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Evolutionary Approach to Model Order Reduction of Discrete Time Systems

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Abstract : In this paper, the authors presented an algorithm for the reduction of high order discrete- time Single Input Single Output (SISO) systems. The proposed order reduction algorithm is based on minimization of Integral Square Error (ISE) by Particle Swarm Optimization (PSO) technique for obtaining the coefficients of both numerator and denominator polynomials of the reduced order models. The proposed algorithm guarantees the stability of Reduced Order Models (ROM) if the original high-order system is stable. The proposed algorithm is having superior features, such as computational simplicity and ease of implementation, when compared to many of the available methods of order reduction of high-order discrete-time systems. The proposed algorithm has been successfully applied to a typical numerical example available in the literature and the results are compared with the results obtained by some of other familiar methods.

Keywords : Particle Swarm Optimization, Bi-linear transformation, Sub optimal Routh Approximation, Bilinear routh approximation, Integral Square error.

1. INTRODUCTION

The approximation of linear high-order systems by a reduced-order model has received considerable attention due to the advantages of reduced computational complexity, increased understanding of the original systems and ease of implementation for controller design. Consequently, a large number of time and frequency-domain simplification techniques have been developed to suit different requirements [3,4]. Amongst them, a frequency-domain method is Padé approximation in which 2k terms of the power series expansion (time moments) of the high-order (n^{th}-order) transfer function are fully retained in reduced-order (kth-order) model.

In some cases, Padé approximant may turn out to be unstable even though the original system is stable. To overcome stability problem, a number of stable reduction methods [5-9] based on retention of only k terms have been developed for discrete-time systems. However, matching of only terms may not generally be sufficient to ensure a good overall time response approximation and it is also important to note that, for overall time response approximation, both time moments and Markov parameters should be considered [10,11]. In [12], a Bilinear Routh Approximation (BRA) method has been

proposed as an extension of RA method [13] to discrete time systems. But it has been found that BRA method may fail to produce good approximants [14] as it again deals with k terms matching. Further improvement over [12] is suggested in [15]. However, the method of [15] does not possess any optimal properties. In a recent publication [16], a Suboptimal bilinear Routh approximation (SBRA) method is presented which is an improvement over BRA method and can be used to improve bilinear Schwarz approximation [17-19]. The SBRA method is based on combining Routh technique and minimization of

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ISE. The last and parameters of BRA method are replaced by new parameters so that the ISE of impulse response of the reduced model is locally minimized without destroying time moments fitting properties of BRA method. However, selecting the denominator coefficients arbitrarily and fixing time moments may bring a loss of considerable degree of freedom in optimization. It may also be noted that the methods [12-19] require bilinear transformation which is not an efficient operation as it involves extra computation and complexity especially for the systems with very high order. Thus, the essential problem is to obtain, avoiding bilinear transformation, a model which retains or near retains a few terms in excess of r terms while preserving stability.

In recent years, particle swarm optimization (PSO) technique has attracted considerable attention among various modern heuristic optimization techniques. Particle swarm optimization (PSO) is a population based stochastic optimization technique, developed by Eberhart and Kennedy [1] in 1995. Like many of the other biologically inspired algorithms (such as genetic algorithm, neural networks and simulated annealing), Particle swarm optimization also have a natural motivation, Two specific inspirations are that of fish schooling and bird flocking. The PSO method is a member of the wide category of Swarm Intelligence methods [2]. In PSO, a collection of particles (or agents) swarm through an N-dimensional space. The rules for how the particles move through the space are based on simple flocking rules that cause the particles to orbit around the best found solution in the hope of finding better one. This algorithm seems to be simple but it is effective and can be used in many types of optimization problems.PSO has been successfully applied in many areas: function optimization, artificial neural network training, fuzzy system control, and other areas where GA can be applied.

In this paper a computationally simple and effective method is proposed for model order reduction of discrete-time systems that preserves the stability. In this paper, the authors presented a technique for order reduction of high order discrete time systems based on minimization of ISE by PSO technique as an extension to the method presented in [11].

