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An FDTD Formulation for Wave Propagation in Biological Dispersive Media

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ABSTRACT

A Finite Difference Time Domain (FDTD) scheme used to model the wave propagation in biological dispersive media has been proposed in this paper. FDTD scheme based on the direct solution of time domain Maxwell's equations has developed and these solutions were approximated by time based linear functions. We are applied FDTD scheme for different dispersive media models such as Cole-Cole, Cole-Davidson, Havriliak–Negami dispersive models of the biological tissues. This proposed scheme is demonstrated by simulating Gaussian pulse propagation in the dispersive medium slabs with air interface at edges. In this proposed formulation, magnetic field components are coupled by the electric field components of Maxwell's equation further development obtained by means of boundary conditions across the interfaces are applied by means of time dependent jump conditions. By calculating reflection coefficient for muscles, bone at air interfaces we obtain promising results and these results plays a key role in microwave medical imaging.

Keywords: finite difference time domain(FDTD), Cole-Cole, Cole-Davidson, Havriliak–Negami, Gaussian pulse, boundary conditions.

1. INTRODUCTION

In electromagnetics, a medium is called dispersive if its permittivity or permeability depends on the wave frequency. When an electromagnetic wave is propagating in a dispersive medium, different frequencies will travel at different speeds. The time domain dispersion simulation is required for many practical applications involving dispersive materials. For example, in the ground penetrating radar that uses the transmittance of pulses to image the subsurface in order to detect buried objects, the geological media are dispersive. Similarly, the microwave imaging has been used for the early detection of breast cancer in which the biological tissues are dispersive. In the near-field optics, the scattering of light by metallic nano-structures is of great interests for the sub-wavelength imaging and sensing. At optical frequencies, metals shall be described as dispersive.

FDTD method has been a main work of the computational electromagnetic (CEM) in the time domain, owing to its simplicity, lack of dissipative error, and having very low cost per grid node. However, comparing with other dispersive approaches, the dispersive FDTD algorithms suffer from a larger accuracy reduction at a dispersive interface, due to a staircase representation for complex geometry. To reduce the staircase error of the dispersive FDTD simulation, several smoothing techniques have been constructed to approximate the permittivities in the vicinity of the interface by an averaged smooth effective permittivity.

At present, many number of empirical relationship for dispersive media including Cole-Cole, Cole-Davidson, Havriliak-Negami have been proposed. FDTD algorithm has been proven to be one of the most powerful algorithm for computing the electromagnetics in time domain to model the wave propagation in complex dispersive media including biological media.

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Bin Guo, et al. [2], presented a new FDTD formulation for modeling the wave propagation in a biological dispersive media by Cole-Cole model by using Z-transform to represent the frequency dependent dielectric properties. Wei-Jun Chen, et al. [3] used Auxiliary Differential Equation(ADE) based FDTD technique to develop a relationship between electric displacement vector to electric field intensity. However, Ioannis T. Rekanos [6] evaluated, FDTD scheme based on ADE for Davidson-Cole model in which fractional derivatives models were occurred. This seems a greater difficulty of this method. To resolve this difficulty, pade approximation has used to approximate the relative complex permitivities. Mohammad-Reza Tofighi [7] has used a novel FDTD to implement the Cole-Cole dispersion model for biological tissues. A novel approach obtained by relaxation time distributions. By using this technique, a fairly accurate result has obtained for complex permitivities at the microwave frequency ranges.

Mohammad A. Alsunaidi et al. [8] has used a ADE based general FDTD algorithm to analyze the dispersive media which is well suited for the materials having different dispersion characteristics. In further, this algorithm successfully simulated for Surface Plasmon Polarities. Myunghyun Ha, et al. [9] proposed Laguerre FDTD to formulate linear dispersive model. Ioannis T. Rekanos [10] explained about the FDTD technique for propagation of wave in the Havriliak-Negami dispersive media. In this kind of dispersive media models, time domain representation of the polarization relation involves time derivatives of fractional order. However, this is an empirical model in which the wide band pulse has propagated inside the dispersive media and the effective results were obtained.

In this paper, Cole-Cole, Cole-Davidson and Havriliak-Negami dispersive model for the biological tissues has demonstrated. The formulation is based on the Yee algorithm for updating the electric field by using previous magnetic field and so on. Based on this updating equations, wave propagation characteristics has analyzed by means of its reflectance and transmittance through the Gaussian pulse propagating inside the dispersive slabs.

