_1,l_2)-l_0\right)\right],$$

$$\langle N_2(l_1)N_2(l_2)\rangle = \left[E_N^2/\mu(1-l_1)(1-l_2)\right]\left[(1+q')^2(1-\max(l_1,l_2))+(q''-q')\ (2+q'+q'')\ (1-\max(l_0,l_1,l_2))\right],$$

$$\langle N_3^2\rangle = \left(E_N^2/\mu\right)\left[(1+q')^2+(q''-q')\ (2+q'+q'')(1-l_0)\right].$$

where $q'=\sigma'^2/E_N$, $q''=\sigma''^2/E_N$, $\mu=T\Omega/2\pi$, and $E_N=N_0\Omega/2\pi$ is the noise $n(t)$ mean power within the bandwidth of the process $\xi(t)$.

Let us introduce the value

$$\varepsilon=\mu^{-1/2}, \tag{11}$$

which is a small parameter, if the condition (4) holds. Then, taking into account Eqs. (9)-(11), we can present the functional (7) in the form of

$$M(\lambda)=\ln\left[\frac{\tilde S_3+\varepsilon\tilde N_3}{\tilde S_2(l)+\varepsilon\tilde N_2(l)}\right]-l\ln\left[\frac{\tilde S_1(l)+\varepsilon\tilde N_1(l)}{\tilde S_2(l)+\varepsilon\tilde N_2(l)}\right]. \tag{12}$$

Here $\tilde S_i(l)=S_i(l)/E_N$, $\tilde N_i(l)=N_i(l)\sqrt{\mu}/E_N$, $i=1,2$ are normalized functions, and $\tilde S_3=S_3/E_N$, $\tilde N_3=N_3\sqrt{\mu}/E_N$ are normalized values.

Firstly, let us assume that stepwise change in the dispersion of the process $\xi(t)$ (1) is absent, i.e. $q'=q''$. In this case, for false-alarm probability α, we have

$$\alpha=P\,[\max_{\lambda\in[\Lambda_1,\Lambda_2]}M(\lambda)>c]=1-P_N(c), \tag{13}$$

where $P_N(c)=P[M(\lambda)<c]$, $\lambda\in[\Lambda_1,\Lambda_2]$.

Applying Eqs. (9)-(11), we overwrite the functional $M(\lambda)$ (12) like that

$$M(\lambda)=\ln\left[\frac{1+\varepsilon N_{30}}{1+\varepsilon N_{20}(l)}\right]-l\ln\left[\frac{1+\varepsilon N_{10}(l)}{1+\varepsilon N_{20}(l)}\right],\ l\in\left[\tilde\Lambda_1,\tilde\Lambda_2\right]. \tag{14}$$

Here the functions $N_{i0}(l)=\tilde N_i(l)/(1+q')$, the variable $N_{30}=\tilde N_3/(1+q')$ and the values $\tilde\Lambda_i=\tilde\Lambda_i/T$, $i=1,2$ are introduced.

Taking into account Eq. (4), we develop Eq. (14) into Maclaurin series on ε (11) and focus on the first two terms of expansion depending on the realization of the observable data $x(t)$ (3). As a result, for $\varepsilon\to 0$ we have

$$M(\lambda)=\varepsilon^2 l(1-l)[\,N_{10}(l)-N_{20}(l)\,]^2/2,\ l\in\left[\tilde\Lambda_1,\tilde\Lambda_2\right]. \tag{15}$$

Within Eq. (15) we execute the change of variables:

$$\theta=\ln[l/(1-l)],\ \theta\in[\Theta_1,\Theta_2],\ \Theta_i=\ln\left[\tilde\Lambda_i/\left(1-\tilde\Lambda_i\right)\right],\ i=1,2, \tag{16}$$

Then, the probability (13) can be defined as

$$P_N(c) = P\left[\mathrm{X}^2(\theta) < 2\mu c\right],\ \theta \in \left[\Theta_1, \Theta_2\right], \tag{17}$$

where $\mathrm{X}(\theta)$ is Gaussian random process with zero mathematical expectation and correlation function $\left\langle \mathrm{X}(\theta_1)\,\mathrm{X}(\theta_2)\right\rangle = \exp\left(-\left|\theta_2 - \theta_1\right|/2\right)$. The problem of determining the greatest maximum characteristics for non-Gaussian random processes is considered, for example, in [8]. Consequently, with the results from [8] in mind, we can write down that

$$P_N(c) = \begin{cases} \exp\left[-(\Theta_2 - \Theta_1)\sqrt{\mu c/\pi}\,\exp(-\mu c)\right], & c \geq 1/2\mu, \\ 0, & c < 1/2\mu. \end{cases} \tag{18}$$

Then the expression for the false-alarm probability (13) gets the form

$$\alpha = \begin{cases} 1 - \exp\left[-(\Theta_2 - \Theta_1)\sqrt{\mu c/\pi}\,\exp(-\mu c)\right], & c \geq 1/2\mu, \\ 1, & c < 1/2\mu. \end{cases} \tag{19}$$

Accuracy of the formula (19) increases with c and ratio

$$m = \tilde{\Lambda}_2\left(1 - \tilde{\Lambda}_1\right)/\tilde{\Lambda}_1\left(1 - \tilde{\Lambda}_2\right). \tag{20}$$

