I J C T A, 6(1) June 2013, pp. 15-27

Backstepping Control Design for A Semiactive Vehicle Suspension System equipped with Magnetorheological Rotary Brake

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ABSTRACT

This paper deals with controller design for vibration reduction of a vehicle suspension system that employs a magnetorheological (MR) damper as the actuator. The experimental setup, "Semi Active Suspention System (SAS)"developed by the Inteco Ltd is the studied system for implementing the controller. The dynamic of this system is nonlinear so a nonlinear feedback controlling method based on Lyapunov stability theory and backstepping method was developed for vibration reduction.The nonlinear behavior of the MR damper is modeled using the Dahl model.The modeling of the system with the backstepping controller was done in Matlab/Simulink environment for the evaluation of the theoretical performance of the system.In addition the laboratory experiment was also carried out for measuring real performance of the SAS system when the controller is connected. Results from both simulation and experiment showed a better reduction in the vibrations of the vehicle body.

I. INTRODUCTION

Vibration is a common and unpredicted phenomena for dynamic and static bodies. These vibrations are unwanted and must be eliminated as quickly as possible to maintain the proper operation of the system by avoiding damages and disturbances to the system.This is achieved by the suspension systems.Basically there are three types of vibration suppression systems. They are active, passive and semi active suspension systems [1].

One of the most common applications of suspension system is the automobile industry. Suspension system in a vehicle is a mechanical system of springs and shock absorbers that connect the wheels and axles to the chassis of a wheeled vehicle. The jobs of these suspension systemsin a vehicle are, carrying the static weight of the vehicle, maximizing the friction between the tires and the road surface, providing steering stability with good handling and ensuring the comfort of the passengers.

One of the better suspension systems currently using for the vibration isolation is the Semi Active Suspension (SAS) System. It consists of a magneto-rheological damper which creates braking torque by changing the viscosity of the Magnetorheological fluid inside the brake according to the applied current or voltage [2]. Magnetorheological fluid is composed of oil and varying percentages of ferrous particles (20-50 microns in diameter) that have been coated with an anti-coagulant material. Therefore Varying the magnetic field strength by changing the input current of voltage has the effect of changing the viscosity of the magnetorheologicalfluid. Change in viscosity of the fluid also changes the torquegenerated by the damper.Thus by controlling the input current, output torque of the MR damper can be controlled so that to make the system stable quickly.

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To make the system stable as soon as possible by decreasing the amplitude of the vibrations an input current or voltage must be properly controlled in response to the system dynamics. For that the system requires a feedback controller. There have been number of several feedback controlling techniques discussed in the literature for vibration reduction in such nonlinear systems. For example proportional integral derivative control, backstepping control, LQR control, LQG control, etc [3].

In this paper a control mechanism for controlling the torque of the MR damper is developed based on the Backstepping control mechanism and simulated it for the SAS system. The experimental task was carried out to measure the performance of the controller in the real system.

II. SEMIACTIVE SUSPENSION SYSTEM

The study of vehicle vibration reduction is made by analyzing the dynamics of the Semi-Active Suspension (SAS) System developed by the Polish Company Inteco Limited. The SAS laboratory model represents a physical system of a quarter car model, and can be used to analyze the vertical dynamics of the car wheel. As shown in Figure 1, the SAS systemconsists of an upper beam which represents the car body, a wheel, rotational MR damper and a spring.

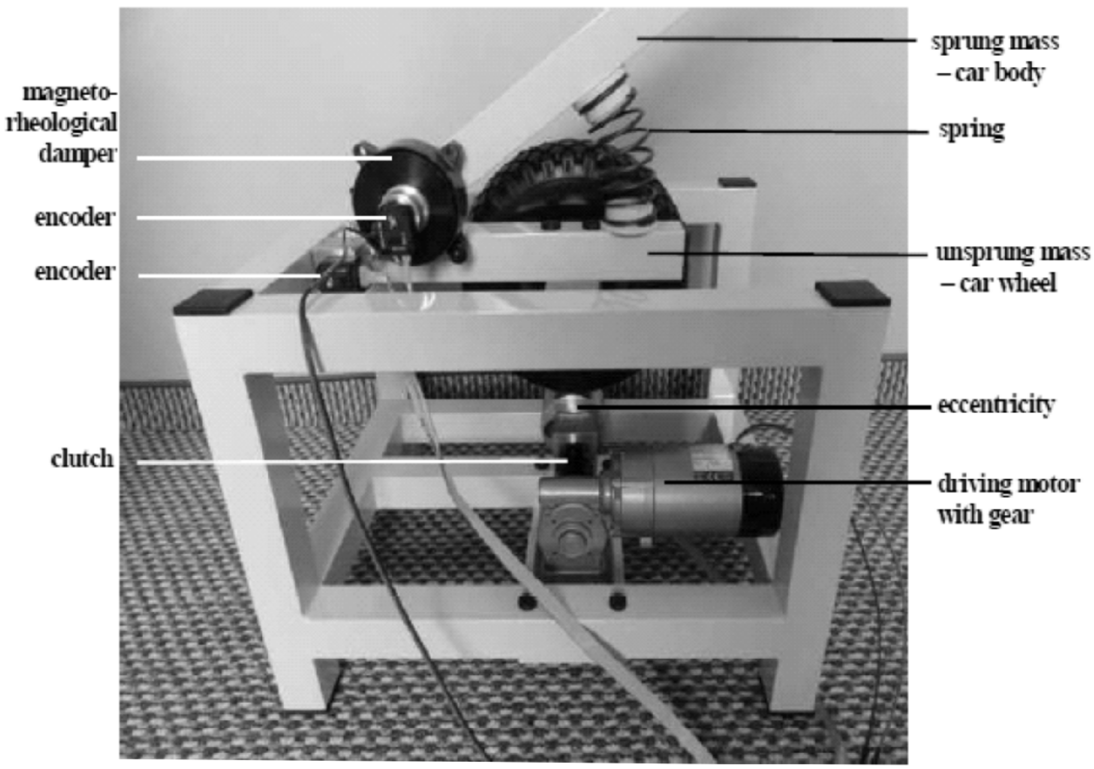


Figure 1: Semiactive Suspension System (SAS)

It is driven by a DCmotor with gear coupled to an eccentric small wheel. Thesuspended car wheel rolls due to the small wheel rotation and oscillates up and down due to the small wheel eccentricity.

