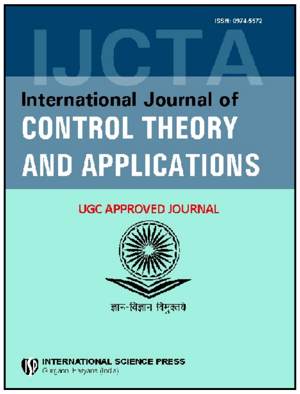


International Journal of Control Theory and Applications

ISSN : 0974-5572

Volume 10 • Number 34 • 2017

Dynamical Analysis of Financial Risk Chaotic System and its Engineering Applications

Yusuf Abdullah[a], Mustafa Mamat[b], Sundarapandian Vaidyanathan[c], *Aceng Sambas[b,d], and Mada Sanjaya WS[e]

[a] *Magister Management Program, Siliwangi University of Tasikmalaya, Indonesia*
[b] *Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Kuala Terengganu, Malaysia*
[c] *Research and Development Centre, Vel Tech University, Avadi, Chennai, India*
[d] *Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, Indonesia*
[e] *Department of Physics, Universitas Islam Negeri Sunan Gunung Djati, Bandung, Indonesia*
* *Corresponding author; Email: acengs@umtas.ac.id*

Abstract: In this work, the effect of the risk management process of ûnancial markets by using tools of nonlinear dynamics is studied. Meanwhile, Various tools of nonlinear theory, such as phase portraits, Lyapunov exponents, Kaplan Yorke dimension and Poincare map, have been used. The mathematical model of financial risk chaotic system is accompanied by an electrical circuit implementation, demonstrating chaotic behavior of the strange attractor. Finally, the results of circuit experimental results and numerical simulation are helpful for better understanding of nonlinear financial risk chaotic systems.

Keywords: financial markets, ûnancial risk chaotic system, Lyapunov exponents, bifurcation diagram, Poincaré map and circuit design.

1. INTRODUCTION

Chaotic system research is popular problem in the nonlinear science ûeld. Chaos has been widely applied to many economic and management, such as business cycle [1-2], volatility of security price [3-4], monetary aggregate [5-6], duopoly game [7-8] and the relationship between R&D spending and production growth [9-10], foreign direct investment [11-12], foreign capital investments [13], a growth cycle in socialism, capitalism and economic Growth [14] etc.

In 1967, Goodwin introduced a fluctuations of the economic aggregates come from endogenous factors [15]. In 1994, Burlando proposed the structure and terms of chaos theory to study ûnancial market risks [16]. In 2009, Pribylová introduced a Van der Pol's equation applied to economic models [17] and Lin (2011) constructed a Dufûng–Holmes model to describe of ûnancial chaos [18] and In recent years, chaotic economics has obtained intensive attention and has been raised to engineering application in understand the complex behavior of the real financial market. Such as a emulating complex business cycles by using an electronic analogue [13], nonlinear financial by using electronic circuit [19] and circuit implementation of a nonlinear fractional incommensurate order ûnancial system [20].

The aim of this study is to examine the chaotic behaviour of a financial risk chaotic model from an electronics analogue. An electronic analogue emulating its behaviour is proposed and its rich dynamic behaviour is studied by means of electronic circuit simulation tools, such as MultiSIM.

Motivated by the above researches, a financial risk chaotic system is proposed in this research, In Section 2, we present a financial risk chaotic system, numerical results in evolving phase portraits, Lyapunov exponent's analysis, Kaplan Yorke dimension and Poincaré map analysis. In Section 3, we present an electronic circuit that implements the a financial risk chaotic system and Finally, Section 4 contains the conclusion remarks.

2. THE FINANCIAL RISK MODEL

In 2013, a third-order dynamical model, describing a risk management process in the financial market, was reported [21]. This nonlinear financial risk chaotic system is studied by the following set:

$$\left.\begin{aligned} x_i &= my_{i-1} - n(x_{i-1} - dy_{i-1}z_{i-1}) \\ y_i &= rx_{i-1} - x_{i-1}z_{i-1} \\ z_i &= x_{i-1}y_{i-1} + kz_{i-1} \end{aligned}\right\} \tag{1}$$

Where x_i, y_i, z_i desribe occurrence value risk, analysis value risk, and control value risk in the current market, respectively. $y_{i-1}z_{i-1}$ represents for the value risk preliminarily controlled by the analysis risk, $x_{i-1}z_{i-1}$ describes the value risk preliminarily controlled by the occurrence risk and $x_{i-1}y_{i-1}$ represents the value risk preliminarily analyzed to the occurrence risk. The parameter *m* denotes the analysis risk efficiency, *n* denotes the risk satisfaction rate, *d* denotes the risk control efficiency, *r* denote the transmission rate of previous risk and *k* denotes for the distortion coefficient of risk control. Three state variables *x*, *y*, and *z* must posistive, because risk in financial markets always exists as the market occurs.