Now let us assume that $q' \neq q''$. In this case, the stepwise change missing probability is written down as

$$\beta = P\,[\max_{\lambda \in \Lambda_1, \Lambda_2} M \;\lambda \; < c] = P\,[\,M_{\lambda \in \Lambda_1, \Lambda_2} \;\lambda \; < c]. \tag{21}$$

Similarly to Eq. (15), we expand the functional $M(\lambda)$ (12) into Maclaurin series on ε and focus on the first term of expansion depending on the realization of the observable data $x(t)$ (3). As a result, for $\varepsilon \to 0$ we have

$$M(\lambda) \approx S(l) + \varepsilon N(l), \tag{22}$$

where

$$S(l) = \ln\left[\tilde{S}_3/\tilde{S}_2(l)\right] - l\ln\left[\tilde{S}_1(l)/\tilde{S}_2(l)\right] \tag{23}$$

– signal and

$$N(l) = \tilde{N}_3/\tilde{S}_3 - (1-l)\tilde{N}_2(l)/\tilde{S}_2(l) - l\tilde{N}_1(l)/\tilde{S}_1(l) \tag{24}$$

– noise components of the functional $M(\lambda)$.

Let us introduce the output SNR as [7]

$$z^2 = \frac{\mu S^2(l_0)}{\left\langle N^2(l_0)\right\rangle} = \mu\frac{\left[1 + q'' + l_0(q' - q'')\right]^2}{l_0(1 - l_0)(q' - q'')^2}\left[\ln\left(\frac{1 + q' + (q'' - q')(1 - l_0)}{1 + q''}\right) - l_0\ln\left(\frac{1 + q'}{1 + q''}\right)\right]^2. \tag{25}$$

From Eq. (25) it follows that SNR $z \to \infty$, if $q' \neq q''$ and $\mu \to \infty$. Therefore, meeting the condition (4) secures the greater SNR value, if the difference $\left|q' - q''\right|$ is not too small.

With $z \to \infty$, the maximum position of the functional $M(\lambda)$ converges to the value λ_0 in mean square [7]. As a result, under great SNR z (25), in order to define the missing probability (21), it is sufficient to study the behavior of the functional $M(\lambda)$ in a near neighborhood of the point λ_0 (l_0) [4, 7]. For that purpose we introduce the designation $\Delta = \max\{|l_1 - l_0|,\ |l_2 - l_0|,\ |l_1 - l_2|\}$. Then, if $\Delta \to 0$, for the signal component (23) and correlation function of the noise component (24), the following asymptotic representations are valid:

$$S(l) = \ln\left[\frac{1+q''+l_0(q'-q'')}{1+q''}\right] - l\ln\left(\frac{1+q'}{1+q''}\right) + \frac{q'-q''}{1+q''}\min(0, l-l_0) + \frac{q'-q''}{1+q'}\max(0, l-l_0) + o(\Delta), \quad (26)$$

$$\langle N(l_1)N(l_2)\rangle = \frac{l_0(1-l_0)(q'-q'')^2}{[1+q''+l_0(q'-q'')]^2} + \frac{(q'-q'')[1+q'+l_0(q'-q'')]}{(1+q'')[1+q''+l_0(q'-q'')]}(\tilde{l}_1+\tilde{l}_2) + \frac{1+q'}{1+q'}|\tilde{l}_2 - \tilde{l}_1| - \left(\frac{1+q'}{1+q''}\right)^2 \max(\tilde{l}_1, \tilde{l}_2) +$$

$$+\min(\tilde{l}_1, \tilde{l}_2) - \frac{(q'-q'')(2+q'+q'')}{(1+q')^2}\max(0, \min(\tilde{l}_1, \tilde{l}_2)) - \frac{(q'-q'')^2[3+q'+2q''+l_0(q'-q'')]}{(1+q')(1+q'')[1+q''+l_0(q'-q'')]}\left[\max(0, \tilde{l}_1) + \max(0, \tilde{l}_2)\right] +$$

$$+\frac{(q'-q'')(2+q'+q'')}{(1+q'')^2}\max(0, \tilde{l}_1, \tilde{l}_2) - \frac{(q'-q'')(2+q'+q'')}{(1+q')(1+q'')}\left[\max(0, \tilde{l}_2 - \max(0, \tilde{l}_1)) + \max(0, \tilde{l}_1 - \max(0, \tilde{l}_2))\right] + o(\Delta),$$

where, $\tilde{l}_i = l_i - l_0$, $i = 1,2$.

Approximations (26) will be consistent mathematically, if the signal component $S(l)$ and dispersion $\langle N^2(l)\rangle$ of the noise component $N(l)$ of the functional $M(\lambda)$ (22) reach their maximum values in the point $l = l_0$. As follows from Eq. (26), the condition $S(l_0) = \max S(l)$ is fulfilled for any values q', q'', l_0, while to satisfy the condition $\langle N^2(l_0)\rangle = \max\langle N^2(l)\rangle$ the following is necessary:

$$\begin{cases} q' < [1+q''(1+l_0)]/l_0, & \text{if } q' > q'', \\ q'' < [1+q'(2-l_0)]/(1-l_0), & \text{if } q'' > q'. \end{cases} \quad (27)$$

Hereinafter we will presuppose that the inequalities (27) hold.