The MR damper incorporated in SAS acts as an interface between the electronic sensors, pre-programmed control algorithms and the mechanical structure of the suspension, using the external damper coil current to adjust the damping. The restoring force in the MR damper depends on the rotary damper velocity and the magnetic field strength.

Figure 2 provides the geometrical view of the SAS system. Using this figure the mathematical modeling equations for the upper lever and the lower lever can be derived as follows [4].

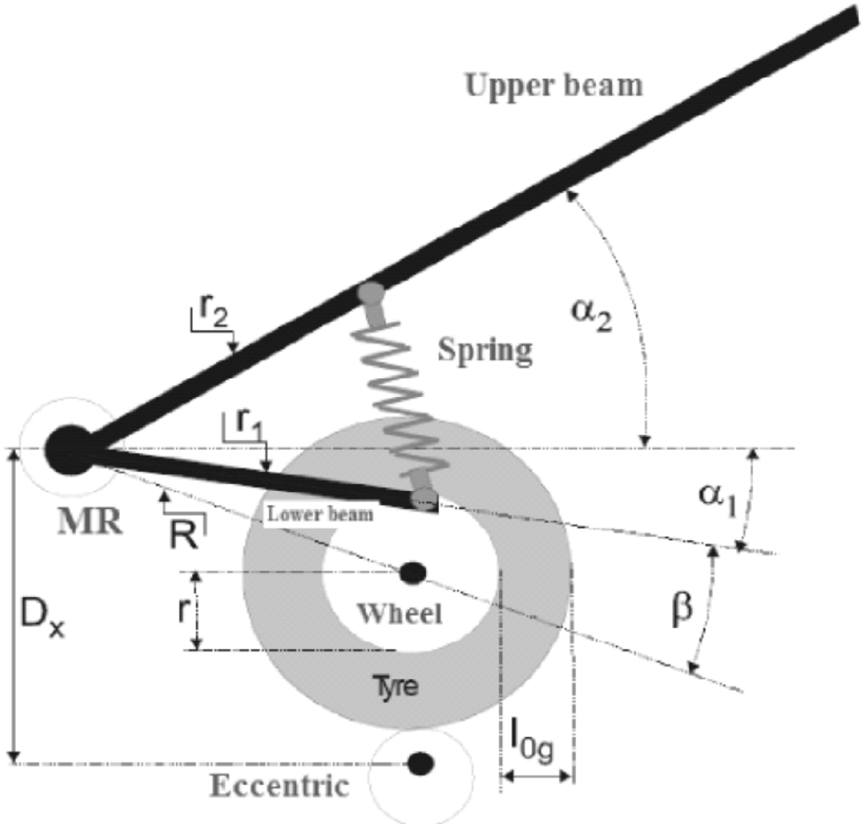


Figure 2: The Geometrical View of the SAS System

The equations of motion of the upper beam are;

$$\dot{\alpha}_2 = \omega_2 \tag{1}$$

$$J_2 \frac{d^2\alpha_2}{dt^2} + k_2 \frac{d\alpha_2}{dt} + M_2 \cos\alpha_2 - r_2 k_S (I_{OS} - I_S) = T_{MR}(i) \tag{2}$$

$$J_2 \dot{\omega}_2 = -k_2\omega_2 - M_2 \cos\alpha_2 + r_2 k_s (l_{os} - I_s) + T_{MR}(i) \tag{3}$$

The equations of motion of the lower beam are;

$$\dot{\alpha}_1 = \omega_1 \tag{4}$$

$$\begin{aligned} &J_1 \frac{d^2\alpha_1}{dt^2} + M_1 \cos(\beta - \alpha_1) + r_1 k_s (I_{os} - I_s) + k_1 \frac{d\alpha_1}{dt} \\ &-k_g R\cos(\beta - \alpha_1)(l_{og} + R\sin(\beta - \alpha_1) + r - D_x + u_{ex}) - f_g \left(\frac{d(D_x - u_{ex})}{dt} - R\cos(\beta - \alpha_1)\frac{d\alpha_1}{dt} \right) = T_{MR}(i) \end{aligned} \tag{5}$$

where; $l_s = \sqrt{(r_1 \cos\alpha_1 - r_2 \cos\alpha_2)^2 + (r_2 \sin\alpha_2 - r_1 \sin\alpha_1)^2}$

J_1, J_2 Moment of inertia of the lower beam and the upper beam with respect to its axis rotation

M_1, M_2 Gravitational moment of the upper and lower beam

k_1, k_2 Viscous friction coefficients of the upper beam and lower beam

α_1 The angle between the upper beam and horizontal line

α_2	The angle between the lower beam and horizontal line
β	The angle between the line drawn through the centers of the tire and MR damper and the horizontal line
r_1, r_2	The distances to the spring mount from the revolute joint along the upper and lower beam
D_x	The distance between the beam pivot and the pivot of the eccentric
R	The distance between the MR damper and the center of the tire
k_s	The elasticity coefficient of the spring
l_{os}	The length of the no-load spring
l_s	The length of the loaded spring
k_g	The elasticity coefficient of the tire
f_g	The absorption coefficient of the tire
u_{ex}	The kinetic sinusoidal excitation
T_{MR}	MR damper torque

III. BACKSTEPPING CONTROLLER DESIGN

The control methodology must be implemented to decrease the vibrations of the body when the tire is subjected to the external excitation force such as bumps or vibrations. Therefore in a backstepping control method we mainly focuson the vibration suppression of the body [5] so that we use only the mathematical model of the body.

