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STOCHASTIC ANALYSIS METHODS IN SDN NETWORKS MODELLING

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Abstract. Stability of SDN is a central problem of the investigation. There are lots of methods to analyse systems, one of them is to use the stochastic analysis. This approach are more accurate and give us the best solution of optimization [1, 2]. In current article we use Kolmogorov equation to find out the optimal parameters of system in various conditions.It also helps us to predict the productivity of the networks and to build the self-defined networks with the ability to rebuilt or restore their operation automatically. To solve this problem, it is necessary to describe all possible conditions for system with automatic rebuilding or restoring and without these characteristics. The important part of the research is to implement the graph theory and on its base to build the mathematical model of the whole system. Then the methods of stochastic analysis using Kolmogorov equation solve the problem of the system elements productivity. At the end of this approach we obtain the accurate probabilities for each condition. The significance of the method in such problems lies in the fact that we can create on it base the automatic algorithm for complex networks with the vast amount of conditions.

1. Introduction

Nowadays the world is changing so fast and its components alters as well. One of the essential element is networks. They bring us a new way of life and make it easier.however, using the networks, communicational systems, we got a list of problems and some of them is based on the efficiency, reliability and other aspects of stability. control and estimation of the network reliability are central problem in networks researches. Thats why, its significant to study the methods and algorithms for researching the network. Most accurate and useful methods imply mathematical apparatus, for instance, mathematical modelling, optimal control theory, graph theory, mathematical statistics and stochastic analysis. Modern networks are complex structures with random events and processes. Therefore, in this context the probability theory can help us to solve the wide spectrum of problems. If to say more concrete, we consider Markov process in our study. Modern communicational systems enroot SDN self-defined networks. They have several advantages in comparison with traditional networks, but, as we understand, they

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also have some disadvantages and our research lies definitely in this field: to find out how to solve problems with these negative aspects.

2. SDN network mathematical modelling

Let consider the SDN network based on principles and architecture used for building those kind of networks. Also let the network consists of one server and two switchers. As well-known, in SDN the switchers have limited option list and all control functions, routing and other operations lies on SDN controller. SDN controller is a technically complex device for the network, so it's important to keep it in online mode and we need to minimize the errors and the faults. Reservation is one of the method that can help us to solve the problem of the faults and unstable processing SDN network.

In previous study [3] we made some assumptions: If one route is offline, the system stays stable; the SDN network is unworkable if SDN controller or all switchers are offline; at the same time only one server is fault; the restore time of controller is more than the restore time of switchers. We also defined the list of conditions for the network: system is online; one switcher is fault; two switchers are fault; restoring switcher if the server is online; restoring two switchers if the server is online; server is offline; server and switcher are offline; server and two switchers are offline; restoring controller; controller from set of servers is unworkable.

When we talk about SDN networks, it's significant to realize in which conditions the system isn't stable. Looking at the list of conditions we can say: system is unworkable in 9th and 10th condition.

3. Transfers between conditions in SDN

Let's first of all look at the links between these defined conditions. (look at [4] or Figure 1 below).

Using all these links and elements of graph theory, we can obtain the state graph for system without reservation ([4]) and state graph for the system with reservation (Figure 2). Notice that system with reservation has one more condition: SDN controller from the set of servers is unworkable.

Further using the algorithm from [4] and the state graph for system with reservation we can write the equations that will help us to define the probabilities for each state in considered Markov process [7, 8, 9].

