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A Comparative Analysis and Design of Fuzzy Logic Based Fractional PID Controller with Variable Order

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Abstract: Fractional order control system is progressed on the concept of Fractional order calculus, which is an extended version of integer order calculus. Controller designing based on this advanced technology offers an extra degree of freedom towards the tuning approach. Fractional order PID controller is the late interpretation of traditional PID controllers. It takes arbitrary real order powers instead of integer order in the place of integrator and differentiator.

In this paper, a new kind of method for designing Fuzzy logic based Fractional PID controller ($PI^{\lambda}D^{\mu}$) is presented for a class of integer order and fractional order systems. After that variable order Fuzzy Fractional PID controller is designed for the same plants and a comparative study is proposed by taking the graphical simulations for different values of λ and μ. Here Fuzzy logic controller is used due to its flexibility towards the parameter variations.

The design methodology and simulation are developed in graphical user interfacing (GUI) platform LabVIEW, which is powerful tool in measurement and automation world having the capability of data acquisition, monitoring, processing and controlling.

Index Terms: Fuzzy logic, PID, Fractional order PID, Traditional PID, Fuzzy Fractional PID, LabVIEW.

1. INTRODUCTION

Fractional calculus is a generalization of integer order calculus which are used widely in control applications. Fractional order system and Fractional order controller have been widely studied now, fractional order controller design is the easiest way for different systems for tuning and is becoming an advanced area of research in control design and simulation platform.

Fractional order PID controllers can achieve better performance and robustness results superior to those obtained with integer PID controllers [1], but the design and tuning of Fractional order controller is bit challenging than integer order controllers since it provides better flexibility and degree of freedom.

Directly controlling and tuning of fractional order control system through conventional PID controller is not possible. So we have to go for further modification of fractional order systems, which can be easily controlled by integer order PID controller. Therefore, we are going for integer order approximation of fractional order systems. Therefore, a fractional order system can be approximated by higher order polynomials having integer differ-integral operators. This method is divided into 2 types [2].

- Continuous Time realization
- Discrete Time realization

Here, we are considering continuous time realization type.

In this paper, we assemble the features of Fuzzy controllers with Fractional PID type. The resulting Fuzzy-Fractional PID (FFPID) controller is examined in terms of its digital interpretation and robustness

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on the control of a Fractional-order plant. The combined advantage of the two controllers results in a better controller with superior robustness and wider domain of application.

There have been a lot of studies in the application of Fuzzy Fractional PID controller. In [3] it was proved the effectiveness and robustness of Fuzzy Fractional PID controllers. Manually tuning of PID controller using fuzzy logic also mentioned in paper [4]. It is applied also in FPGA application [5]. Using Genetic algorithm this is also used in design of an active magnetic bearing system [6]. Many other applications could be found in [7], [8], [9], [10], [11], and [12].

The remaining part of the paper is organized as follows: Section II denotes the introduction to Fractional calculus, Section III presents the Fractional order system designing and approximation towards integer order model, Fuzzy Fractional model designing and simulation is given in Section IV, Results and analysis shown in Section V and Section VI provides the Conclusion and Future work of the project.

2. FRACTIONAL ORDER SYSTEM ANALYSIS

A. Fractional Calculus

It deals with the derivatives and integrals of non-integer orders. There are different definition proposed by Cauchy, Riemann-Liouville and Grunwarld-Letnikov. These definitions are given below,

$$\mathrm{D}^{\gamma} f(t) = \frac{\Gamma(\gamma+1)}{2\Pi j}\int_{c}\frac{f(\tau)}{(\tau-t)^{\gamma+1}}\,d\tau \tag{1}$$

Equation (1) is Cauchy's Fractional order integration formula. Hence C is the smooth curve encircling the single-valued function $f(t)$.

$${}_{a}\mathrm{D}_{t}^{\alpha} f(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left[\frac{t-a}{h}\right]}(-1)^{j}\binom{\alpha}{j} f(t-jh) \tag{2}$$

Equation (2) is the Grunwald-Letnikov definition. Here, $w_{j}^{\alpha} = (-1)^{j}\binom{\alpha}{j}$ represents the co-efficient of the polynomial $(1-z)^{\alpha}$. This can be obtained by,

$$w_{0}^{\alpha} = 1,\ w_{j}^{\alpha} = \left(1-\frac{\alpha+1}{j}\right) w_{j-1}^{\alpha}\ j = 1, 2, 3 \ldots$$

$${}_{a}\mathrm{D}_{t}^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_{a}^{t}(t-\tau)^{\alpha-1} f(\tau)\,d\tau \tag{3}$$

Equation (3) is the Riemann-Liouville fractional order differentiation. Here $0 < \alpha < 1$ and a is the Initial time instance.