2. MATHEMATICAL FORMULATION

General Maxwell's equations for lossless and dispersive medium are given by,

$$\frac{\partial D(r,\, t)}{\partial t} = \nabla \times H(r,\, t) - J(r,\, t)$$

$$\frac{\partial H(r,\, t)}{\partial t} = -\frac{1}{\mu_0} \nabla \times E(r,\, t)$$

a one-dimensional space where there are only variations in the *x* direction. Assume that the electric field only has a *z* component. In this case Faraday's law can be written

$$-\mu \frac{\partial H}{\partial t} = \nabla \times E = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & 0 & E_z \end{vmatrix} = -\hat{a}_y \frac{\partial E_z}{\partial x}$$

Where μ is permeability of a medium. Ampere's law is given by,

$$\in \frac{\partial E}{\partial t} = \nabla \times H = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & H_y & 0 \end{vmatrix} = \hat{a}_z \frac{\partial H_y}{\partial x}$$

Where ε is permittivity of a medium From these two Maxwell's equations, we can derive two scalar equations that are,

$$\mu \frac{\partial H}{\partial t} = \frac{\partial E_z}{\partial x}$$

$$\in \frac{\partial E}{\partial t} = \frac{\partial H_y}{\partial x}$$

In the first equation, spatial derivative of an electrical field is given in terms of magnetic fields in temporal derivatives which is used advance the magnetic field in time domain. On the other hand second equation is the spatial derivative of magnetic field in terms of temporal derivative of electric field which will be used in the advancement of the electric field in time domain. By this way, the electric and magnetic fields were updated consequently and this process is denoted as leap-frog method.

2.1. Cole-Cole dispersion model

Schwan and Foster[19] explained in detail about various interaction mechanism in biological tissues causing dispersion due to its complex permittivity from Extreme Low frequency to Microwave frequency ranges. The complex permittivity can be expressed by,

$$\varepsilon = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{(1 + (j\omega\tau_0)1 - \alpha)} + \frac{\sigma}{j\omega\varepsilon_0}$$

Here, ε_∞ is optical permittivity, σ is conductivity, and τ_0 is relaxation time.

2.2. Davidson-Cole model

The complex relative permittivity of the Davidson-Cole dispersive model in angular frequency is given by,

$$\varepsilon_r(\omega) = \varepsilon_\infty + \frac{\Delta\varepsilon}{(1 + j\omega\tau_0)^\beta},\ 0 < \beta < 1$$

Where, $\Delta\varepsilon = \varepsilon_s - \varepsilon_\infty$ and ε_s is static permittivity and ε_∞ is optical permittivity τ_0 is relaxation time and β is the measure of dispersion broadening parameter. This relative complex permittivity can be expressed in another form in terms of integrals,

$$\varepsilon_r(\omega) = \varepsilon_\infty + \int_0^\infty \frac{G(\tau)}{(1 + j\omega\tau)} d\tau$$

Where, $G(\tau)$ is distribution function that should satisfying the condition, $\int_0^\infty G(\tau) = 1$.

2.3. Havriliak-Negami model

This model is being considered as a general expression for universal law. The most general approximation of general frequency dependent complex permittivity is given by,

$$\varepsilon_r(\omega) = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{\left[1 + (j\omega\tau_0)^{1-\alpha}\right]^{1-\beta}}$$

ε_s is static permittivity and ε_∞ is optical permittivity τ_0 is relaxation time and α and β are the dispersion control parameters which should satisfy, $0 \le \alpha \le 1$ and $0 \le \beta \le 1$. For $\alpha = 0$, $0 \le \beta \le 1$ the medium will act as Cole-Davidson dispersion model and for $\beta = 0$, $0 \le \alpha \le 1$ the medium behave as Cole-Cole dispersion model.

3. NUMERICAL RESULTS

In this paper, FDTD scheme has been validated by simulating the electromagnetic propagation in all the three kinds of biological dispersive media described by Cole-Cole, Cole-Davidson and Havrilik-Negami models. To obtain accurate modeling one dimensional problem was simulated where, the dispersive region surrounded $z \ge 0$ is Perfectly Matched Layer.