PSO ALGORITHM

In PSO each individual is referred to as particle. Each particle flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also flying experience of other particles. In PSO, each particle strives to improve itself by imitating traits from their successful peers. Further, each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it.

BASIC PSO FLOW

The basic flow for the PSO algorithm begin by initiating our population of particles. We provide each particle with a random location within the N-dimensional space and a random velocity for each dimension. We then evaluate each particle's fitness for the given problem. If the fitness is better than the particle's best fitness (pbest), we save the location vector for the particle. If the particle's fitness is better than the global best fitness (gbest), we save this particle's location vector as gbest. Finally, we update the particle's velocity and look at the next particle in the population. If our global fitness meets our exit criteria, we end the run and provide the location vector as the solution to the given problem.

PARTICLE MOMENT

Two basic equations govern the motion of particles in the swarm. The moment equation eq 1 provides for the actual motion of the particle using their specific velocity vector. The velocity update equation eq 2 provides for velocity vector adjustment given the two competing forces (gbest and pbest)

$$X_n = X_n + V_n \tag{1}$$

$$V_n = V_n + c_1 * \text{rand}()*(g_{\text{best},\,n} - X_n) + C_2 * \text{rand}() * (p_{\text{best},n} - X_n) \tag{2}$$

rand() = random number generator in between 0 and

c_1, c_2 = cognitive and social acceleration, respectively

V_n = velocity of nth particle

X_n = position of nth particle gbest,

n = global best position

$P_{best, n}$ = best position of n^{th} particle ever visited by it

2. REDUCTION PROCEDURE

Let the transfer function of original high order discrete time system of order 'n' be,

$$G_n(z) = \frac{a_c + a_1 + \dots\dots\dots + a_m z^m}{b_0 + b_1 z + \dots\dots\dots + b_m z^n}; m \le n$$

By using Bilinear Transformation $z = \frac{1+w}{1-w}$ $G_n(z)$ can be converted into $G_n(w)$ as

$$G_n(w) = G_n(w) = \frac{N(w)}{D(w)} = \frac{A_0 + A_1 w + \dots\dots\dots\dots + A_n w^n}{B_0 + B_1 w + \dots\dots\dots\dots + B_n w^n}$$

Let the Reduced Order Model (ROM) of order 'r' in w-domain be defined as

$$G_n(w) = G_r(w) = \frac{P_0 + P_1 w + \dots\dots\dots\dots\dots\dots + P_{r-1} w^{r-1}}{Q_0 + Q_1 w + \dots\dots\dots\dots\dots + Q_r w^r}; r < n \quad (2,1)$$

The deviation of ROM response from the original system is given by an error index 'E' known as Integral Square Error (ISE)

$$E = \int_0^\infty [y(t) - y_r(t)]^2 \, dt \quad (2.2)$$

where $y(t)$ = step response of $G_n(w)$

$y_r(t)$ = step response of $G_r(w)$

In a d-dimensional search space the particle updates its velocity and position with the flowing equations.

$$W = \frac{W_{max} - W_{min}}{maxgen} \; x \, gen$$

$$V_{kd}^{(t+1)} = W \, v_{kd}^t + c_1 \, rand() \, [p_{bestkd} - X_{kd}^{(t)}] + c_2 \, rand() \, [g_{bestd} - X_{kd}^{(t)}]$$

Where

W = inertia weight,

W_{max} = maximum value of inertia weight,

W_{min} = minimum value of inertia weight,

gen = current iteration,

k = 1, 2, 3,………………swarm size or population size,

d = 1, 2, 3………………..no. of variables *i.e.* '$2r$',

c_1, c_2 = Cognitive and Social acceleration respectively,

rand() = random numbers uniformly distributed in the range (0,1),

v_{kd}^{t} = velocity of d^{th} variable of k^{th} population in t^{th} iteration,

v_{kd}^{t} = position of d^{th} variable of k^{th} population in t^{th} iteration,