Let's define;

The viscous friction damping torque, $M_{d2} = k_2\omega_2$

The gravitational force torque, $M_{g2} = M_2 \cos\alpha_2$

Spring torque, $M_{s2} = r_2 k_s (l_{os} - l_s)$

Then the equation of the upper beam can be written as,

$$\dot{\alpha}_2 = \omega_2 \tag{6}$$

$$\dot{\omega}_2 = J_2^{-1}(-M_{d2} - M_{g2} + M_{s2} + T_{MR}(i)) \tag{7}$$

The equilibrium point of the system can be found when,

$$\left(\dot{\alpha}_2, \dot{\omega}_2\right) = (0,0)$$

It was found as, $(\alpha_{2,eq}, \omega_{2,eq})$ = (0.55 rad, 0) when T_{MR} (0) = 0.

Defining the new coordinates z_1 and z_2 by shifting the origin to the equilibrium point.

$$(z_1, z_2) = (\alpha_2 - \alpha_{2,eq}, \omega_2 - \omega_{2,eq}) = (\alpha_2 - \alpha_{2,eq}, \omega_2)$$

In the new coordinate system equation of the upper beam becomes,

$$\dot{z}_1 = z_2 \tag{8}$$

$$\dot{z}_2 = J_2^{-1}(-M_{d2} - M_{g2} + M_{s2}) + J_2^{-1}T_{MR} \tag{9}$$

In order to apply the backstepping technique let's define standard backstepping variables.

$$e_1 = z_1 \qquad \dot{e}_1 = z_2$$
$$e_2 = z_2 + h_1 e_1,\ h_1 > 0 \qquad \dot{e}_2 = \dot{z}_2 + h_1 \dot{e}_1$$
$$e_2 = z_2 + h_1 z_1 \qquad \dot{e}_2 = \dot{z}_2 + h_1 z_2$$

Lyapunov's direct method of stability is used to determine a rule for the stability of the system. Candidate Lyapunov candidate function [6];

$$V = \frac{1}{2}V_1^2 + \frac{1}{2}V_2^2 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \tag{10}$$

Taking the derivative of V in (10) results in

$$\dot{V} = -h_1 e_1^2 - h_2 e_2^2 + e_2((\alpha_2 - \alpha_{2,eq})(1 + h_1 h_2) + J_2^{-1}(-M_{d2} - M_{g2} + M_{s2})$$
$$+ J_2^{-1} T_{MR} + \omega_2 (h_1 + h_2))$$

Then $\dot{V} \le -h_1 e_1^2 - h_2 e_2^2 < 0$ for given that $h_1, h_2 > 0$.

If, $e_2 \begin{pmatrix} (\alpha_2 - \alpha_{2,eq})(1 + h_1 h_2) + J_2^{-1}(-M_{d2} - M_{g2} + M_{s2}) + J_2^{-1} T_{MR} \\ + \omega_2 (h_1 + h_2) \end{pmatrix} \le 0$ Then, $\dot{V} < 0$. Therefore according to the Lyapunov second method of stability the system is stable. Then, $(e_1, e_2) \to (0,0)$ and $(z_1, z_2) \to (0,0)$. This implies that, $(\alpha_2, \omega_2) \to (\alpha_{2eq}, 0)$.

In order the system to be stable let's define the control law as

$$T_{MR} = -J_2(\alpha_2 - \alpha_{2,eq})(1 + h_1 h_2) + (M_{d2} + M_{g2} - M_{s2}) - \omega_2 J_2 (h_1 + h_2) \tag{11}$$

The controller must command the MR damper to produce the torque derived above to suppress the vibrations of the body as soon as possible. The MR damper can be controlled by the input current or voltage. Therefore the appropriate current or voltage must be found to give as the input to the MR damper to produce the required torque.

IV. MR DAMPER

The MR damper is a type of semi-active damper where the viscosity of the MR fluid is controlled by varying the amount of current supplied and hence changes the level of damping [2]. These dampers are intrinsically nonlinear, so one of the challenging aspects of applying this technology is the development of accurate models to describe their behavior for control design and evaluation purposes.

To describe the hysteretic behavior of MR dampers, several models have been proposed in the literature, including the Bingham model, Dahl model, the Bouc-Wen phenomenological model, Lugre Model, neural networks, etc. [7], [8].These models can be classified into two main categories as parametric and non-parametric. Non-parametric models are able to model the MR damper behavior in such a way that the model parameters do not necessarily have physical meanings. Some of the nonparametric models are neural networks and neuro-fuzzy. In parametric models, their parameters have some physical meaning. These models consist of some mechanical elements such as linear viscous, friction, springs, etc. Examples for nonparametric models are Bingham viscoplastic Model, Bouc–Wen model, Dahl model and LuGre friction model.In this paper Dahl model is selected to model the MR damper [9].

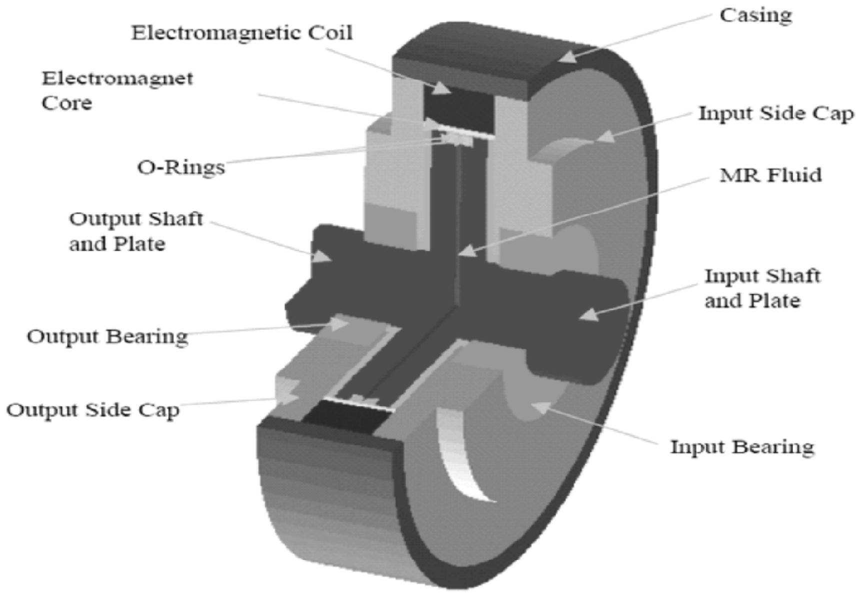


Figure 3: MR Damper

The governing equations of the MR damper are;

$$T_{MR} = K_x(i)\ \dot{\theta} + K_y(i)\ z \tag{12}$$

With;

$$\dot{z} = \alpha\ (\dot{\theta} - |\dot{\theta}|\ z) \tag{13}$$

Where; θ is the angle, K_x is the damping coefficients which depend linearly on the current *(i)* and the z is the hysteretic variable. Parameters K_y and α control the shape of the hysteresis curve.