We introduce the differential functional

$$\varsigma_x(l) = [M(l) - M(x)]/\sigma_S, \quad l, x \in \Lambda_\delta. \quad (28)$$

Here $\sigma_S^2 = l_0(1-l_0)(q'-q'')^2/\mu[1+q''+l_0(q'-q'')]^2$, $\Lambda_\delta = [l_0 - \delta, l_0 + \delta]$, and δ is fixed and it is chosen so small that, under $\Delta < \delta$, the expressions (26) can be approximated by the dominant terms of the asymptotic expansions with the required accuracy. Then, for $z >> 1$ (25), it is possible to present the missing probability (21) in the form of

$$\beta \approx P[\sup_{l \in [l_0-\delta, l_0+\delta]} \varsigma_{l_0}(l) < c/\sigma_S - \kappa_0] = P_S(c/\sigma_S), \quad (29)$$

where, $\kappa_0 = M(l_0)/\sigma_S$.

By applying the Doob's theorem in the wording [9], similarly to [10], it is easy to show that, within the interval Λ_δ and under $\mu \to \infty$, the process $\varsigma_x(l)$ is the asymptotic Gaussian Markov random process of the diffusion type, for which the drift K_1 and diffusion K_2 coefficients are defined by the following expressions

$$K_1 = \frac{1}{\sigma_S}\begin{cases} a_1, & l \le l_0, \\ -a_2, & l > l_0, \end{cases} \quad K_2 = \frac{1}{\sigma_S^2}\begin{cases} b_1, & l \le l_0, \\ b_2, & l > l_0, \end{cases} \tag{30}$$

where,

$$a_1 = \frac{q' - q''}{1+q''} - \ln\left(\frac{1+q'}{1+q''}\right), \; a_2 = \frac{q'' - q'}{1+q'} + \ln\left(\frac{1+q'}{1+q''}\right), \; b_1 = \frac{(q'-q'')^2}{\mu(1+q'')^2}, \; b_2 = \frac{(q'-q'')^2}{\mu(1+q')^2}, \tag{31}$$

At the same time, as follows from Eq. (26), the realizations of the process $\varsigma_{l_0}(l)$ within the intervals $[l_0 - \delta, l_0]$, $(l_0, l_0 + \delta]$ are not correlated, and therefore they are statistically independent, as being asymptotic Gaussian ones. Then, for probability $P_S(\kappa)$ (29) we have

$$P_S(\kappa) = F_1(\kappa - \kappa_0)\; F_2(\kappa - \kappa_0), \; F_1(\kappa) = P\,[\sup_{l_0-\delta \le l \le l_0} \varsigma_{l_0}(l) < \kappa], \; F_2(\kappa) = P\,[\sup_{l_0 < l \le l_0+\delta} \varsigma_{l_0}(l) < \kappa]. \tag{32}$$

The random variable κ_0 in Eq. (29) is, under $\mu \to \infty$, asymptotic Gaussian random value with mathematical expectation z (25) and unit dispersion. In view of the latter remark, the probability (32) is determined as

$$P_S(\kappa) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\kappa} F_1(\kappa - y)\; F_2(\kappa - y)\; \exp\left[-(y-z)^2/2\right] dy, \tag{33}$$

The probabilities $F_1(\kappa)$, $F_2(\kappa)$ can be found using the Markovian properties of the process $\varsigma_{l_0}(l)$. As a result, following [10], for the functions $F_1(\kappa)$, $F_2(\kappa)$ we obtain

$$F_1(\kappa) = \Phi\left[(a_1\delta + \sigma_S\kappa)/\sqrt{b_1\delta}\right] - \exp(-2\sigma_S a_1 \kappa/b_1)\,\Phi\left[(a_1\delta - \sigma_S\kappa)/\sqrt{b_1\delta}\right], \tag{34}$$
$$F_2(\kappa) = \Phi\left[(a_2\delta + \sigma_S\kappa)/\sqrt{b_2\delta}\right] - \exp(-2\sigma_S a_2 \kappa/b_2)\,\Phi\left[(a_2\delta - \sigma_S\kappa)/\sqrt{b_2\delta}\right],$$

where, $\Phi(x) = \int_{-\infty}^{x} \exp(-t^2/2)dt/\sqrt{2\pi}$ is the probability integral [11], and a_1, a_2, b_1, b_2 are defined from Eqs. (31).

Substituting Eqs. (34) into Eq. (33) and using the asymptotic formula [12] for the function $\Phi(x)$ under $z \to \infty$: $\Phi(x) \xrightarrow[x\to\infty]{} 1 - \exp(-x^2/2)/\sqrt{2\pi}x$ and then neglecting higher-order infinitesimal terms on z, after carrying out the integration, we get for the missing probability β (29):

$$\beta \approx \Phi\left(\frac{c}{\sigma_S} - z\right) - \exp\left[\frac{\psi_-^2 z^2}{2} + \psi_- z\left(z - \frac{c}{\sigma_S}\right)\right]\Phi\left[\frac{c}{\sigma_S} - z(\psi_- + 1)\right] - \exp\left[\frac{\psi_+^2 z^2}{2} + \psi_+ z\left(z - \frac{c}{\sigma_S}\right)\right] \times$$

$$\times \Phi\left[\frac{c}{\sigma_S} - z\left(\psi_+ + 1\right)\right] + \exp\left[\frac{z^2\left(\psi_- + \psi_+\right)^2}{2} + z\left(\psi_- + \psi_+\right)\left(z - \frac{c}{\sigma_S}\right)\right]\Phi\left[\frac{c}{\sigma_S} - z\left(\psi_- + \psi_+ + 1\right)\right]. \qquad (35)$$

Here $\psi_- = 2\sigma_S a_1/zb_1$, $\psi_+ = 2\sigma_S a_2/zb_2$. The accuracy of the formula (35) increases with μ and z.