4. CONCLUSION

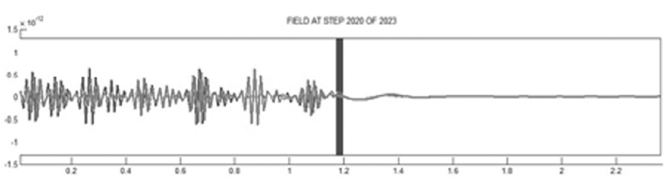
FDTD method for biological dispersive media by Cole-Cole, Cole-Davidson and Havriliak-Negami model has presented in this paper. In this paper, we modeled for wave propagation through the medium has analyzed. From the wave propagation diagrams we conclude that, the thickness of the medium plays a great

Table 1
Properities of Biological Media

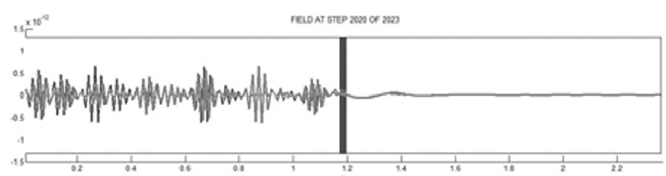
Tissue Type	*σ (Conductivity)*	*Thickness*
Muscle	0.777	20mm
Skull	0.015	6.93 inches

Table 2
Parameters of The Different Dispersive Models

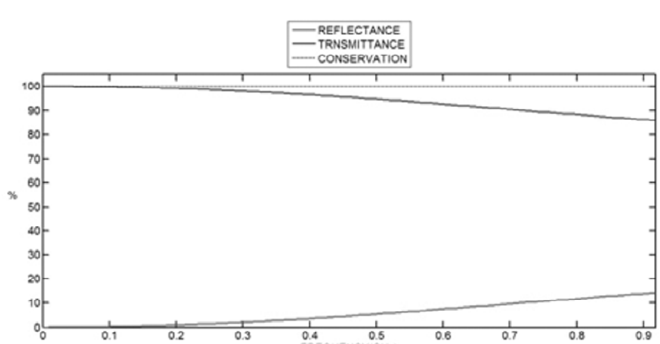
Relation	α	β	t *(ps)*	ε_s	ε_∞
Cole-Cole	0.3	0	153	50	2
Davidson-Cole	0	0.15	153	50	2
Haviriliak-Negami	0.3	0.15	153	50	2



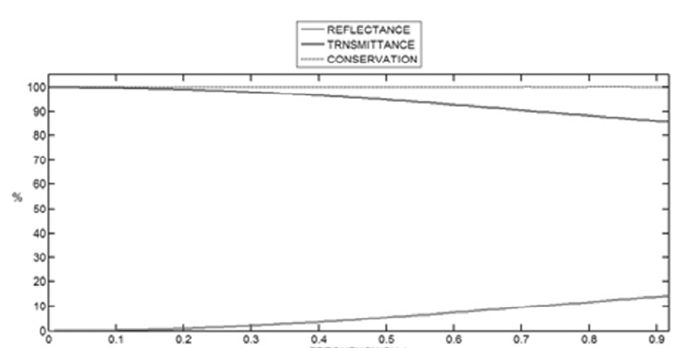
Wave propagation in muscle by Cole-Cole model.



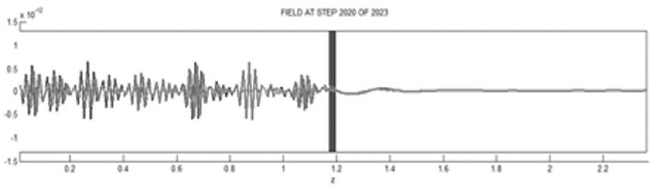
Wave propagation in muscle by Davidson-Cole model.



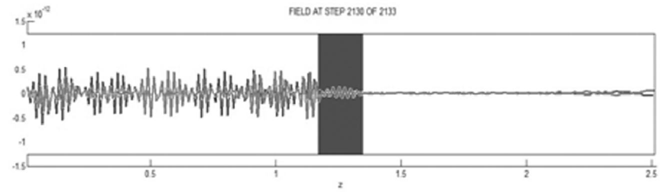
Reflectance and Transmittance in muscle byCole-Cole model.