3. FRACTIONAL ORDER SYSTEM DESIGNING AND APPROXIMATION

The general Fractional order system of a single variable can be represented as,

$$G(s) = \frac{b_1 s^{\gamma 1} + b_2 s^{\gamma 2} + \ldots b_m s^{\gamma m}}{a_1 s^{\eta 1} + a_2 s^{\eta 2} + \ldots a_{n-1} s^{\eta n-1} + a_n s^{\eta n}} \tag{4}$$

Here, a_i and b_i are real numbers. Orders γ_i, η_i of numerator and denominator are also real numbers.

By considering Grunwald-Letnikov definition in equation (2), the discrete form can be obtained as,

$$ {}_aD_t^{\eta t}\, y(t) \approx \frac{1}{h^{\eta i}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} w_j^{\eta i}\, y_{t-jh} \tag{5}$$

By taking the above considerations we can get the step response of the fractional order system and for frequency response we can take Bode analysis directly.

For integer order approximation we have taken Oustaloup's recursive Filter, which gives very good fitting to the fractional order elements within a chosen frequency band.

The filter can be written as [13],

$$ C(s) = K \prod_{-N}^{N} \frac{s + w_{p_n}}{s + w_{z_n}} \tag{6}$$

Here, Poles (w_{p_n}), Zeros (w_{z_n}) and gain (K) can be calculated as,

$$ w_{p_n} = w_b \left(\frac{w_h}{w_b}\right)^{\frac{(k+\mathrm{N}+1)/2(1-\gamma)}{2\mathrm{N}+1}}; $$

$$ w_{z_n} = w_b \left(\frac{w_h}{w_b}\right)^{\frac{(k+\mathrm{N}+1)/2(1+\gamma)}{2\mathrm{N}+1}}; $$

$$ \mathrm{K} = w_h^{\gamma} $$

Here, γ = Order of differentiation

$2N + 1$ = Order of the filter

Frequency range = (w_b, w_h)

B. Fractional Order PID Controller Designing

In this section, we are implementing the designing of Fractional Order PID Controller, which is controlling by giving the K_p, K_i, K_d, λ and μ value manually.

The fractional-order controller of PID-type, usually named $PI^{\lambda}D^{\mu}$ controller, may be given as,

$$ C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{7}$$

Where K_p, K_i and K_d are the proportional, integral and derivative gains, and usually λ and μ are the fractional orders. Figure 1 shows the block diagram of the $PI^{\lambda}D^{\mu}$ equation which is implemented using LabVIEW.

As per it is manually controlled so let us take the values of $K_p = 3$, $K_i = 2$, $K_d = 5$, $\lambda = 1.78443$ and $\mu = 1.60479$ whose response is shown in Figure 2.

4. FUZZY FRACTIONAL MODEL DESIGNING AND SIMULATION

Fuzzy Control adopts human logical thinking when making decisions. It applies the IF–THEN during computation algorithms. Number of rules can be selected according to the application and control resolution required. In closed loop application, two input parameters were essential that is the magnitude of error $e(t)$