The current dependent parameters K_x and K_y described by following equations.

$$\begin{aligned} K_x &= K_a + K_b\ i \\ K_y &= K_1 + K_2\ i \end{aligned} \tag{14}$$

The Dahl model of the MR damper was modeled in the Matlab/Simulink an shown in the Figure 4.

Damper must produce the damping torque of;

$$T_{MR,C} = -J_2(\alpha_2 - \alpha_{2,eq})(1 + h_1 h_2) + (M_{d2} + M_{g2} - M_{s2}) - \omega_2 J_2 (h_1 + h_2)$$

Damper Torque given from the Dahl model;

$$T_{MR,C} = (K_a + K_b\ i)\ \dot{\theta} + (K_1 + K_2\ i) z \tag{15}$$

Therefore the control current that should be sent to the MR damper is,

$$i = \frac{T_{MR,C} - K_a\dot{\theta} - K_1 z}{K_b\dot{\theta} + K_2 z} \quad \text{or}$$

$$i = \frac{\begin{array}{c} -J_2(\alpha_2 - \alpha_{2,eq})(1 + h_1 h_2) + (M_{d2} + M_{g2} - M_{s2}) \backslash \\ -\omega_2 J_2 (h_1 + h_2) - K_a\dot{\theta} - K_1 z \end{array}}{K_b\dot{\theta} + K_2 z} \tag{16}$$

This control law is modeled in Matlab/Simulink as in the Figure 5.