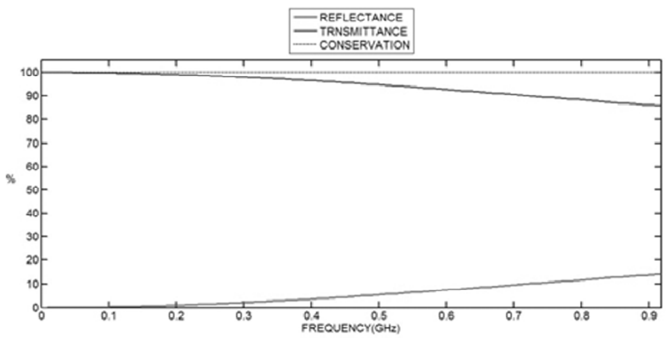
Reflectance and Transmittance in muscle Davidson-Cole model.



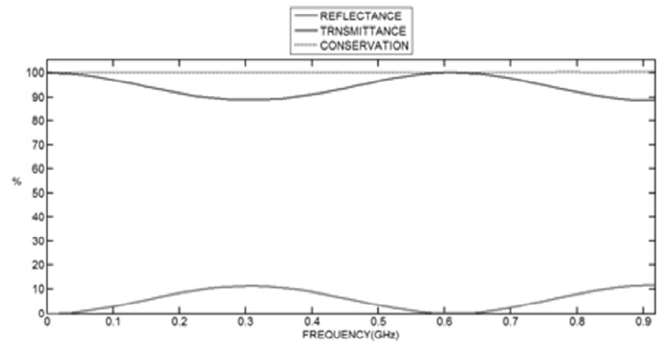
Wave propagation in muscle by Havriliak-Negami model.



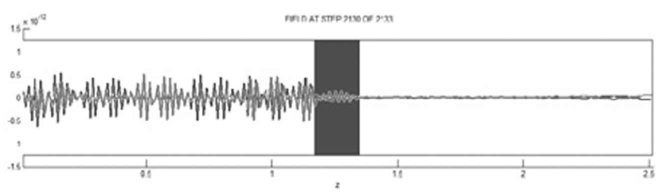
Wave propagation in skull by Davidson-Cole model.



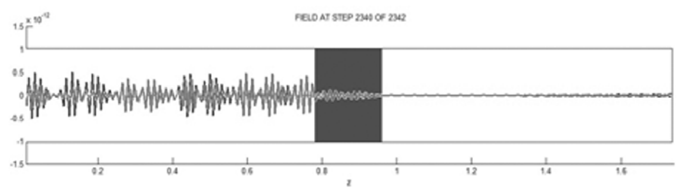
Reflectance and Transmittance in muscle by Havriliak-Negami model.



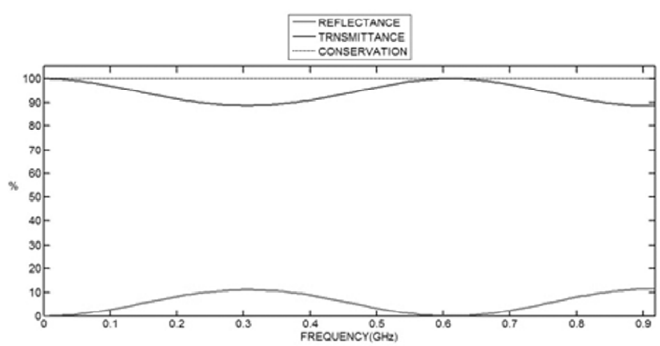
Reflectance and Transmittance in skull by Davidson-Cole model



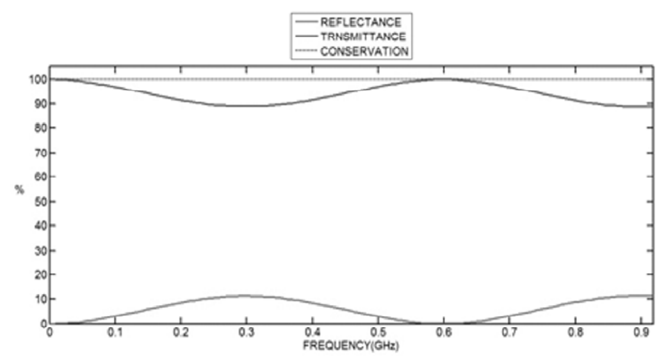
Wave propagation in Skull by Cole-Cole model



Wave propagation in skull by Havriliak-Negami model.



Reflectance and Transmittance in skull by Cole-Cole model



Reflectance and Transmittance in skull by Havriliak-Negami model.

role in the wave propagation. For a higher thickness medium reflection gets higher. This will be inferred from the Reflectance and Transmittance plots for different kinds of dispersive media.

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