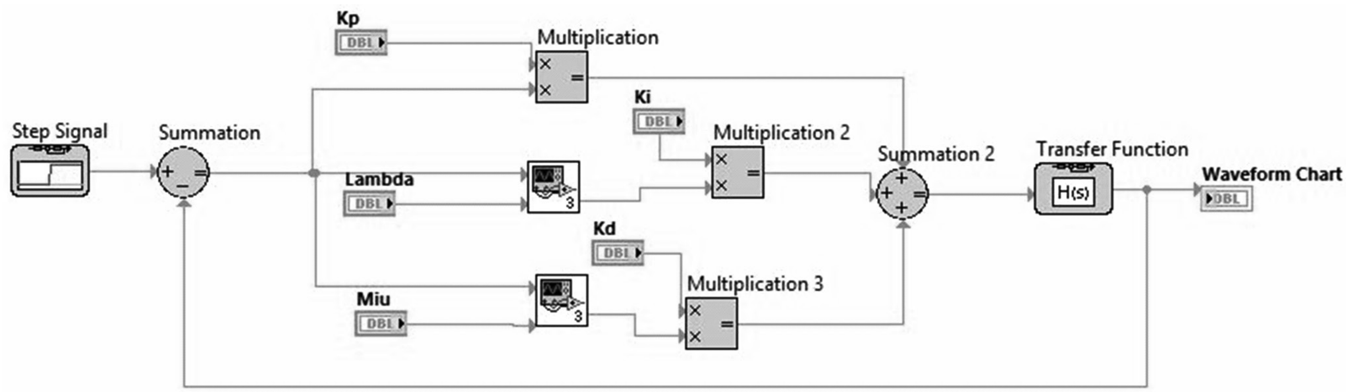

Figure 1: Block diagram of Fractional Order PID Controller using LabVIEW

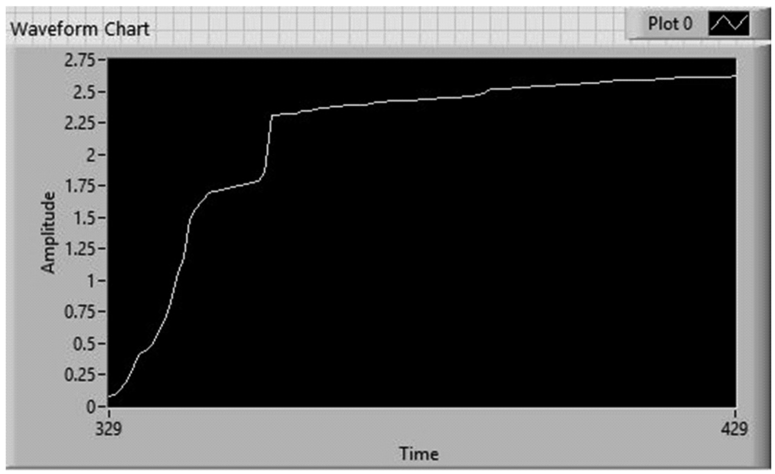

Figure 2: Step response of Fractional Order PID Controller at given values using LabVIEW

and change in error $\frac{de(t)}{dt}$. Based on these values desired control signal can be delivered. Fuzzy will tune the PID parameters K_p, K_i & K_d according to the current value of error and change in error.

The Linguistic variables we have taken were defined as Negative Big(NB), Negative Medium(NM), Negative Small(NS), Zero(ZO), Positive Small(PS), Positive Medium(PM) & Positive Big(PB).

This Figure 3 shows block diagram of the variable order Fuzzy fractional order Controller.