Let, $m = d$, $n + 1 = d$, $nd = 1$ and $k = 1 - b$. the discrete system (1) can be transformed into the following continuous form [21]:

$$\left.\begin{aligned} \dot{x} &= \delta(y - x) + yz \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - bz \end{aligned}\right\} \tag{2}$$

The system (2) is chaotic when the parameter values are taken as $d = 10$, $r = 28$ and $b = 8/3$. We take the initial conditions of the system (1) as (10, 10, 10). In this study the system's chaotic behavioris investigated numerically by employing a fourth order Runge-Kutta algorithm. In the MATLAB 2010, ODE45 solver yielding a fourth-order Runge-Kutta integration solution has been used. Figure 1 shows the 3-D phase portrait of the financial risk chaotic system (2). Figures 2, 3 and 4 show the 2-D projection of the financial risk chaotic system (2) on the (*x*, *y*), (*x*, *z*) and (*x*, *z*) planes, respectively.

The dynamics behavior of the financial risk chaotic system can be characterized with its Lyapunov exponents which are computed numerically by Wolf algorithm proposed in Ref. [42]. The Lyapunov exponents of the financial risk chaotic system are obtained as $L_1 = 1.251$, $L_2 = 0.0020064$ and $L_3 = -14.9197$, while the Kaplan-

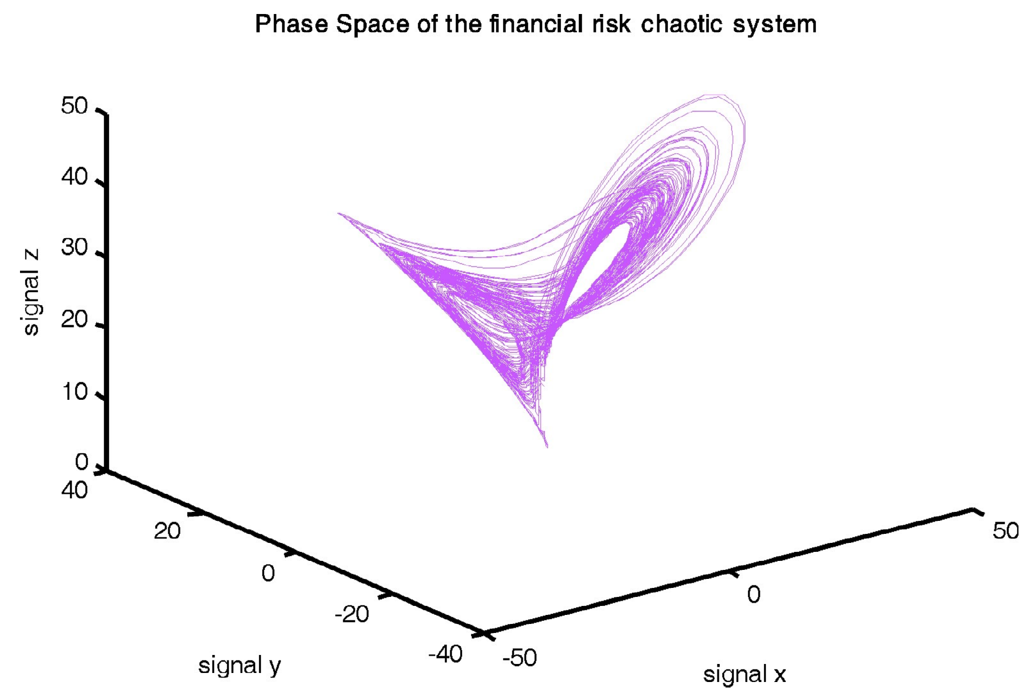


Figure 1: 3-D phase portrait of the financial risk chaotic system

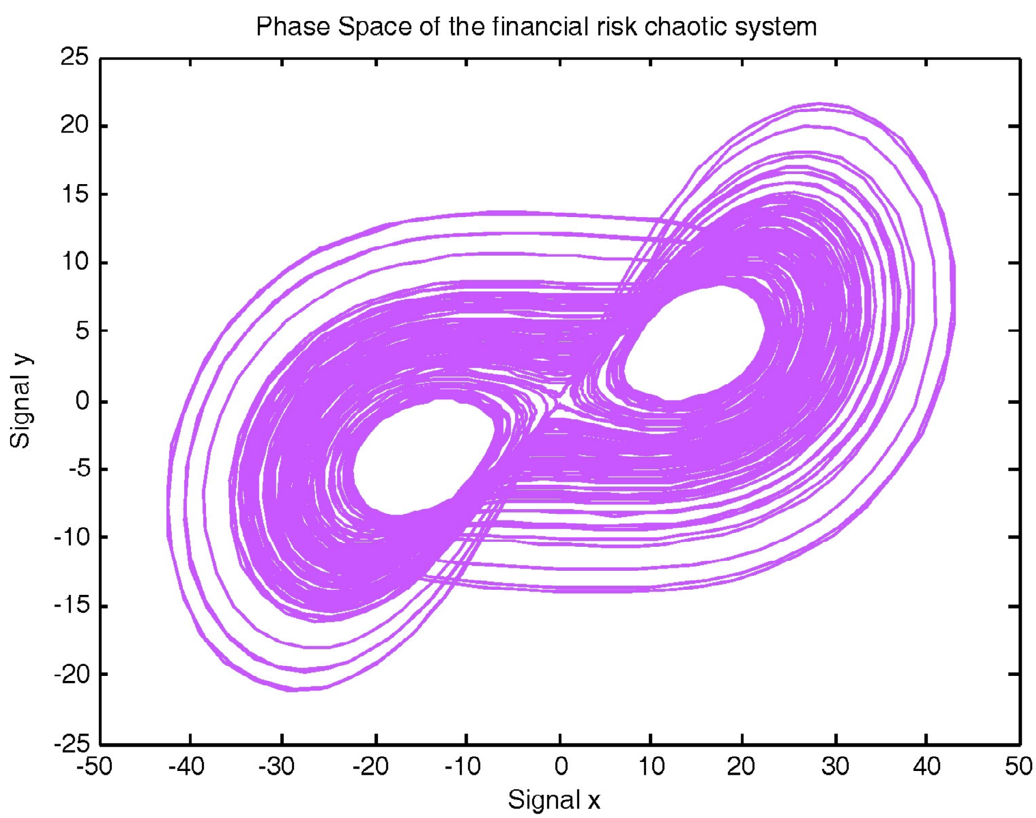


Figure 2: 2-D projection of the financial risk chaotic system on the (x, y) plane