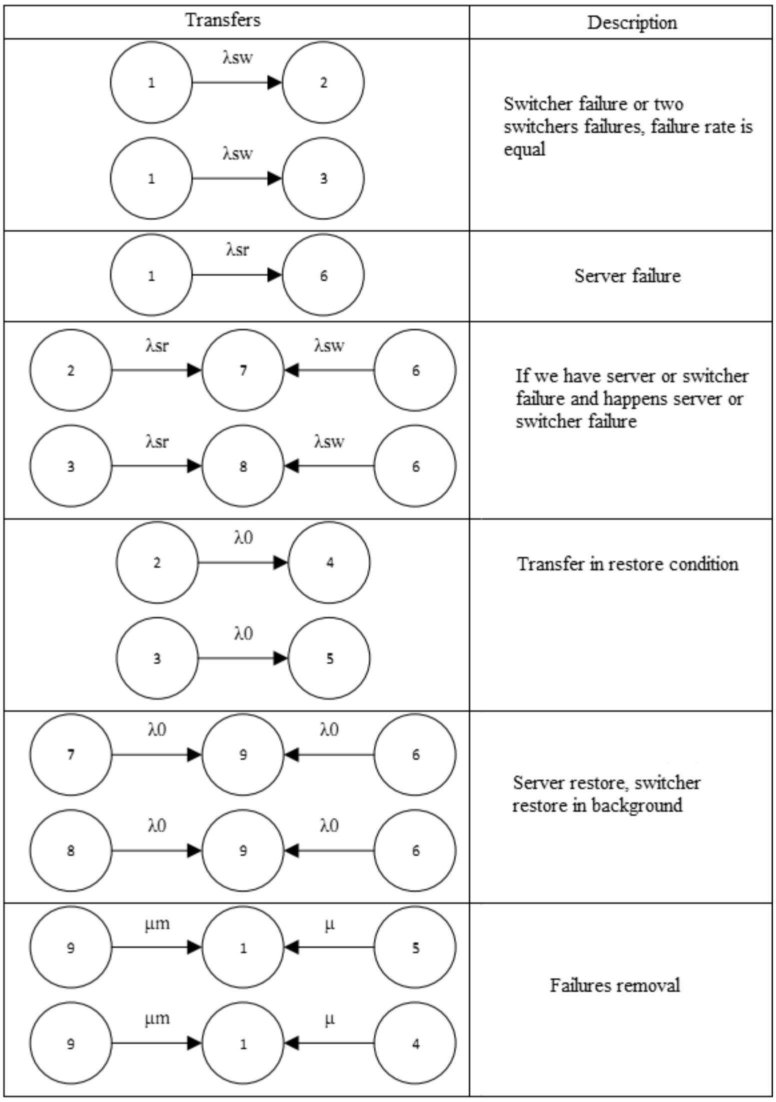

FIGURE 1. Transfers between conditions in SDN with description

Therefore, we obtain the following system of Kolmogorov differential equations:

$$\begin{cases} \frac{dP_1(t)}{dt} = -(2\lambda_{sw} + \lambda_{sr})P_1 + \mu P_3 + \mu P_4 + \mu P_8, \\ \frac{dP_2(t)}{dt} = \lambda_{sw} P_1 - (\lambda_0 + \lambda_{sr})P_2, \\ \frac{dP_3(t)}{dt} = \lambda_{sw} P_1 - (\lambda_0 + \lambda_{sr})P_3, \\ \frac{dP_4(t)}{dt} = \lambda_0 P_2 - \mu P_4, \\ \frac{dP_5(t)}{dt} = \lambda_0 P_3 - \mu P_5, \\ \frac{dP_6(t)}{dt} = \lambda_{sr} P_1 - (2\lambda_{sw} + \lambda_0 + \lambda_{sr})P_6, \\ \frac{dP_7(t)}{dt} = \lambda_{sr} P_2 + \lambda_{sw} P_6 - (\lambda_0 + \lambda_{sr})P_7, \\ \frac{dP_8(t)}{dt} = \lambda_{sr} P_3 + \lambda_{sw} P_6 - (\lambda_0 + \lambda_{sr})P_8, \\ \frac{dP_9(t)}{dt} = \lambda_0 P_6 + \lambda_0 P_7 - \lambda_0 P_8 - \mu P_9 + \lambda_0 P_1 0, \\ \frac{dP_1 0(t)}{dt} = \lambda_{sr} P_6 + \lambda_{sr} P_7 - \lambda_{sr} P_8 - \lambda_0 P_1 0. \end{cases} \tag{3.1}$$