3. ESTIMATING THE STEPWISE CHANGE POINT AND THE DISPERSION OF THE RANDOM PROCESS

Let us suppose now that the stepwise change of the dispersion of the random process $\xi(t)$ (1) is realized with the probability 1 within the interval $[\Lambda_1, \Lambda_2]$. And let us state that it is necessary to measure the change-point time λ_0 jointly with the values of the power parameters σ'^2, σ''^2. The synthesis of the joint estimation algorithm is to be conducted by means of maximum likelihood method. Using Eqs. (5), for maximum likelihood estimates (MLEs) λ_m, σ'^2_m, σ''^2_m of the unknown parameters λ_0, σ'^2, σ''^2, we obtain

$$\lambda_m = \underset{\lambda \in [\Lambda_1, \Lambda_2]}{\arg\sup}\, M(\lambda), \quad \sigma'^2_m = M_1(\lambda_m) - E_N, \quad \sigma''^2_m = M_2(\lambda_m) - E_N. \qquad (36)$$

Let us determine the characteristics of the estimates (36). We presuppose that the condition $z >> 1$ (25) is satisfied and thus the normalized estimate $l_m = \lambda_m/T$ (36) is situated in a near δ-neighborhood Λ_δ of the point l_0 (28) with the probability tending to 1. Then, the conditional distribution function $F_0(x|l_0)$ of the estimate l_m can be presented in the form of

$$F_0(x|l_0) = P[\, l_m < x] = P\left[\max_{l<x} \varsigma_x(l) > \max_{l \geq x} \varsigma_x(l)\right], \; l, x \in \Lambda_\delta, \qquad (37)$$

where, $\varsigma_x(l)$ is the Markov random process of the diffusion type (28) with drift and diffusion coefficients (30).

Using the representation from (37) and referring to the results of the studies [10, 12], for the conditional probability density $w_0(l|l_0)$, bias $b_0(l_m|l_0) = \langle l_m - l_0 \rangle$ and variance $V_0(l_m|l_0) = \left\langle (l_m - l_0)^2 \right\rangle$ of the estimate l_m (36), in the conditions of high posterior accuracy, we obtain:

$$w_0(l|l_0) = \begin{cases} \left(2a_1^2/b_1\right)\, \Psi\left(2a_1^2(l-l_0)/b_1, 1/R\right), & l - l_0 \leq 0, \\ \left(2a_2^2/b_2\right)\, \Psi\left(2a_2^2(l-l_0)/b_2, R\right), & l - l_0 > 0, \end{cases} \qquad (38)$$

$$b_0(l_m|l_0) = \left[a_1^2 b_2 R(R+2) - a_2^2 b_1 (2R+1)\right] \Big/ 2a_1^2 a_2^2 (R+1)^2,$$

$$V_0(l_m|l_0) = \left[a_1^4 b_2^2 R\left(2R^2 + 6R + 5\right) + a_2^4 b_1^2 \left(5R^2 + 6R + 2\right)\right] \Big/ 2a_1^4 a_2^4 (R+1)^3.$$

Here $\Psi(x, y) = \Phi\left(\sqrt{|x|/2}\right) - 1 + (2y+1)\exp\left[|x| y(y+1)\right]\left[1 - \Phi\left((2y+1)\sqrt{|x|/2}\right)\right]$, $R = a_1 b_2 / a_2 b_1$, and a_1, a_2, b_1, b_2 are defined from Eqs. (31). From this, it particularly follows that the estimate of the stepwise change time l_m is conditionally biased under finite SNR, generally. The accuracy of the formulas (38) increases with $\mu_{\min}$ (4) and z (25).

Under small values z ($z \to 0$), the decision statistics $M(\lambda)$ (7), (12) can be approximately presented in the form of (15). In Eq. (15) we make the change of variables (16) and move from the normalized estimate l_m (36) to the estimate

$$\theta_m = \arg\max_{\theta\in[\Theta_1,\Theta_2]} X^2(\theta), \tag{39}$$

Here $X(\theta)$ is determined in the same way as in Eq. (17).

According to [7], the position of the maximum of the stationary random process is described by the uniform probability density. Then, for conditional probability density $w(\theta|\theta_0)$ of the random variable θ_m (39), where $\theta_0 = \ln[l_0/(1-l_0)]$, we can write down: $w(\theta|\theta_0) = 1/(\Theta_2 - \Theta_1)$, $\theta \in [\Theta_1, \Theta_2]$. Hence, taking into account the transformation (16), under small z for the conditional probability density $w_a(l|l_0)$, bias $b_a(l_m|l_0)$ and variance $V_a(l_m|l_0)$ of the normalized MLE l_m (36), we get

$$w_a(l|l_0) = 1/l(1-l)\ln m, \; l \in [\tilde{\Lambda}_1, \tilde{\Lambda}_2], \tag{40}$$

$$b_a(l_m|l_0) = 1 - l_0 - \ln(\tilde{\Lambda}_2/\tilde{\Lambda}_1)/\ln m,$$

$$V_a(l_m|l_0) = l_0^2 + \left[(1-2l_0)\ln\left((1-\tilde{\Lambda}_1)/(1-\tilde{\Lambda}_2)\right) + \tilde{\Lambda}_1 - \tilde{\Lambda}_2\right]/\ln m,$$

where, m is defined from Eq. (20).