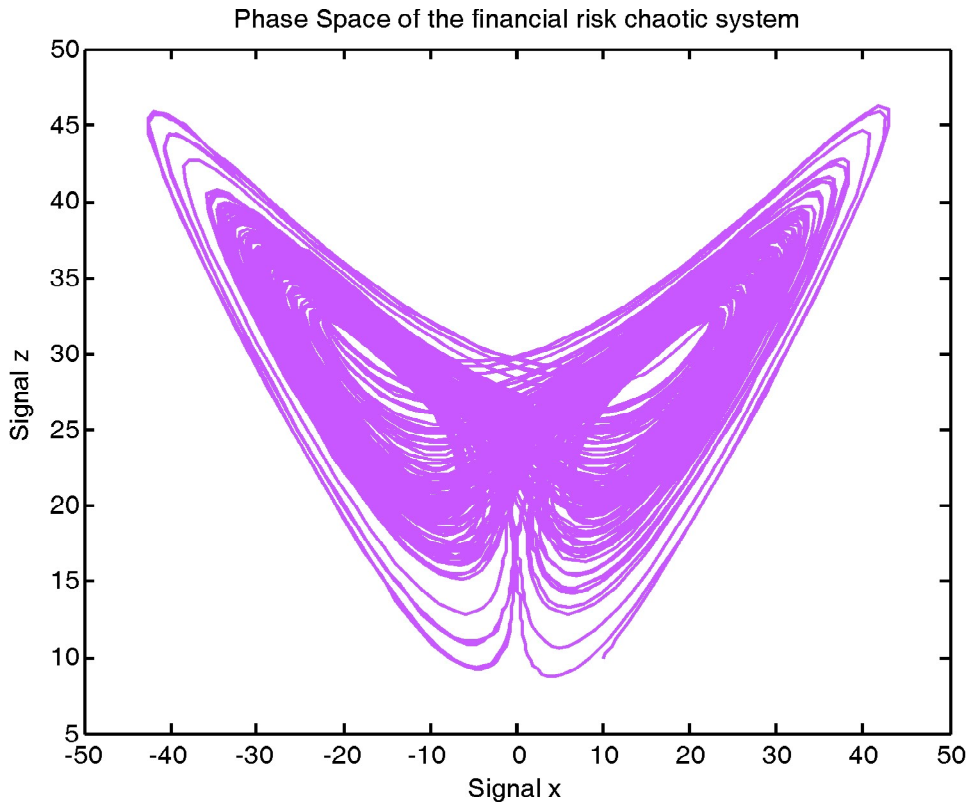


Figure 3: 2-D projection of the financial risk chaotic system on the (x, z) plane

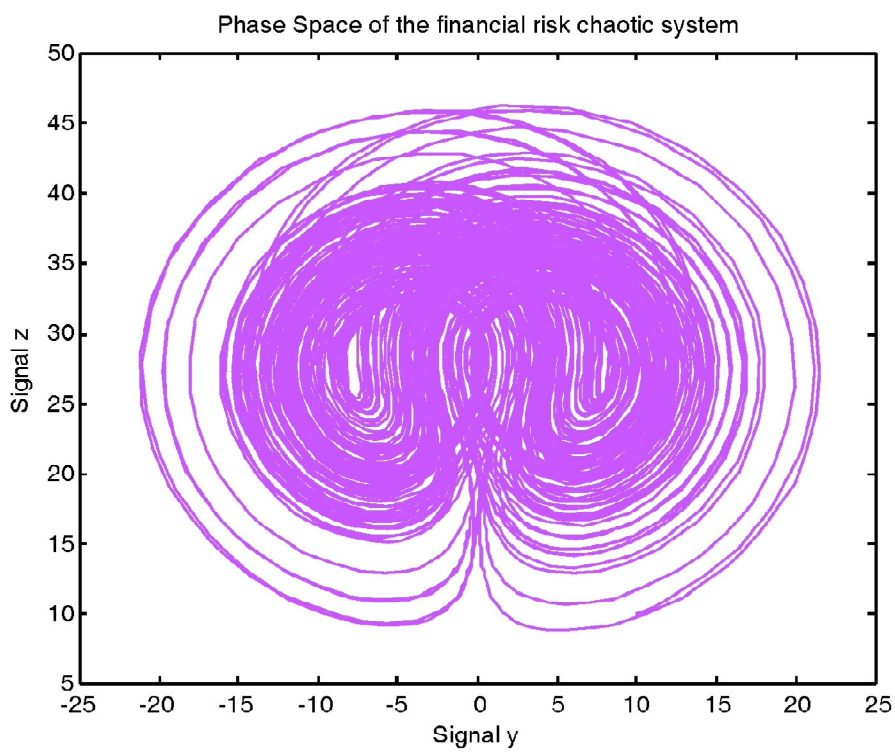


Figure 4: 2-D projection of the financial risk chaotic system on the (y, z) plane