P_{bestkd} = best value of d^{th} variable among all the population,

P_{bestkd} = best value of d^{th} variable,

In the present study PSO algorithm is used to minimize the objective function 'E' given in eqn (2.2) and the parameters to be determined are the coefficients of numerator and denominator polynomials of reduced order model $G_k(w)$ as given in eqn (2.1), subject to following constraints

1. To have stable reduced model

$$d_i > 0 \; ; \; i = 0, 1, 2, \ldots\ldots.(r-1)$$

2. To have steady state error to zero in the approximation, the condition proposed is

$$d_0 = \frac{B_0}{A_0} x C_o$$

Once obtaining $G_r(w)$, applying inverse bilinear transformation $w = \frac{z-1}{z+1}$ to $G_r(w)$ the reduced model in z-domain $G_r(z)$ is obtained as

NUMERICAL EXAMPLE

Consider the discrete time system given by its transfer function [8]

$$G(z) = \frac{p_0 + p_1 z + \ldots\ldots\ldots\ldots + p_r z^r}{q_0 + q_1 z + \ldots\ldots\ldots\ldots + q_r z^r}$$

By using Bilinear transformation $z = \frac{1+w}{1-w}$, the transfer function G(z) can be written as

$$G(w) = \frac{2z^4 + 1.8z^3 + .8z^2 + 0.1z - 0.1}{z^4 - 1.2z^3 + 0.3z^2 + 0.1z + 0.02}$$

APPLICATION OF PROPOSED METHOD

It is proposed to apply the method suggested here using particle swarm optimization to obtain second order model. Parameters considered for the proposed PSO method of model reduction are

Maxgen = 500; Swarm Size or Population size = 20; Limits for variable = [0.001, 100]

By simulating the digital computer algorithm developed for the proposed reduction technique based on PSO, the second order reduced model is obtained as

$$R_2(w) = \frac{1.3091w + 4.6}{w^2 + 0.2437w + .0.054}$$

By applying inverse bilinear transformation $w = \frac{z-1}{z+1}$, the transfer function $R_2(w)$ can be written as

$$R_2(z) = \frac{2.439z^2 + 2.26z - 0.179}{1.2977z^2 - 1.892z + 0.8103}$$

The integral square error (ISE) between the original system and its reduced order model $R_2(z)$ is ISE = 0.123622.The step responses of the original system G(z) and its reduced order model $R_2(z)$

(proposed) are compared in Fig.2

COMPARISON WITH OTHER METHOD

1. The second order reduced model is obtained using “Suboptimal Bilinear Routh Approximation method” [9] as:

$$R_2^{'}(z) = \frac{2.54733z^2 + 2.36893z - 1.6465}{1.27367z^2 - 1.92181z + 0.80453} \quad \text{(SUBOPTIMAL BRAM)}$$

2. The second order reduced model is obtained using “Bilinear Routh Approximation method” [7] as:

$$R_2^{''}(z) = \frac{2.69474\,z^2 + 4.08421\,z - 2.62857}{1.34737\,z^2 - 1.90075\,z + 0.75190} \quad \text{(BRAM)}$$

3. The second order reduced model is obtained using “Improved Bilinear Routh Approximation method” [8] as:

$$R_2^{'''}(z) = \frac{2.77569z^2 + 5.02618z - 3.16789}{1.38784z^2 - 1.88919z + 0.72297} \quad \text{(BRAM)}$$

The step responses of the original system G(z) and its reduced order models $R_2(z)$

(proposed), $R_2'(z)$, $R_2''(z)$, $R_2'''(z)$ are compared in fig 2. It can be observed from fig 2 that the proposed method gives better approximation of the original system when compared to BRAM, IBRAM and Suboptimal BRAM methods.

Table. 1.