For the arbitrary values of z, the distribution of the estimate l_m is found in the form of

$$w(l|l_0) = P_0 w_0(l|l_0) + (1-P_0) w_a(l|l_0), \tag{41}$$

where, $w_0(l|l_0)$, $w_a(l|l_0)$ are defined from Eqs. (38), (40), $P_0 = P[H_S > H_N]$, and H_N, H_S are the random variables corresponding to the maxima of the functional $M(\lambda)$ (7) when normalized MLE l_m (normalized maximum position) is subject to the distribution law $w_0(l|l_0)$ (38), or of $w_a(l|l_0)$ (40), accordingly.

The probability P_0 can be determined by applying either the two-dimensional probability density $w_2(u,v)$, or the distribution function $F_2(u,v) = P[H_S < u, H_N < v] = \int_0^u \int_0^v w_2(u',v')\, du'dv'$ of the random variables H_N, H_S as

$$P_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{u} w_2(u,v)\, dv du = \int_{-\infty}^{\infty} \left[\frac{\partial F_2(u,v)}{\partial u}\right]\Bigg|_{v=u} du\,. \tag{42}$$

As it follows from Eqs. (15) and (22)-(24), the random variables H_N, H_S are uncorrelated. Then Eq. (42) can be approximately presented in the form of

$$P_0 \approx \int_0^{\infty} P_N(u)\, dP_S(u), \qquad (43)$$

where, $P_N(u)$, $P_S(u)$ are determined according to Eqs. (18), (35). Substituting the explicit form of the functions $P_N(u)$, $P_S(u)$ into Eq. (43), for the probability P_0 we obtain

$$\begin{aligned} P_0 = \frac{z}{\sigma_S} \int_{1/2\mu}^{\infty} \exp\left[-(\Theta_2-\Theta_1)\sqrt{\frac{\mu u}{\pi}}\exp(-\mu u)\right] \Bigg\{ \psi_- \exp\left[\frac{\psi_-^2 z^2}{2} + \psi_- z\left(z - \frac{u}{\sigma_S}\right)\right] \times \\ \times \Phi\left[\frac{u}{\sigma_S} - z(\psi_- + 1)\right] + \psi_+ \exp\left[\frac{\psi_+^2 z^2}{2} + \psi_+ z\left(z - \frac{u}{\sigma_S}\right)\right] \Phi\left[\frac{u}{\sigma_S} - z(\psi_+ + 1)\right] - \\ -(\psi_- + \psi_+)\exp\left[\frac{z^2(\psi_- + \psi_+)^2}{2} + z(\psi_- + \psi_+)\left(z - \frac{u}{\sigma_S}\right)\right] \Phi\left[\frac{u}{\sigma_S} - z(\psi_- + \psi_+ + 1)\right]\Bigg\} du\,. \end{aligned} \qquad (44)$$

By means of Eqs. (41), (44), we can write the analytical expressions for the conditional bias $b(l_m|l_0)$ and the variance $V(l_m|l_0)$ of the estimate l_m in case of the arbitrary SNR z values as follows:

$$b(l_m|l_0) = P_0 b_0(l_m|l_0) + (1-P_0)\; b_a(l_m|l_0),\; V(l_m|l_0) = P_0 V_0(l_m|l_0) + (1-P_0)\; V_a(l_m|l_0), \qquad (45)$$

where, $b_0(l_m|l_0)$, $V_0(l_m|l_0)$, $b_a(l_m|l_0)$, $V_a(l_m|l_0)$ are determined from Eqs. (38), (40). The accuracy of the formulas (44), (45) increases with $\mu_{\min}$ (4), m (20), z (25).

Now let us determine the characteristics of the estimates $\sigma_m'^2$, $\sigma_m''^2$ (36). In the study [13], it is shown that the accuracy of the MLEs of the regular parameters (in the present case – dispersions) does not asymptotically (with increasing SNR) depend on the presence of the unknown discontinuous parameter (in the present case – stepwise change point). It means that, in case of the greater values of $\mu_{\min}$ (4), the conditional biases and variances of the MLEs $\sigma_m'^2$, $\sigma_m''^2$ (36) coincide asymptotically with the conditional biases and variances of the estimates of the dispersions of the random process $\xi(t)$ with a priori known stepwise change point. Then, supposing that $l_m = l_0$ in Eq. (36), directly averaging over all the possible realizations of the observable data $x(t)$ (3) at the fixed values σ'^2, σ''^2, and taking into account Eq. (4) for the characteristics of the estimates (36), we now find

$$b_0\left(\sigma_m'^2 \middle| \sigma'^2\right) = \left\langle \sigma_m'^2 - \sigma'^2 \right\rangle = 0,\; V_0\left(\sigma_m'^2 \middle| \sigma'^2\right) = \left\langle \left(\sigma_m'^2 - \sigma'^2\right)^2 \right\rangle = 2\pi E_N^2 (1+q')^2 / \Omega\lambda_0\,, \qquad (46)$$

$$b_0\left(\sigma_m''^2 \middle| \sigma''^2\right) = \left\langle \sigma_m''^2 - \sigma''^2 \right\rangle = 0,\; V_0\left(\sigma_m''^2 \middle| \sigma''^2\right) = \left\langle \left(\sigma_m''^2 - \sigma''^2\right)^2 \right\rangle = 2\pi E_N^2 (1+q'')^2 / \Omega(T-\lambda_0).$$

The accuracy of the formulas (46) increases with $\mu_{\min}$ (4) and *z* (25).