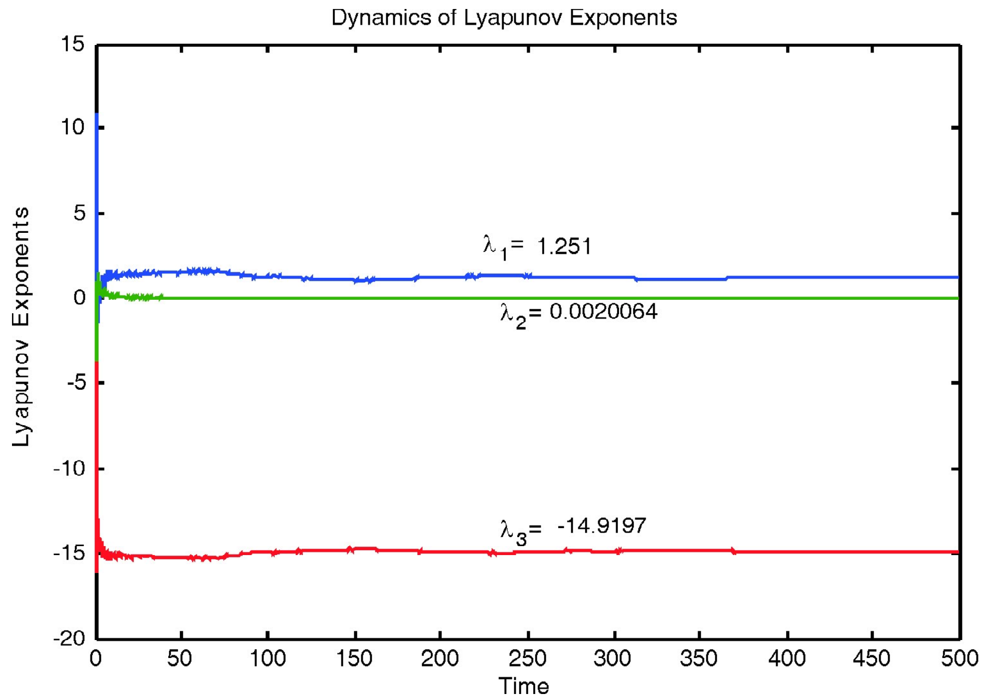


Figure 5 The dynamics of Lyapunov exponents of financial risk chaotic system for *d* = 10, *r* = 28 and *b* = 8/3, using MATLAB 2010