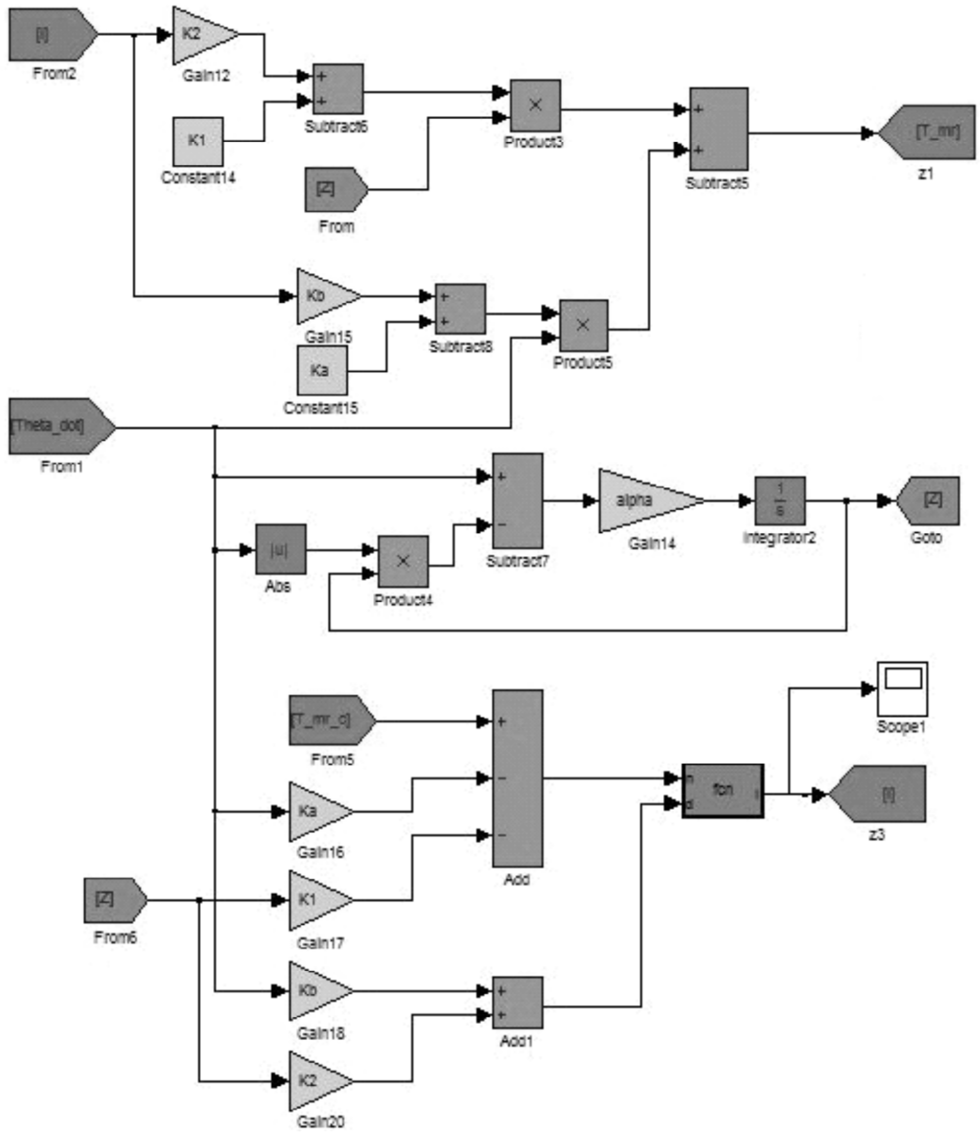


Figure 4: MR Damper Model

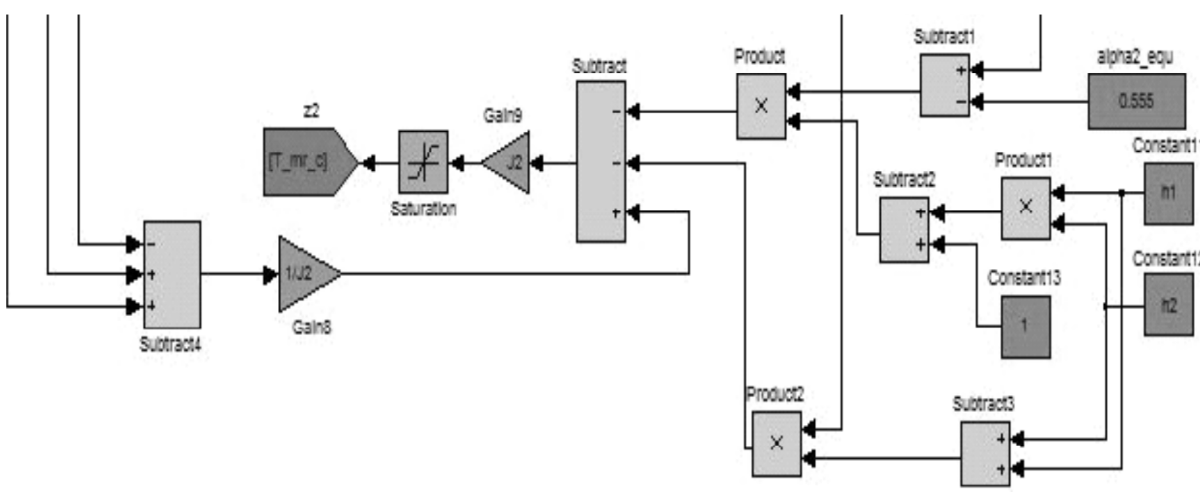


Figure 5: Backstepping Controller

V. SIMULATION RESULTS

The SAS system with the backstepping controller is modeled in the Matlab/Simulink and the Simulink diagram is given in Figure 6.

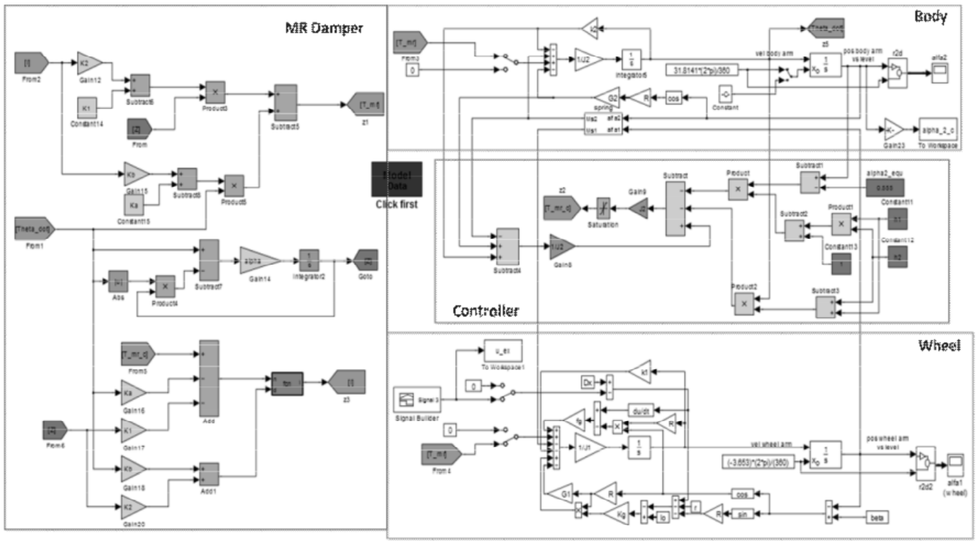
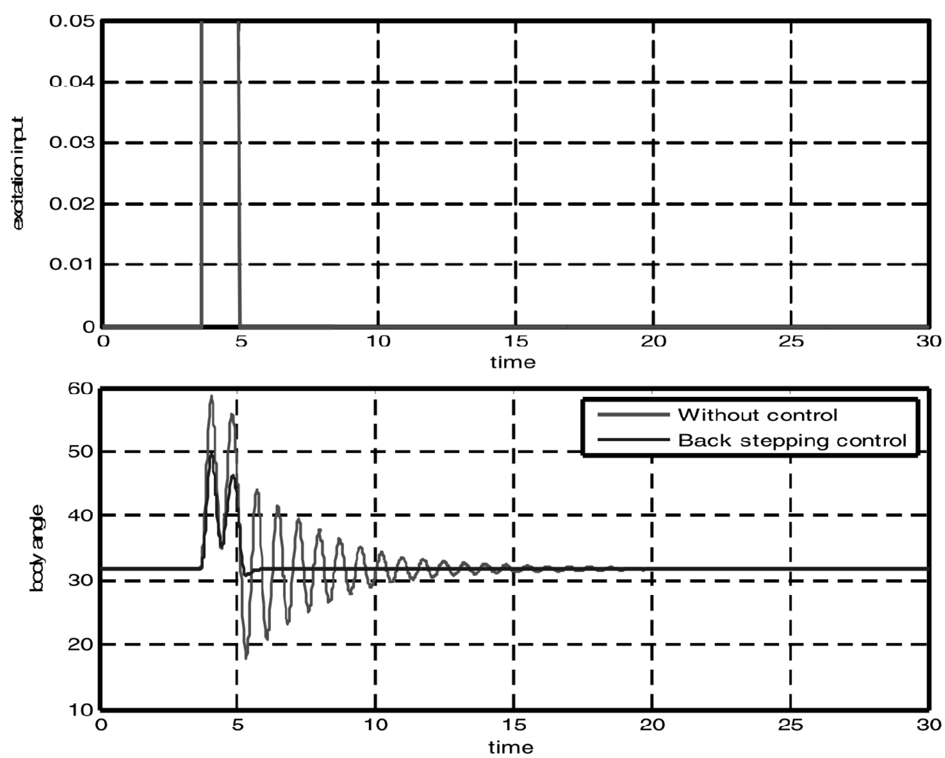