REDUCTION METHOD	***Reduced Order Model***	***ISE***
Proposed	$R_2(z) = \frac{2.439z^2 + 2.26z - 0.179}{1.2977z^2 - 1.892z + 0.8103}$	0.123622
Suboptimal BRAM	$R_2^{'}(z) = \frac{2.54733z^2 + 2.36893z - 1.6465}{1.27367z^2 - 1.92181z + 0.80453}$	2.52363
BRAM	$R_2^{''}(z) = \frac{2.69474\,z^2 + 4.08421\,z - 2.62857}{1.34737\,z^2 - 1.90075\,z + 0.75190}$	5.687473
IBRAM	$R_2^{'''}(z) = \frac{2.77569z^2 + 5.02618z - 3.16789}{1.38784z^2 - 1.88919z + 0.72297}$	9.171604

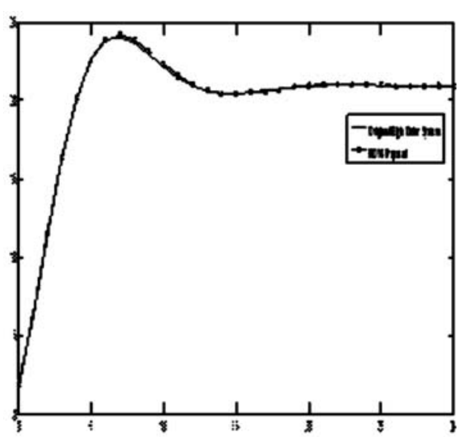

Fig. 2. Step responses Comparison of G(z) and $R_2(z)$.

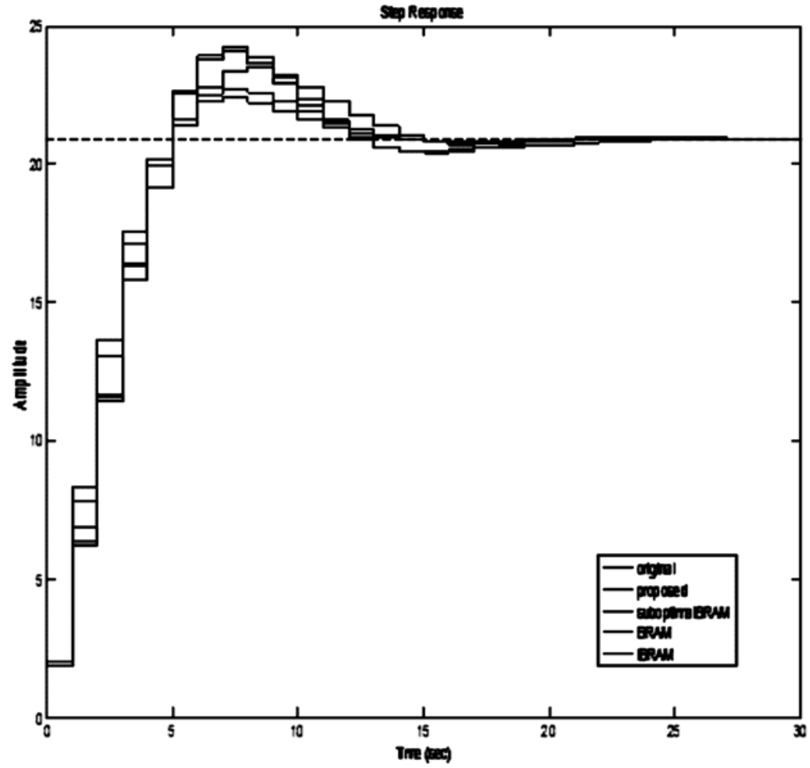

Fig. 3. Comparison of step responses of G(*z*), $R_2(z)$, $R_2'(z)$, $R_2''(z)$ and R2'''(*z*)

3. CONCLUSIONS

A new method for the reduction of high-order discrete- time systems is proposed to overcome some of the drawbacks of existing methods of discrete- time systems reduction. The proposed order reduction method is based on Particle Swarm Optimization technique for obtaining the coefficients of both numerator and denominator of the reduced order models. The proposed method retains the stability of the original high-order system in the reduced order models. The new method is observed to be computationally simple compared to many of the available methods of order reduction of high-order discrete-time systems. The flexibility and efficacy of the proposed method is shown through typical numerical examples considered from the literature The results are compared with the results obtained by some of the familiar methods and successfully verified to show the superiority of the proposed method over other methods.

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