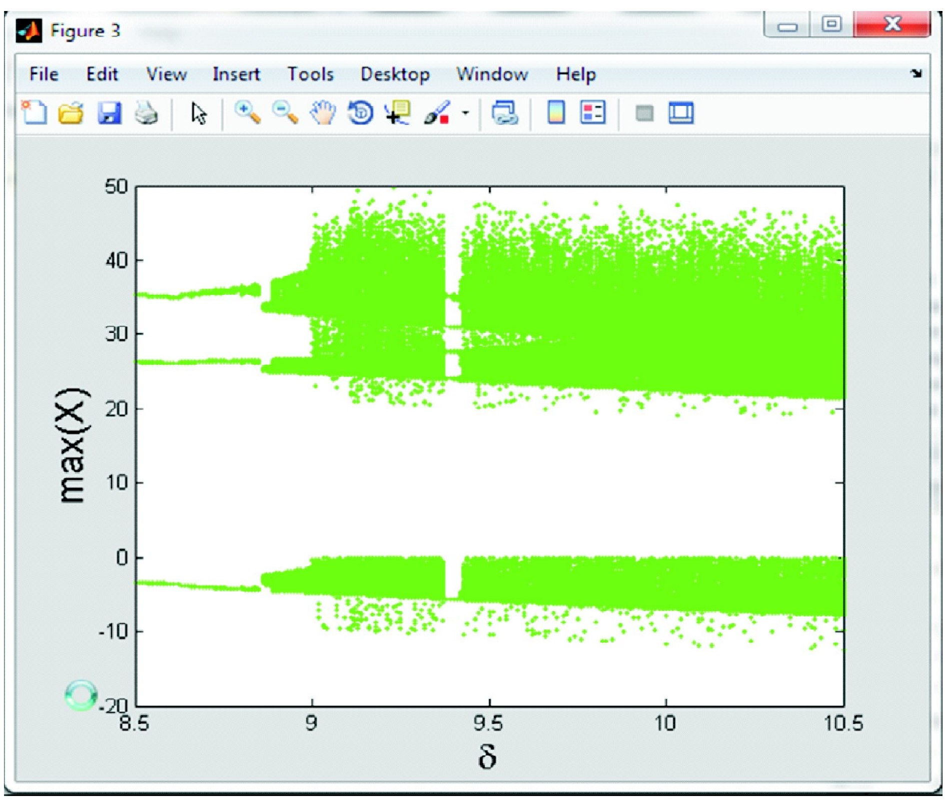


Figure 6: Bifurcation diagram of max(X) versus d, for r = 28, b = 8/3

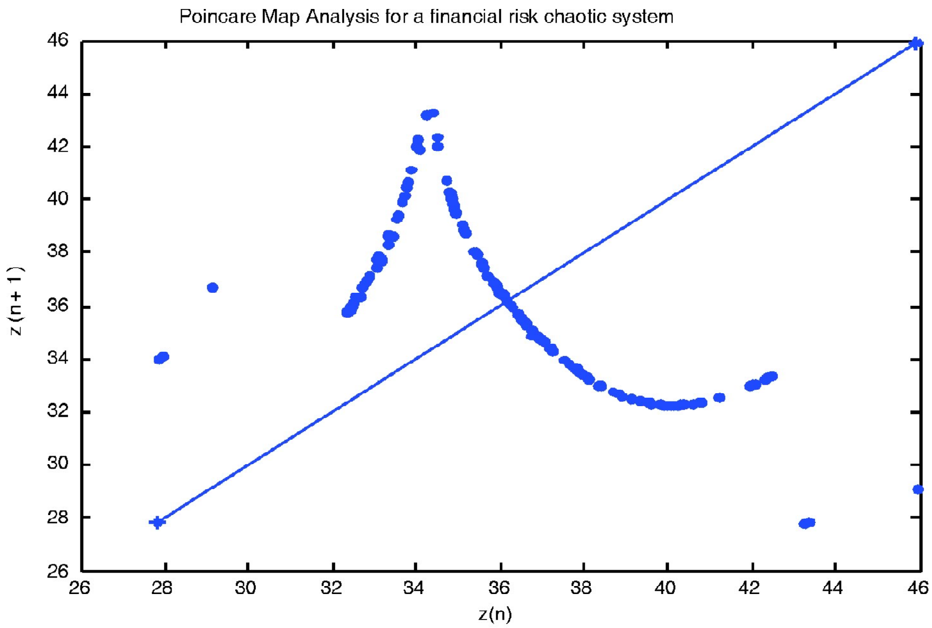


Figure 7: Poincare map in the plot given the maxima of $x(n + 1)$ against those of $x(n)$ space plane when for $d = 10$, $r = 28$ and $b = 8/3$, using MATLAB 2010

Yorke dimension of the financial risk chaotic system is obtained as $D_{KY} = 3.0839$. Figure 5 shows the Lyapunov exponents of financial risk chaotic system for constant parameter $\delta = 10$, $r = 28$ and $b = 8/3$. The bifurcation diagrams of the occurrence value risk x versus the analysis risk efficiency δ, see Figure 6. As the value of $\delta < 8.9$ the system remains always in periodic state. While for $\delta \leq 8.9$ a complex behavior is chaotic attractor. In addition, the Poincare map of the system in Figure 7 also reflects properties of chaos.

3. CIRCUIT REALIZATION OF THE FINANCIAL RISK CHAOTIC SYSTEM

In this section, we introduce an electronic analogue modeling of the financial risk chaotic system. The circuit in Figure 8 has been designed following an approach based on operational amplifiers TL082CD [23-27] where the state variables x, y, z of the system (2) are associated with the voltages across the capacitors C_1, C_2 and C_3, respectively. The nonlinear term of system (2) are implemented with multiplier AD633. By applying Kirchhoff's circuit laws, the corresponding circuital equations of designed circuit can be written as:

$$\left.\begin{aligned} \dot{x} &= \frac{1}{C_1R_1}y - \frac{1}{C_1R_2}x + \frac{1}{10C_1R_3}yz \\ \dot{y} &= \frac{1}{C_2R_4}x - \frac{1}{C_2R_5}y - \frac{1}{10C_2R_6}xz \\ \dot{z} &= \frac{1}{10C_3R_7}xy - \frac{1}{C_3R_8}z \end{aligned}\right\} \tag{3}$$

We choose With $R_1 = R_2 = R_3 = R_6 = R_7 = 1\ K\Omega$, $R_4 = 350\ \Omega$, $R_8 = 3.75\ K\Omega$, $R_5 = R_9 = R_{10} = R_{11} = R_{12} = 10\ K\Omega$, $C_1 = C_2 = C_3 = 10$ nF. The circuit has three integrators by using Op-amp TL082CD in a feedback loop and three multipliers IC AD633. The supplies of all active devices are ±9 Volt. With MultiSIM 10.0, we obtain the experimental observations of system (2) as shown in Figure 9. As compared with Figure 1 a good qualitative agreement between the numerical simulation and the MultiSIM 10.0 results of the financial risk chaotic system is confirmed.