4. RESULTS OF THE STATISTICAL SIMULATION

In order to establish the borders of applicability for the found asymptotically exact formulas for detection and estimation characteristics, we demonstrate the statistical computer simulation of the algorithms (7), (36), using a procedure presented in [10]. For the reduction of the computer time expenditure, it has been supposed that the process $\nu(t)$ (1) is narrowband [4], i.e. the condition $\vartheta >> \Omega$ is satisfied. It allows us to apply representation of the function $y(t)$ (5) through their low-frequency quadratures [10] and to form the functionals $M_1(\lambda)$, $M_2(\lambda)$ (8), as the sum of the two independent random processes, as well as the functional M_3 (8), as the sum of the two independent random variables:

$$M_1(\lambda)=[M_{11}(\lambda)+M_{12}(\lambda)]/2\lambda\,,\; M_3=(M_{31}+M_{32})/2T\,, \tag{47}$$
$$M_2(\lambda)=[TM_3-\lambda M_1(\lambda)]/(T-\lambda)=[M_{31}+M_{32}-M_{11}(\lambda)-M_{12}(\lambda)]/2(T-\lambda).$$

Here

$$M_{1i}(\lambda)=\int_0^{\lambda} y_i^2(t)\,dt\,,\; M_{3i}=\int_0^{T} y_i^2(t)\,dt\,,\; y_i(t)=\int_{-\infty}^{\infty} x_i(t')h_0(t-t')\,dt'\,,$$
$$x_i(t)=[\,\sigma'+(\sigma''-\sigma')\theta(t-\lambda_0)\,]\nu_i(t)+n_i(t),\; i=1{,}2\,,$$

$\nu_i(t)$ and $n_i(t)$ are statistically independent centered Gaussian random processes with the spectral densities $G_\nu(\omega)=(2\pi/\Omega)[\theta(\omega+\Omega/2)-\theta(\omega-\Omega/2)]$ and $G_n(\omega)=N_0$, respectively; $h_0(t)$ is the function whose spectrum $H_0(\omega)$ fulfils the condition $|H_0(\omega)|^2=(\Omega/2\pi)G_\nu(\omega)$, while $\theta(\cdot)$, σ', σ'' is defined in the same way as in Eq. (1).

During modeling within the period $[0{,}1]$ of the normalized time $\tilde{t}=t/T$, with discretization step Δ, the samples $\tilde{y}_{in}=\tilde{y}_i(n\Delta)$ are formed of the normalized random process realizations $\tilde{y}_i(\tilde{t})=y_i(t)\sqrt{T/N_0}$, $i=1{,}2$ (47). This allows obtaining the stepwise approximations for the normalized functionals $\tilde{M}_1(l)=TM_1(\lambda)/N_0$, $\tilde{M}_2(l)=TM_2(\lambda)/N_0$, $\tilde{M}_3=TM_3/N_0$ (8) and decision statistics $M(l)$ (7) in the form of

$$\tilde{M}_1(l)=\frac{\Delta}{2l}\sum_{n=0}^{N-1}\left(\tilde{y}_{1k}^2+\tilde{y}_{2k}^2\right),\; \tilde{M}_2(l)=\left[\tilde{M}_3-l\tilde{M}_1(l)\right]/(1-l), \tag{48}$$
$$\tilde{M}_3=\tilde{M}_1(1),\; M(l)=\ln\left[\tilde{M}_3/\tilde{M}_2(l)\right]-l\ln\left[\tilde{M}_1(l)/\tilde{M}_2(l)\right].$$

Here $N=\operatorname{int}\{l/\Delta\}$, $\operatorname{int}\{\cdot\}$ is an integer part, and l is the normalized current value (9) of the abrupt change point. When we choose the discretization step equal to $\Delta=10^{-4}$, the mean square error of approximations (48) of the functionals (47) does not exceed 10 %. Samples of the processes $\tilde{y}_{in}$, $i=1{,}2$, $n=\overline{0,\operatorname{int}\{1/\Delta\}}$ are generated in terms of the sequence of the independent Gaussian random numbers by a moving summation method [4, 10]:

$$\tilde{y}_{in} = \sum_{k=n-p}^{n+p-1} \left(\tilde{\xi}_{ik} + \frac{\alpha_{ik}}{\sqrt{\Delta}} \right) H_{nk} \,,\; \tilde{\xi}_{ik} = \frac{1}{\pi} \sqrt{\frac{q_k}{\Delta}} \sum_{m=0}^{2p} H_{mp} \beta_{im+k} \,. \qquad (49)$$

Here α_{ik}, β_{ik} are the independent Gaussian random numbers with zero mathematical expectations and unit dispersions, $H_{nk} = \sin[\pi\mu\Delta(n-k)]/\pi(n-k)$, $q_k = q'$, if $k < \text{int}\{l_0/\Delta\}$, and $q_k = q''$, if $k \geq \text{int}\{l_0/\Delta\}$, while q', q'', l_0 are defined in the same way as in Eqs. (10).