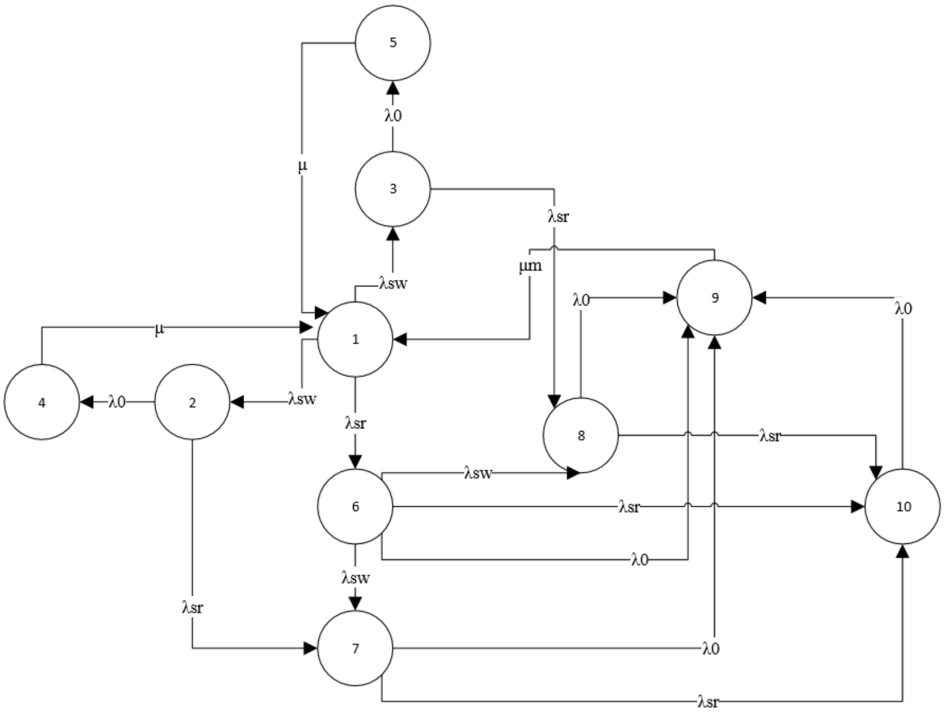

FIGURE 2. State graph for SDN with reservation

The solution of this system could be obtained by well-known algorithm for differential equations systems. However, we can simplify the calculation if notice: in this network there is a stationary Markov process, so then $dP_i = 0$, i.e. probabilities aren't changing through the time.

Now we overwrite the left-side of the equations to zero and put in system normalization condition:

$$\sum_{i=0}^{n} P_i = 1.$$

Thus, we obtain:

$$\begin{cases} -(2\lambda_{sr})P_1 + \mu P_3 + \mu P_4 + \mu P_8 = 0, \\ \lambda_{sw}P_1 - (\lambda_0 + \lambda_{sr})P_2 = 0, \\ \lambda_{sw}P_1 + (\lambda_0 + \lambda_{sr})P_3 = 0, \\ \lambda_0 P_2 - \mu P_4 = 0, \\ \lambda_0 P_3 - \mu P_5 = 0, \\ \lambda_{sr}P_1 - (2\lambda_{sw} + \lambda_0 + \lambda_{sr})P_6 = 0, \\ \lambda_{sr}P_2 + \lambda_{sw}P_6 - (\lambda_0 + \lambda_{sr})P_7 = 0, \\ \lambda_{sr}P_3 + \lambda_{sw}P_6 - (\lambda_0 + \lambda_{sr})P_8 = 0, \\ \lambda_{sr}P_6 + \lambda_{sr}P_7 - \lambda_0 P_8 - \mu P_9 + \lambda 0 P_1 0 = 0, \\ \lambda_{sr}P_6 + \lambda_{sr}P_7 + \lambda_{sr}P_8 - \lambda 0 P_1 0 = 0. \end{cases} \tag{3.2}$$

The last system of differential equations can be simplified, because some of probabilities are equal.