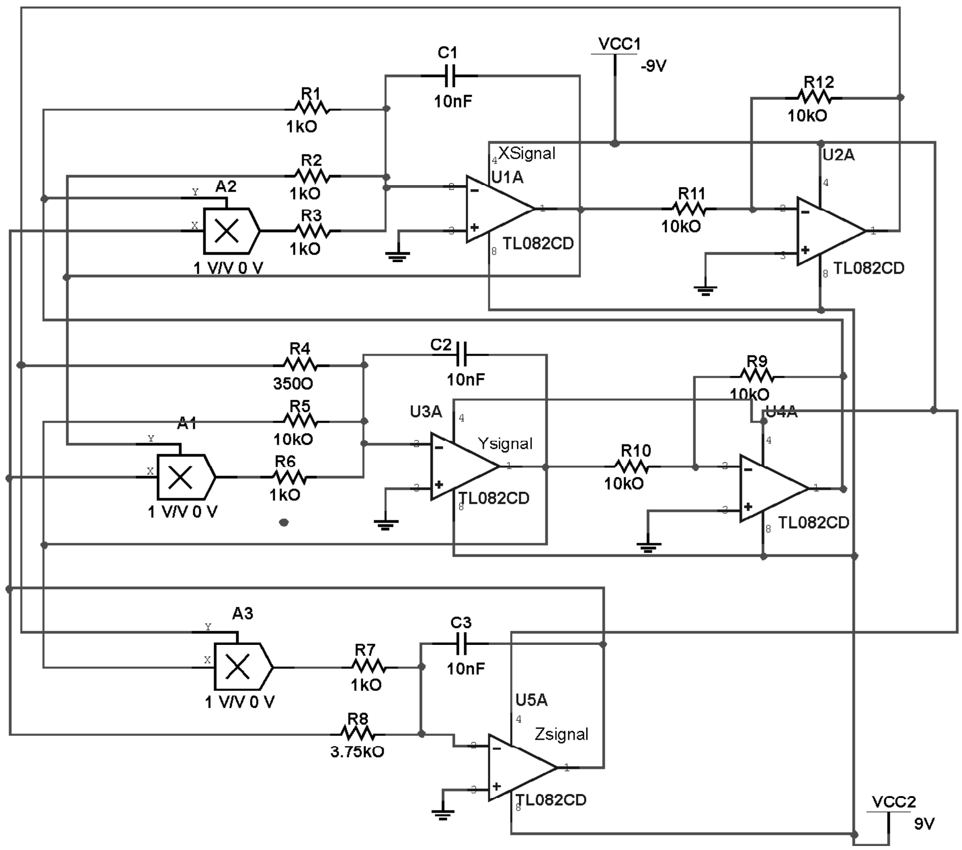
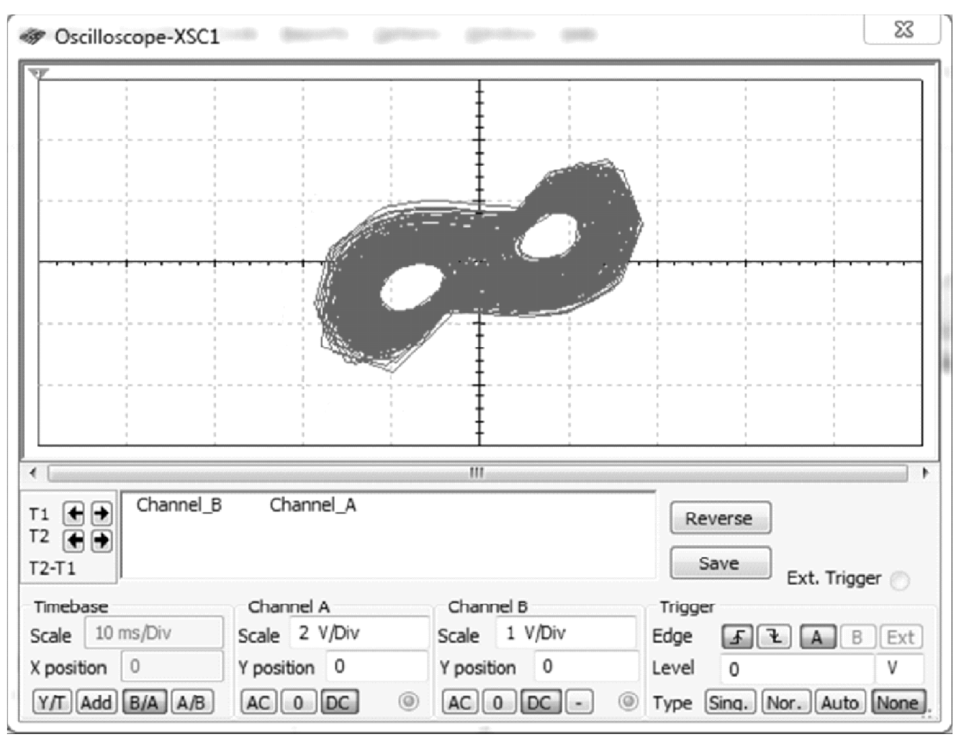
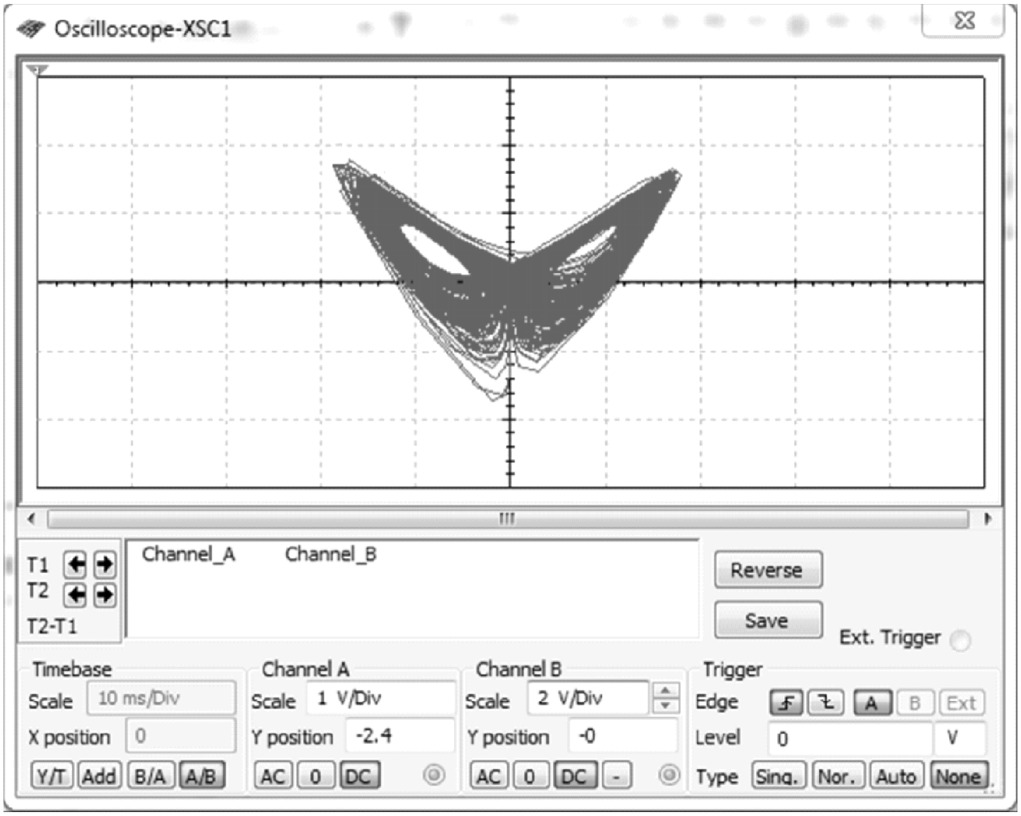