Figure 6: SAS System with the Backstepping Controller

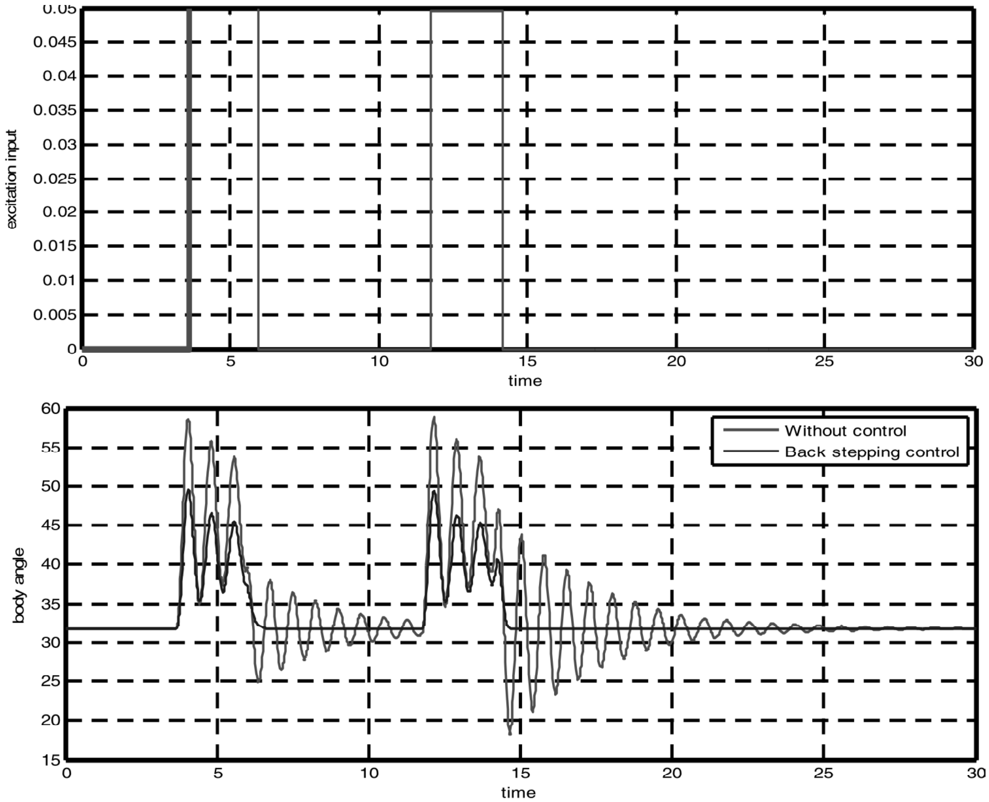
The following MR damper parameters for the Dahl model are selected for the Simulation.

$$K_1 = 5,\ K_2 = 1.5,\ K_a = 0.001,\ K_b = 0.001,\ \alpha = 5$$

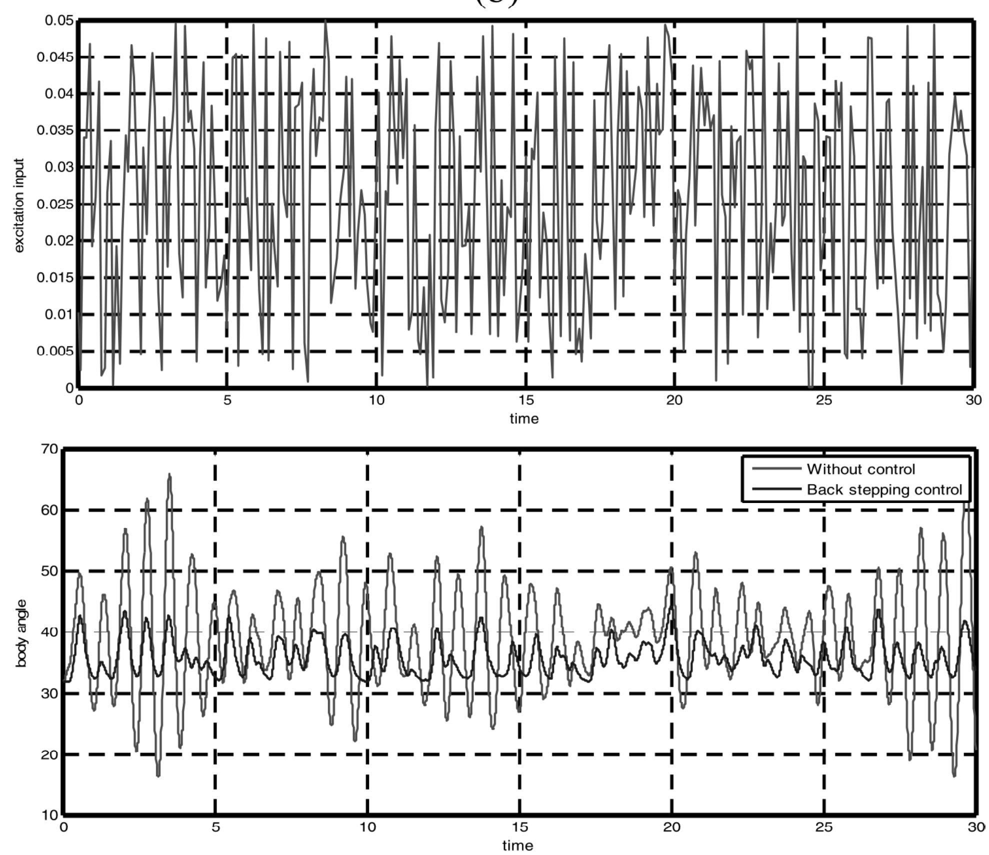
Backstepping control was applied to the SAS model with the parameters $h_1 = 10$ and $h_2 = 10$. Vibration response of the body was obtained for different excitation input and the results are illustrated in the Figure 7. According to Figure 7, it can be clearly seen that, when the backstepping controller is present the body stabilizes more quickly by reducing the amplitude of the vibration.