In the sums (49), the number of summands corresponds to the value $p = 200$. It provides a relative deviation of the generated sample dispersion from the modeled process dispersion to be not greater than 5 % [10], under $\mu \leq 500$. Formation of the independent Gaussian numbers with parameters (0,1) is implemented using the standard generator of independent random numbers, uniformly distributed within the interval [0,1], by means of the Cornish-Fisher method [10].

By realizations of the processes $\tilde{M}_1(l)$, $\tilde{M}_2(l)$, $M(l)$ derived from formulas (48), (49), according to Eqs. (7) (in case of the generalized threshold c), (36) the normalized estimates l_m (37), $q'_m = \sigma'^2_m / E_N$, $q''_m = \sigma''^2_m / E_N$ are defined, and also the decision on the presence, or the absence of the stepwise change of the process $\xi(t)$ dispersion is made. Further, the detection and measurement experimental characteristics are found.

Some results of the statistical simulation are presented in Figures 2-5 where corresponding theoretical dependences are also shown. Each experimental value is obtained by processing no less than $3 \cdot 10^4$ realizations of $x(t)$ (3) under $l_0 = 0.75$, $\tilde{\Lambda}_1 = 0.05$, $\tilde{\Lambda}_2 = 0.95$. Thus, with the probability of 0.9, the confidence intervals boundaries deviate from the experimental values no greater than by 10 %.

In Figure 1, by solid lines, the theoretical dependences (19) of the false-alarm probability α from a threshold c are drawn. The curve 1 is calculated for $\mu = 50$, 2 – 100, 3 – 200, 4 – 500. Experimental values for $\mu = 50$, 100, 200, 500 are designated by squares, crosses, rhombuses and circles. In Figure 2 solid lines represent the theoretical dependences of the missing probability $\beta(\Delta q)$ (35). Here $\Delta q = q'' - q'$ is the magnitude of stepwise change of the process $\xi(t)$ dispersion. For certainty it is supposed that $q' = 0.1$. The threshold c is determined from Eq. (19) by Neumann-Pirson criterion according, to the level of the false-alarm probability set equal to 0.01.

In Figure 3 by solid lines the dependences of the conditional variance $V_l(\Delta q) = V(l_m | l_0)$ (45) are plotted, and by dashed lines – the conditional variance $V_{0l}(\Delta q) = V_0(l_m | l_0)$ (38) of the normalized estimate of the stepwise change point l_m, as the functions of the variable Δq under $q' = 0.1$. Finally, in Figures 4 and 5 we deal with the theoretical dependences (46) of the normalized conditional variances $V_{q'} = V_0\left(\sigma'^2_m \middle| \sigma'^2\right) / E_N^2$, $V_{q''} = V_0\left(\sigma''^2_m \middle| \sigma''^2\right) / E_N^2$ of the estimates σ'^2_m, σ''^2_m (36) from the values q', q''.