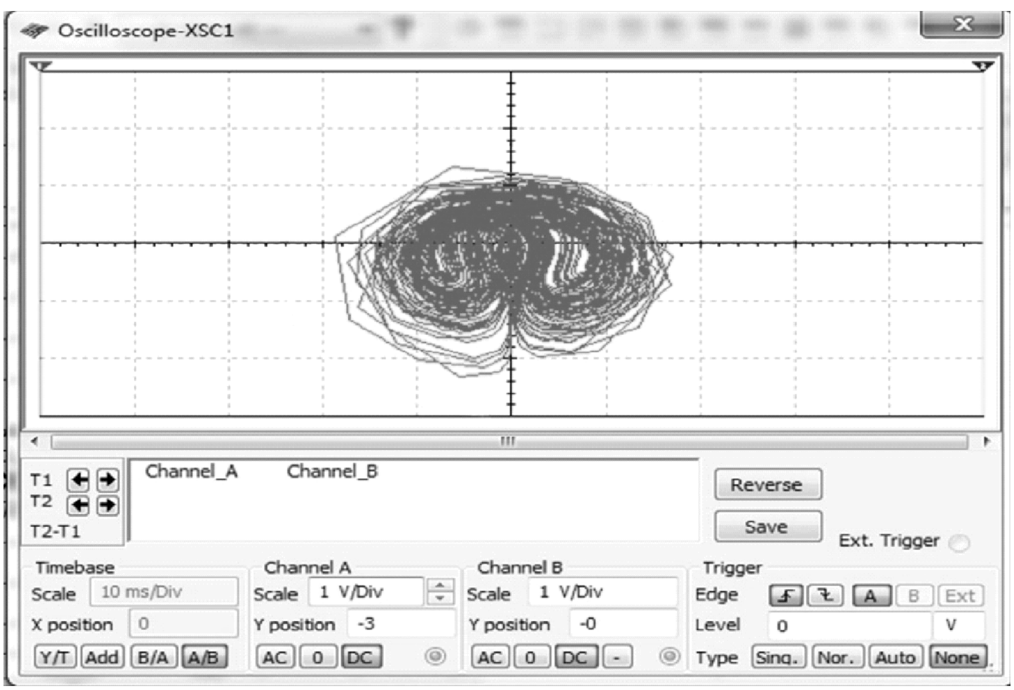
Figure 8: Schematic of the proposed financial risk chaotic system by using MultiSIM 10.0.



(a)



(b)



(c)

Figure 9: Various projections of the financial risk chaotic system using MultiSIM in *x-y* plane, (b) *x-z* plane and (c) *y-z* plane

4. CONCLUSION

A nonlinear dynamic of financial risk chaotic system based on the risk management process is constructed and analyzed. The fundamental properties of the system such as Lyapunov exponents and Poincaré map as well as its phase portraits were described in detail. The model describes the time variation of three state variables: the occurrence value risk (x), the analysis value risk (y) and the control value risk (z). Interesting features of this financial risk system, such as a chaotic behavior and periodic behavior were investigated. Moreover, it is implemented via a designed circuit with MultiSIM and numerical simulation using MATLAB, showing very good agreement with the simulation result.

REFERENCES

[1] Hommes, C. H., 1993, Periodic, almost periodic and chaotic behaviour in Hicks' non linear trade cycle model. *Economics Letters*, 41(4), 391-397.

[2] Stutzer, M. J., 1980, Chaotic dynamics and bifurcation in a macro model. *Journal of Economic Dynamics and Control*, 2, 353-376.