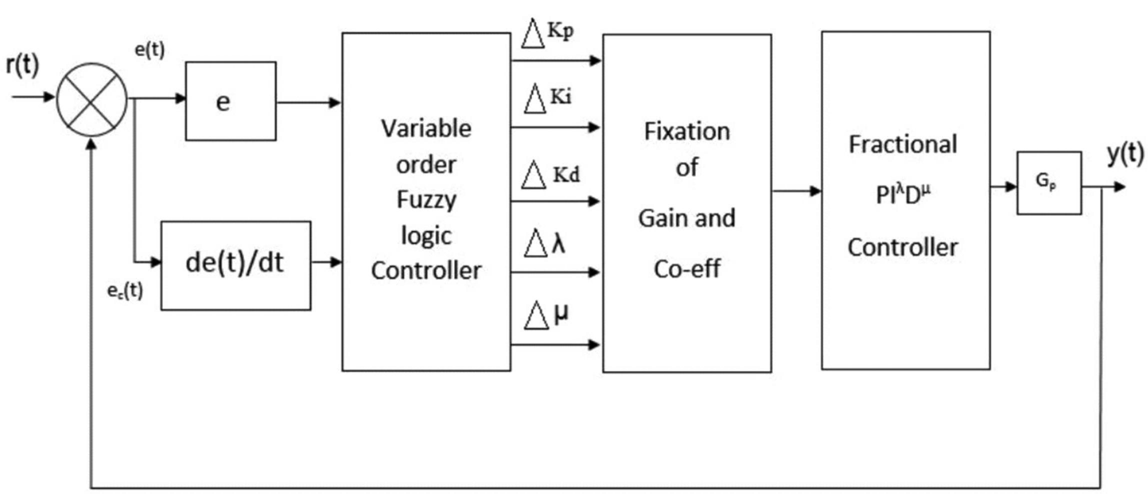

Figure 3: Block diagram of the variable order Fuzzy fractional order Controller

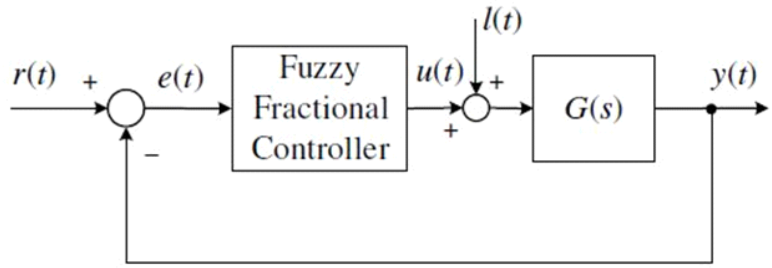

Figure 4: Fuzzy Fractional PID Control system

5. RESULT ANALYSIS

Here we have discussed about the Fuzzy fractional PID Controller which will be designed from the fuzzy rule base made according to the input and output variables. Here we have taken the input variables as error and change in error. The output variables are taken as K_p, K_i, K_d, λ and μ. One of the Fuzzy rule base is shown in Table 1. Which is done for K_p. Like this for output variables the rule base should be prepared. Figure 5 and 6 represents the membership function of the Input and output variables. The range was set and classified into several membership functions and designated with its own linguistic variables. The input out relationship is also shown in the Figure 7.

Here after the making of Fuzzy rule we have to draw the block diagram of Fuzzy Fractional PID controller which is shown in Figure 8. The output response is shown in Figure 9.

Table 1
Fuzzy Rule Base Table for K_p

e	e_c						
	NB	*NM*	*NS*	*ZO*	*PS*	*PM*	*PB*
NB	PB	PB	PM	PM	PS	ZO	ZO
NM	PB	PB	PM	PS	PS	ZO	NS
NS	PM	PM	PM	PS	ZO	NS	NS
ZO	PM	PM	PS	ZO	NS	NM	NM
PS	PS	PS	ZO	NS	NS	NM	NM
PM	PS	ZO	NS	NM	NM	NM	NB
PB	ZO	ZO	NM	NM	NM	NB	NB