(a)



(b)



(c)

Figure 7: Simulation Results of the Beam Vibrations for Different Excitation Input

VI. LABORATORY EXPERIMENT

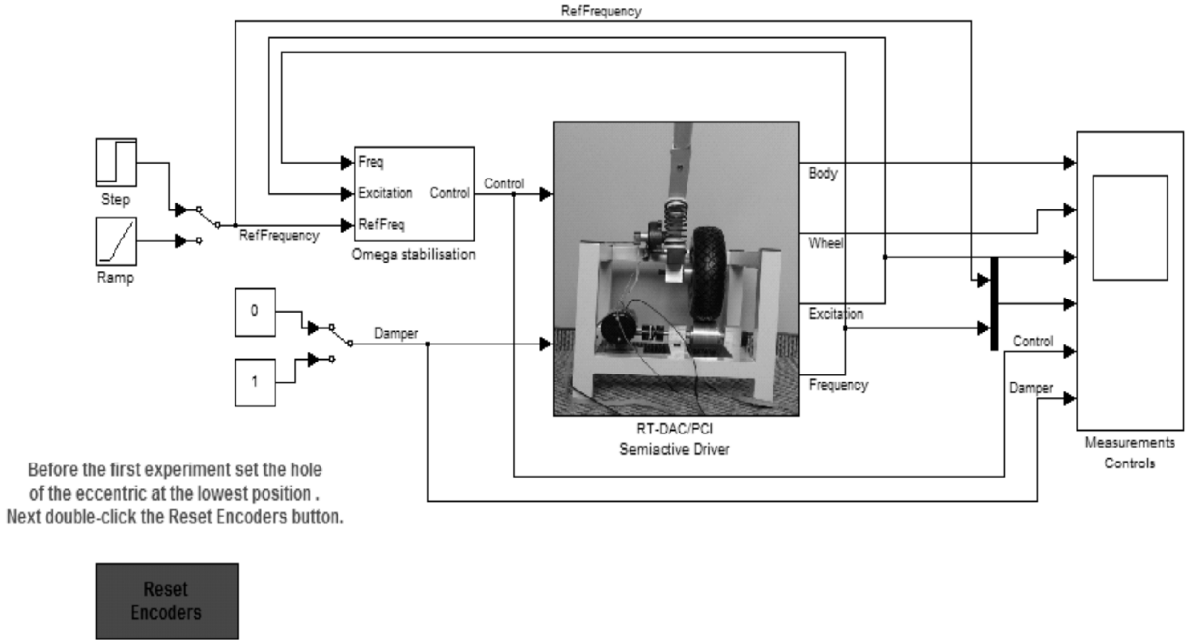


Figure 8: Simulink Interface that Communicates with the SAS Physical Model

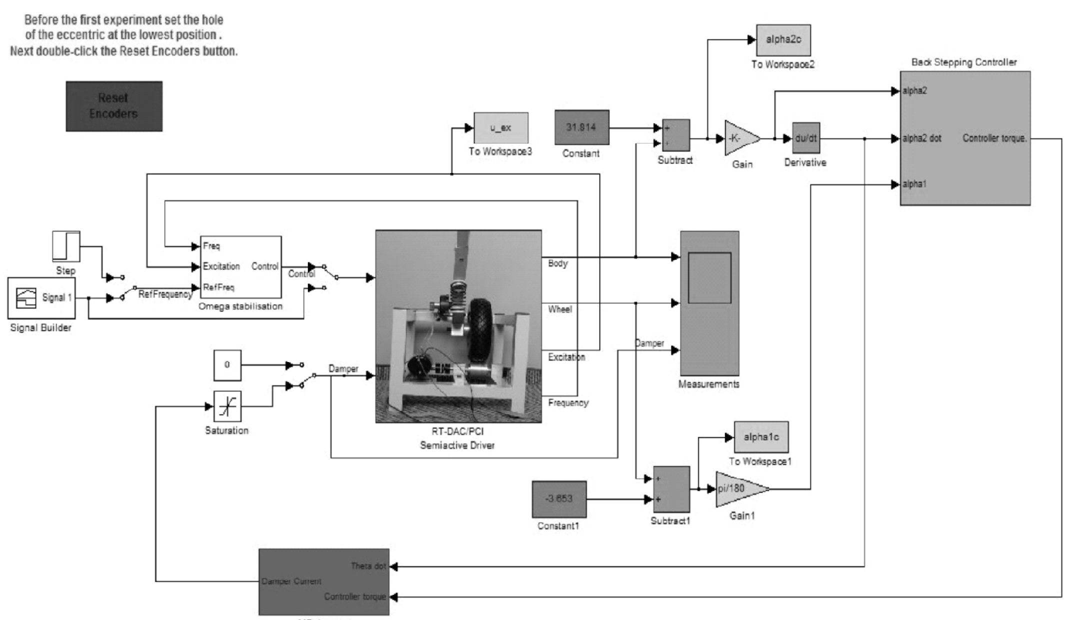
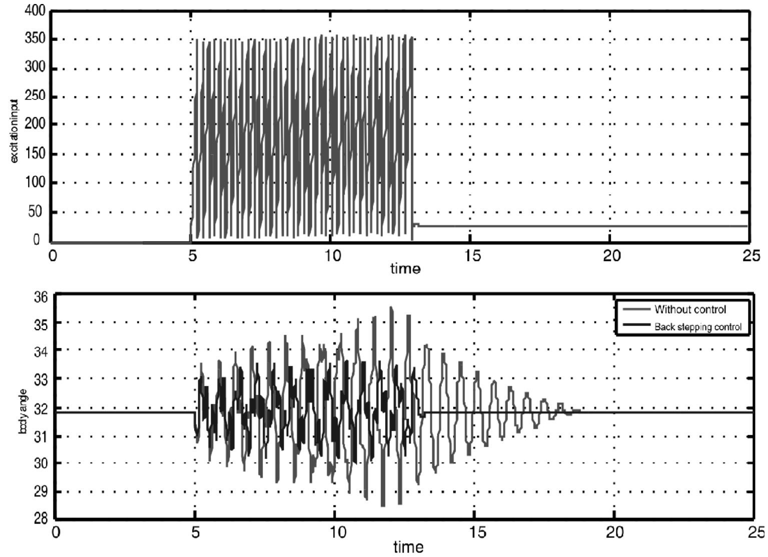