[3] Scheinkman, J. A., and LeBaron, B., 1989, Nonlinear dynamics and stock returns. *Journal of business*, 311-337.

[4] Peters, E. E., 1996, *Chaos and order in the capital markets: a new view of cycles, prices, and market volatility*, John Wiley & Sons.

[5] Barnett, W. A., and Hinich, M. J., 1992, Empirical chaotic dynamics in economics. *Annals of Operations Research*, 37(1), 1-15.

[6] Barnett, W., and Chen, P., 1988, Deterministic chaos and fractal attractors as tools for nonparametric dynamical econometric inference: with an application to the divisia monetary aggregates. *Mathematical and Computer Modelling*, 10(4), 275-296.

[7] Agiza, H. N., and Elsadany, A. A., 2004, Chaotic dynamics in nonlinear duopoly game with heterogeneous players. *Applied Mathematics and Computation*, 149(3), 843-860.

[8] Dubiel-Teleszynski, T., 2011, Nonlinear dynamics in a heterogeneous duopoly game with adjusting players and diseconomies of scale. *Communications in Nonlinear Science and Numerical Simulation*, 16(1), 296-308.

[9] Feichtinger, G., and Kopel, M., 1993, Chaos in nonlinear dynamical systems exemplified by an R & D model. *European journal of operational research*, 68(2), 145-159.

[10] Whitby, S., Parker, D., and Tobias, A., 2001, Non-linear dynamics and duopolistic competition: a R&D model and simulation. *Journal of Business Research*, 51(3), 179-191.

[11] Hsiao, F. S., and Hsiao, M. C. W., 2004, The chaotic attractor of foreign direct investment-Why China?: A panel data analysis. *Journal of Asian Economics*, 15(4), 641-670.

[12] Findlay, R., 1978, Relative backwardness, direct foreign investment, and the transfer of technology: a simple dynamic model. *The Quarterly Journal of Economics*, 92(1), 1-16.

[13] Bouali, S., Buscarino, A., Fortuna, L., Frasca, M., and Gambuzza, L. V., 2012, Emulating complex business cycles by using an electronic analogue. *Nonlinear Analysis: Real World Applications*, 13(6), 2459-2465.

[14] Goodwin, R. H., 1967, *A Growth Cycle. In Socialism, Capitalism and Economic Growth*, CH Feinstein Ed.

[15] Goodwin, R. M., 1967, *A Growth Cycle*, Cambridge University Press, Cambridge.

[16] Burlando, T., 1994, Chaos and risk management. *Risk management*, 41(4), 54.

[17] Pribylova, L. (2009). Bifurcation routes to chaos in an extended van der Pol's equation applied to economic models. *Electronic Journal of Differential Equations*, (53), 1-21.

[18] Lin, Z. O. U., Chao-qun, M. A., and Zhang, H., 2011, Study of the Duffing-Holms Model of Financial Chaos and its Controlling Methods. *Journal of Hunan University (Natural Sciences)*, 12, 018.

[19] Volos, C. K., Kyprianidis, I. M., Stavrinides, S. G., Stouboulos, I. N., Magafas, L., & Anagnostopoulos, A. N., 2011, Nonlinear financial dynamics from an engineer's point of view. *Journal of Engineering Science and Technology Review*, 4(3), 281-285.

[20] Hajipour, A., and Tavakoli, H., 2016, Analysis and circuit simulation of a novel nonlinear fractional incommensurate order financial system. *Optik-International Journal for Light and Electron Optics*, 127(22), 10643-10652.

[21] Xiao-Dan, Z., Xiang-Dong, L., Yuan, Z., and Cheng, L, 2013, Chaotic dynamic behavior analysis and control for a financial risk system. *Chinese Physics B*, 22(3), 030509.

[22] Wolf A, Swift J. B., Swinney H. L and Vastano J. A., 1985, Determining Lyapunov exsponents from a time series, *Physica D*, 16, 285-317.

[23] Li, X. F., Chu Y. D., Zhang J. G and Chang Y. X., 2009, Nonlinear dynamics and circuit implementation for a new Lorenz-like attractor. *Chaos, Solitons and Fractals*. 41, 2360–2370.

[24] Sambas, A., Vaidyanathan, S., Mamat, Sanjaya, W.S.M and Rahayu, D. S. (2016). A 3-D Novel Jerk Chaotic System and Its Application in Secure Communication System and Mobile Robot Navigation. In *Advances and Applications in Chaotic Systems* (pp. 283-310). Springer International Publishing.

[25] Sambas, A., Sanjaya W.S.M., Mamat, M., and Prastio, R. P., 2016), Mathematical Modelling of Chaotic Jerk Circuit and Its Application in Secure Communication System. In *Advances in Chaos Theory and Intelligent Control* (pp. 133-153). Springer International Publishing.

[26] Sambas A, Sanjaya W.S.M., Mamat M, Salleh Z and Mohamad F. S., 2015, Secure Communications Based on the Synchronization of the New Lorenz-like Attractor Circuit, *Advanced Studies in Theoretical Physics,* 9 (8), 379 – 394.

[27] Sambas, A., Mamat, M., and WS, M. Sanjaya. Dynamics, Adaptive Backstepping Control, Synchronization and Circuit Implementation of the Sprott MO_5 Chaotic System, International Journal of Control Theory and Applications. 9(38), 365-382.