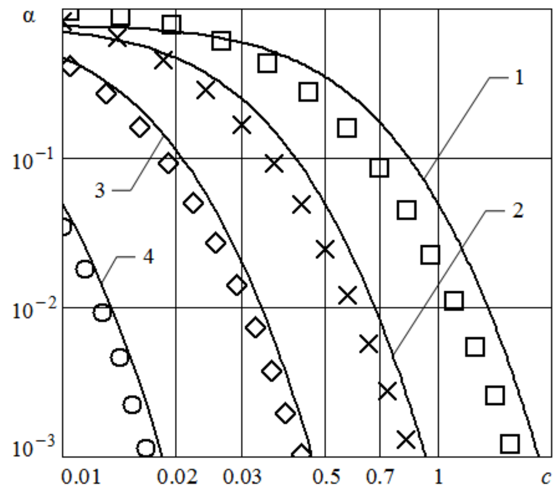

Figure 1: False-alarm probability

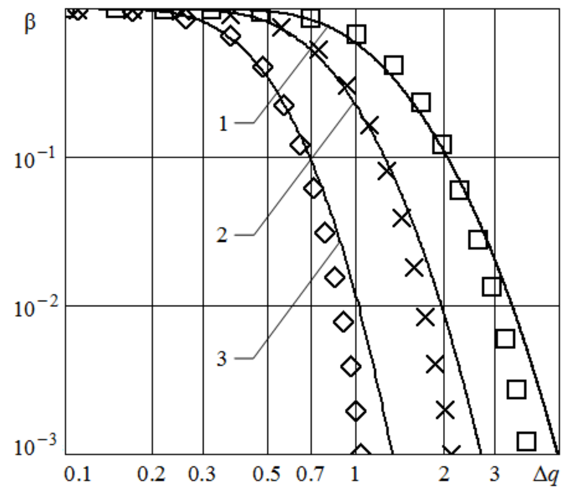

Figure 2: Missing probability

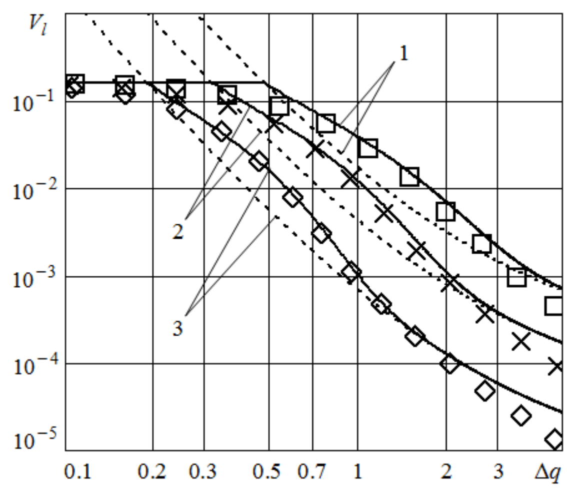

Figure 3: Variance of the normalized estimate of the stepwise change point

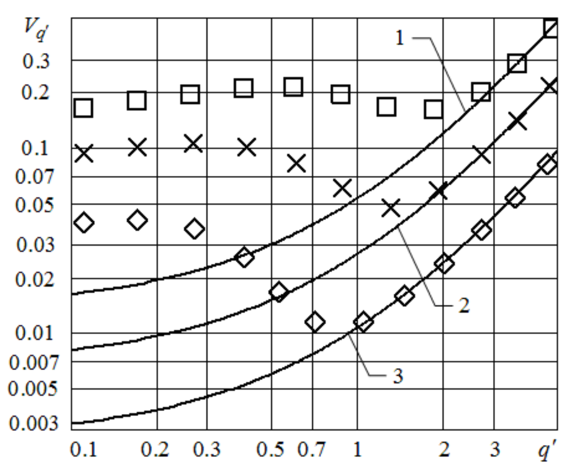

Figure 4: Normalized variance of the estimated dispersion before stepwise change

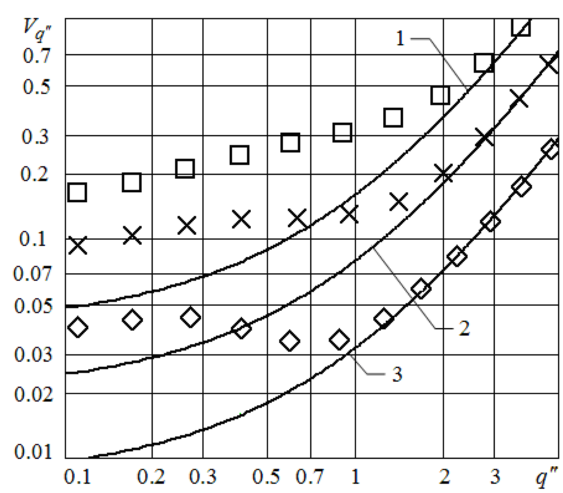

Figure 5 Normalized variance of the estimated dispersion after stepwise change

In Figures 2-5 the curves 1 are calculated for $\mu = 100$, 2 – 200, 3 – 500. Experimental values of the detection and measurement characteristics corresponding to the curves 1-3 are designated by squares, crosses, rhombuses.

From the conducted analysis and Figures 1-5, it follows that the theoretical dependences for the probabilities α (19), β (35) and the variance $V\left(l_m \middle| l_0\right)$ (45) already agree quite successfully with the experimental data, at least, under $\mu \geq 100$, $|\Delta q| \geq 0$, $\tilde{\Lambda}_1 \geq 0.05$, $\tilde{\Lambda}_2 \leq 0.95$. And if $z > 3.5\ldots4$ (25), then the simpler formula (38) can be used for calculating the variance of the estimate of the stepwise change point. When $|\Delta q| > 3$, the deviation of the experimental values of the variance $V\left(l_m \middle| l_0\right)$ is observed from the corresponding theoretical dependences obtained while using Eqs. (38) or (45). It is the result of the formulas for the functional $M(\lambda)$ and its characteristics (26) having been found on the assumption that the sizes of order of the correlation time of the process $\xi(t)$ are negligible. Hence, when MLE l_m variance decreases to the size of order $\mu_{\min}^{-2}$ (4), the calculation errors in Eqs. (38), (45) becomes considerable. Formulas (46) for the variances of the estimates $\sigma_m'^2$, $\sigma_m''^2$ satisfactorily approximate the experimental data under $z > 3.5\ldots4$ (25), when the distribution and the characteristics of the estimate of the stepwise change point l_m are described by the expressions (38).

5. CONCLUSION

In order to detect the stepwise change point in the fast-fluctuating Gaussian process and to measure its jumping and constant parameters, the maximum likelihood method can be effectively applied. This approach allows us to obtain the algorithms for determining stepwise change in the statistical characteristics of the random process in the conditions of the parametrical prior uncertainty, while neglecting the values of the order of the correlation time of the analyzed random process. These algorithms are technically the simplest ones in comparison with the common analogues. We apply the local Markov approximation method to write down the closed analytical expressions for the efficiency characteristics of the specified algorithms.

We used the statistical simulation to established that the obtained theoretical results successfully agree with the corresponding experimental data in a wide range of the observable data realization parameters values. Additional researches show that the detectors and the measurers synthesized by means of the introduced approach can also be used in the analysis of the stepwise changes of the statistical characteristics of the non-Gaussian high-frequency random processes and bring no great losses in performance.

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