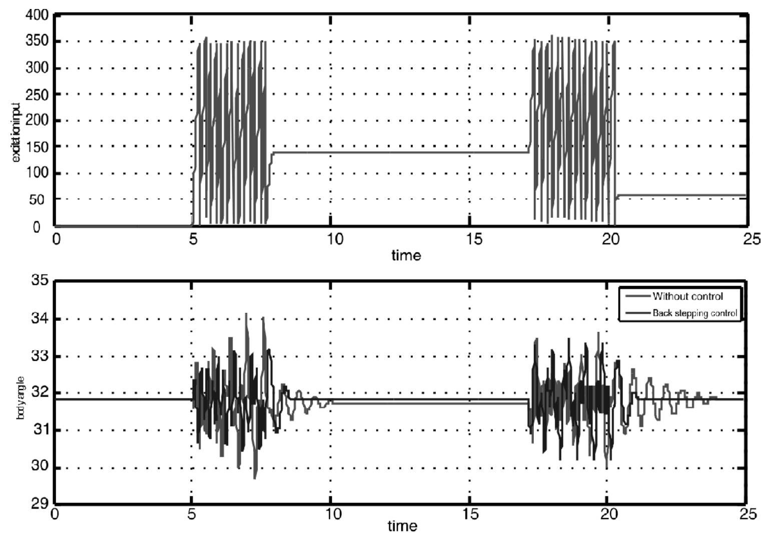
Figure 9: Real Time Experimental Simulink Model of the SAS with Back Stepping Controller

Vibration analysis was done in the experimental setupof the Semi Active Suspension system. The SAS system has been designed to operate with an external, PC-based digital controller. The control computer communicates with the level sensors, values and pump by a dedicated I/O board and the power interface. The I/O board is controlled by the real time software which operates in Matlab/Simulink/RTWT rapid prototyping environment. Figure 8 gives the inbuilt Simulink interface that communicates with the SAS physical model without any feedback controller for controlling the vibrations.

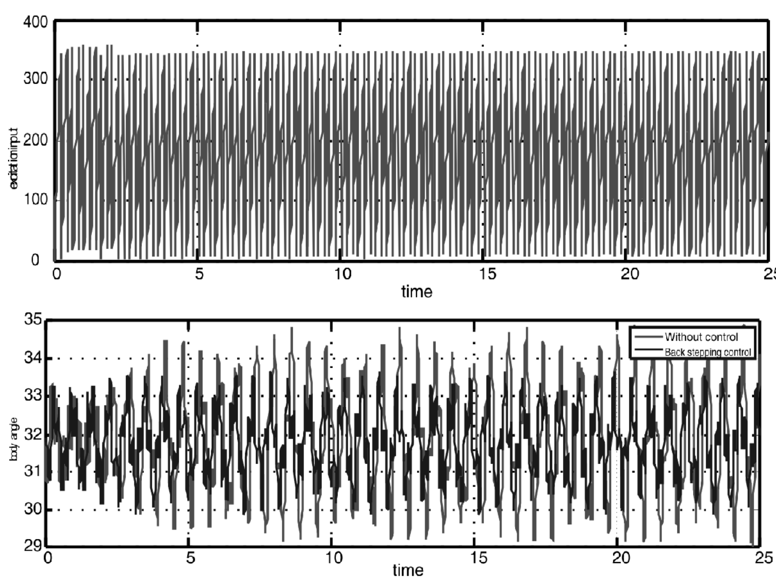
The Back stepping controller designed in Matlab/Simulink was added to thisSimulink model as illustrated in Figure 9.



(a)



(b)



(c)

Figure 10: Angle of the Upper Beam for Different Excitation Input

Different signals were given as the reference rotatingfrequency of the eccentric wheel to introduce vibrations to the Wheel. Then the vibration response of the upper beam was obtained with and without connecting the Back stepping controller. The results are illustrated in Figure 10.

The vibration response was also obtained for a freely released beam after suppression. A variation of the body angle forthe controlled and uncontrolled case is given in the Figure 11.

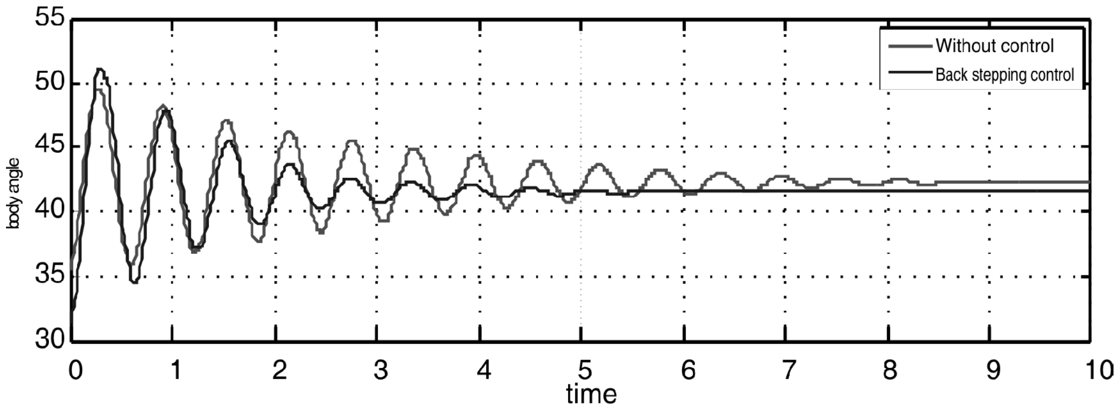


Figure 11: Experimental Results of the Body Angle for Releasing the Beam after Suppression

According to the results obtained from the experiment we can see that, the amplitudes of the vibrations were decreased when the back stepping controller is connected and the body stabilizes more quickly with the controller. In the experimental set up maximum input current for the MR damper is 1.5A. Therefore optimal control cannot be achieved due to the current limitation in the experimental setup as the results obtained with the simulations in Matlab.

VII. CONCLUSION

In this study we have developed a controller for suppressing vibrations in a body of a quarter car model. The experimental setup “Semi Active Suspension System” is the model which represents the quarter car model. It uses a spring and an MR damper as the vibration suspension devices. As the output torque of the MR damper can be controlled by controlling it’s input current the controller was developed to control the current. The controller is modeled based on the Lyapunov stability theory and backstepping technique.

The proposed controller was tested in Matlab/Simulink and with the real time experimentsfor various disturbance signals. The obtained results show a good performance of the backstepping controller in the simulation as well as the laboratory experiment.

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