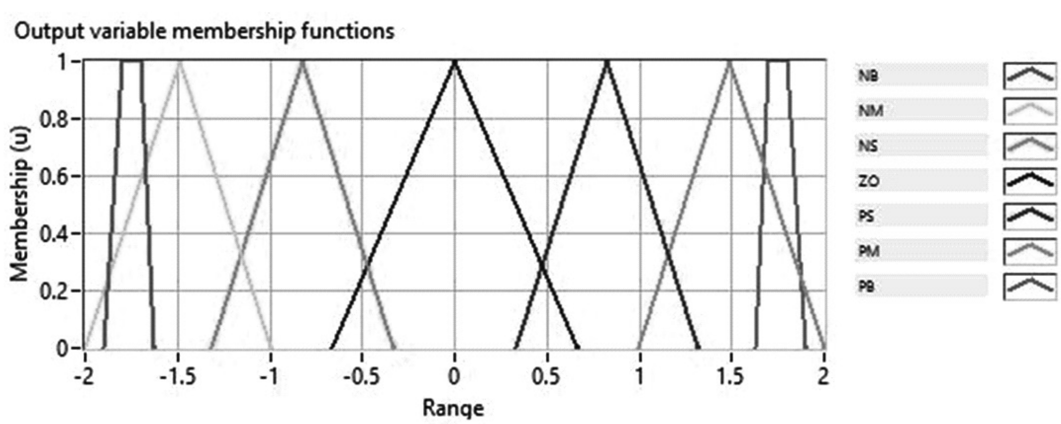

Figure 5: Membership Function diagram of Output variables

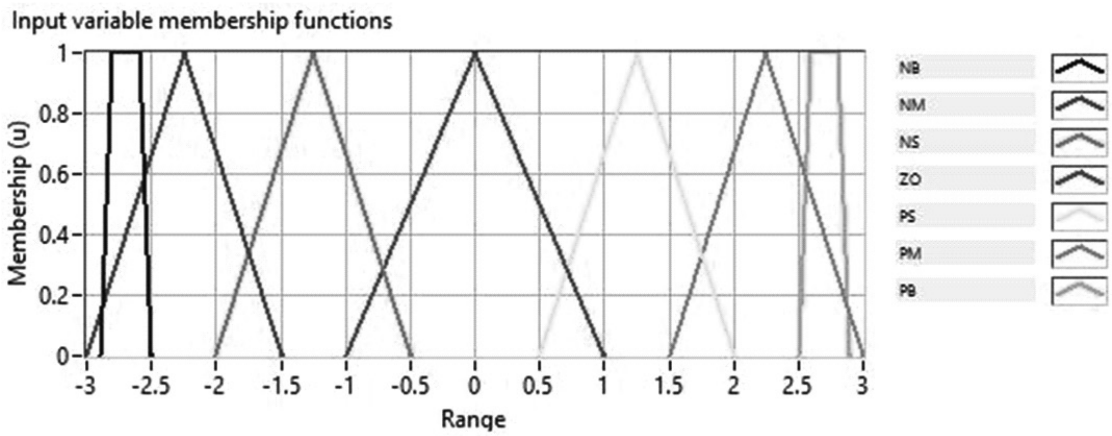

Figure 6: Membership Function diagram of Input variables

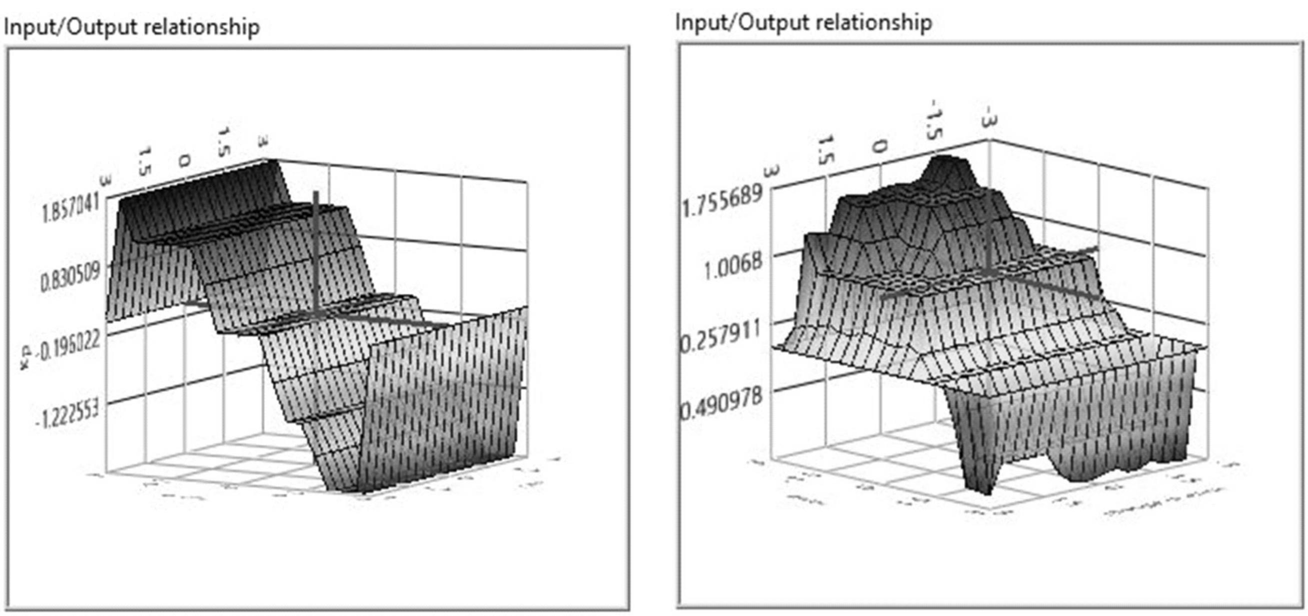

Figure 7: Input output relationships of Fuzzy rules

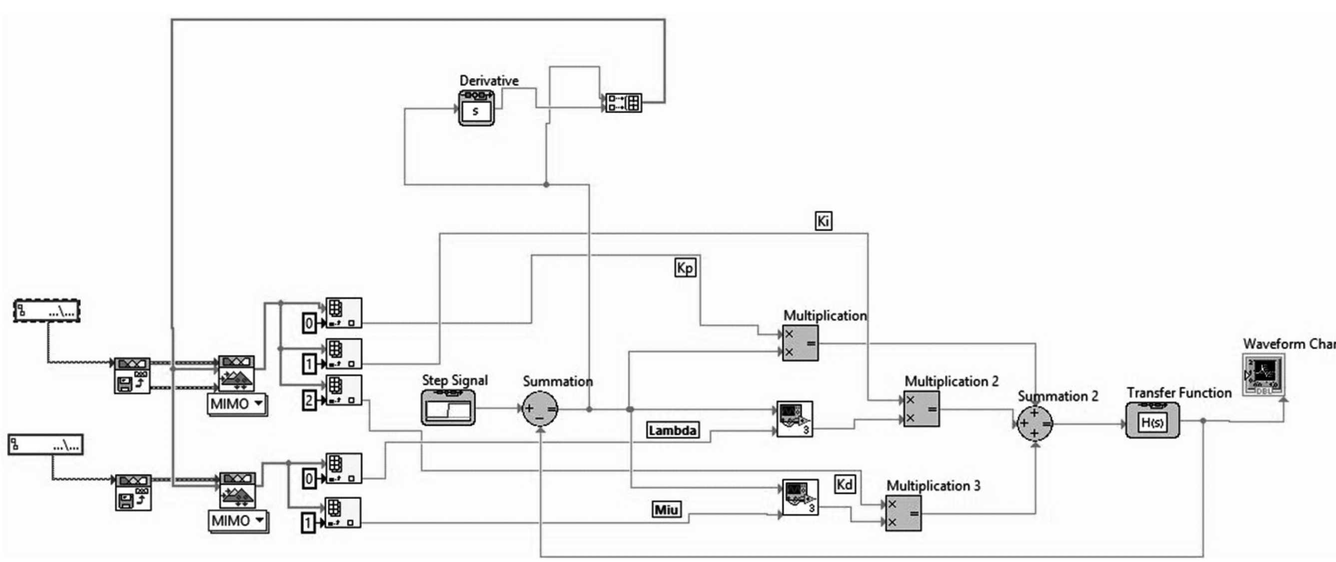

Figure 8: Block diagram of Fuzzy Fractional PID controller Using LabVIEW

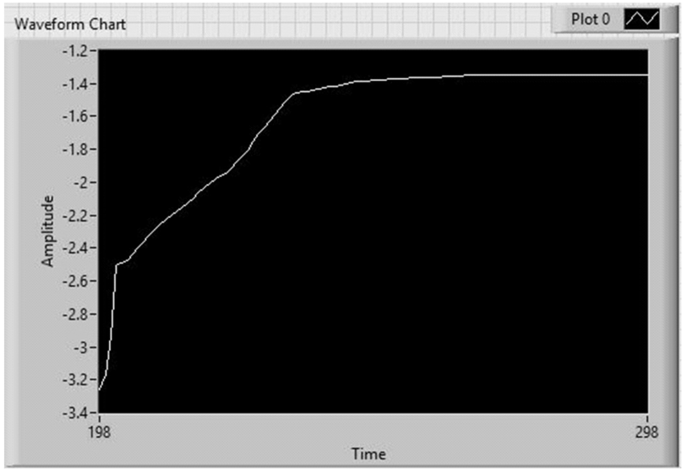

Figure 9: Step response of Fuzzy Fractional PID Controller using given Fuzzy rules

6. CONCLUSION AND FUTURE WORK

In this paper we have developed the Fuzzy Fractional PID Controller. The proposed technique of Fractional Order PID Controller is a manual tuning method which is controlling by adjusting the K_p, K_i, $K_{d,,}$ λ and μ value. The future work of the project is to design the Fuzzy Fractional PID Controller for an auto tuning Fractional Order PID Controller by using LabVIEW Software. Here LabVIEW is efficient for easy programming and real time implementation. It has the capability of data acquiring, processing, monitoring and controlling.

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