Thereby, we have:

$$\begin{cases} \lambda_{sw}P_1 - (\lambda_0 + \lambda_{sr})P_2 = 0, \\ \lambda_0 P_2 - \mu P_4 = 0, \\ \lambda_{sr}P_1 - (2\lambda_{sw} + \lambda_0 + \lambda_{sr})P_6 = 0, \\ \lambda_{sr}P_2 + \lambda_{sw}P_6 - (\lambda_0 + \lambda_{sr})P_7 = 0, \\ \lambda_0 P_6 + 2\lambda_0 P_7 - \mu P_9 + \lambda_0 P_1 0 = 0, \\ \lambda_{sw}P_6 + 2\lambda_{sr}P_7 - \lambda_0 P_1 0 = 0, \\ P_1 + 2P_2 + 2P_4 + P_6 + 2P_7 + P_9 + P_1 0 = 1. \end{cases} \tag{3.3}$$

To find out the solution, we involve Mathcad where define matrix X and vector Y. Moreover, if we have the intensity points, we obtain the accurate probabilities for each state, illustrated on Figure 3.

P_i \ λ_0	0.5	1	2	4	5	10	15	20	24
P_1	0.999978755080811	0.999976525300732	0.999972065770406	0.99996314682908	0.99995868741808	0.999936390959674	0.999914095495552	0.999891801025647	0.999873966165514
P_2	9.678590892341742*10^-7	1.935713292823544*10^-6	3.871407043200608*10^-6	7.742735917677192*10^-6	9.678371042307647*10^-6	1.935625354999978*10^-5	2.903364755625712*10^-5	3.871055309425739*10^-5	4.645172584980608*10^-5
P_3	9.678590892341742*10^-7	1.935713292823544*10^-6	3.871407043200608*10^-6	7.742735917677192*10^-6	9.678371042307647*10^-6	1.935625354999978*10^-5	2.903364755625712*10^-5	3.871055309425739*10^-5	4.645172584980608*10^-5
P_4	7.742872713873395*10^-6	7.742853171294174*10^-6	7.742814086401216*10^-6	7.742735917677192*10^-6	7.742696833846119*10^-6	7.742501419999912*10^-6	7.742306015001899*10^-6	7.742110618851476*10^-6	7.741954308301014*10^-6
P_5	7.742872713873395*10^-6	7.742853171294174*10^-6	7.742814086401216*10^-6	7.742735917677192*10^-6	7.742696833846119*10^-6	7.742501419999912*10^-6	7.742306015001899*10^-6	7.742110618851476*10^-6	7.741954308301014*10^-6
P_6	2.941107427222461*10^-7	5.88218862158984*10^-7	1.176427231281225*10^-6	2.352812491257784*10^-6	2.940989382533247*10^-6	5.881716460204763*10^-6	8.822181259333107*10^-6	1.17623838062337*10^-5	1.411435703983217*10^-5
P_7	5.693281575550584*10^-13	2.277304008412807*10^-12	9.109147060012178*10^-12	3.643603646300351*10^-11	5.693087590533343*10^-11	2.277148825727233*10^-10	5.123390895974683*10^-10	9.10790568139968*10^-10	1.311498699278956*10^-9
P_8	5.693281575550584*10^-13	2.277304008412807*10^-12	9.109147060012178*10^-12	3.643603646300351*10^-11	5.693087590533343*10^-11	2.277148825727233*10^-10	5.123390895974683*10^-10	9.10790568139968*10^-10	1.311498699278956*10^-9
P_9	3.529343614584669*10^-6	3.529342576684652*10^-6	3.529340500886445*10^-6	3.529336349297349*10^-6	3.529334273506462*10^-6	3.529323894588615*10^-6	3.529313515731758*10^-6	3.52930313693589*10^-6	3.529294833943107*10^-6
P_{10}	8.650349452310621*10^-14	3.460137745688239*10^-13	1.384053470088641*10^-12	5.536200854895788*10^-12	8.650303659661937*10^-12	3.460101111747004*10^-11	7.785181709569678*10^-11	1.384024163219997*10^-10	1.992985417020808*10^-10
ΣP_i	1	1	1	1	1	1	1	1	1

FIGURE 3. Current probabilities for the states

Thus, every complex network can be simulated by mathematical modelling and applying the stochastic analysis to investigation of efficiency and reliability networks. As a result, we obtain accurate characteristics.

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