International Journal of Mathematical Sciences
Vol. 18, Nos. 1-2, January-June 2019, pp. 125-136

AN INTERVAL-VALUED ITERATIVE GOAL PROGRAMMING APPROACH TO SOLVE MULTIOBJECTIVE FRACTIONAL PROGRAMMING PROBLEMS

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Abstract: This paper presents a goal programming approach for solving multiobjective fractional programming problems in which coefficients of the objectives as well as the system constraints are considered as interval- valued.

In the model formulation of the problem, first the interval-valued system constraints are converted into the equivalent crisp system by using interval programming approach. Then, the target intervals for goal achievement of the respective objectives are determined by considering the individual best and worst objective values in the decision making environment. The fractional objective goals are transformed into linear goals by employing the iterative parametric method which is an extension of Dinkelbach approach. In the solution process, the goal achievement function, termed as 'regret function', is formulated for minimizing the unwanted deviational variables to achieve the goals in their specified ranges and thereby arriving at most satisfactory solution in the decision making environment.

To illustrate the proposed approach one numerical example is solved.

Keywords - Fractional Programming, Goal Programming, Interval Arithmetic, Interval Programming, Iterative approach.

1. INTRODUCTION

In the real-world decision making situations, decision makers (DMs) are frequently faced with the three types of uncertainties in the area of inexact programming, which are stochastic programming (SP), fuzzy programming (FP) and interval programming (IP).

The SP based on the probability theory, initially introduced by Charnes and Cooper [5], has been studied [14, 16] extensively in the past and applied to the various real-life problems [12, 13] by Keown *et al*. (1978, 1980) and others.

On the other hand, FP based on the theory of fuzzy sets, initially introduced by Zadeh [26] in 1965, has been studied [1,7, 8, 27] deeply during the last 45 years from the viewpoint of the potential use to daily life problems [2] with imprecise data.

Now, in a certain decision situation, it has been realized that parameter values are found to be neither probabilistic nor fuzzy, but they are rather in the form of intervals with certain lower- and upper- bounds.

To overcome such a situation, IP approach [4, 10, 11, 14], based on interval arithmetic [10,17], has appeared as a prominent tool for solving decision problems with interval-valued parameter sets.

IP approaches to decision problems in inexact environment have been deeply studied by Birtran [3] and Steuer [24] in the past. Mainly, two types of methodological aspects are used to solve the IVP problems. The first one is based on the satisfying philosophy of GP and the second one is a traditional method of optimization. GP approaches [9, 20, 21] to IP problem have been introduced by Inuguichi and Kume [10] in 1991. The potential use of IP approach to mobile robot path planning [7] and portfolio selection [8] has been studied in the past. The methodological development made in the past has been surveyed by Oliveira and Antunes [18] in 2007. However, the IP approach to multiobjective fractional programming problems (MOFPPs) is yet to be circulated in the literature.

In this article, a GP solution approach to MOFPPs with interval valued objectives together with interval valued system constraints is presented. In the model formulation of the problem, the interval valued system constraints are transformed into equivalent crisp system constraints by using the interval inequality relation which was first introduced by Tong Shaocheng [25] in 1994 and further developed by Sengupta et al. [23] in 2001. Then, the target intervals for goal achievement in the interval goal programming approach are determined by considering the individual best and worst objective values of each objective in the decision making horizon. The interval-valued objectives with target intervals are transformed into standard goals in the conventional GP formulation by using interval arithmetic operation rule [10, 17] and then introducing under- and over deviational variables to each of them. To avoid the computational complexity of using the conventional fractional programming approach [15] to MOFPP, an iterative parametric approach [19] which is an extension of Dinkelbach approach [6] is used to convert the fractional objective goals into linear goals to solve the problem by using linear GP methodology.

In solution process, both the aspects of GP, *minsum* GP [21] as well as *minimax* GP [9] for minimizing the (unwanted) deviational variables are taken into consideration with a view to minimize the overall regret in the context of achieving the goal values within the specified ranges of the target intervals in the decision making horizon.

The proposed approach is illustrated by a numerical example.

2. PROBLEM FORMULATION

The generic form of an interval valued MOFPP with interval valued system constraints can be stated as:

Find $X(x_1, x_2, \ldots\ldots, x_n)$

so as to:

$$\text{Maximize } Z_k(\mathbf{X}) = \frac{\sum_{j=1}^{n}[a_{kj}^L, a_{kj}^U]x_j + [\alpha_k^L, \alpha_k^U]}{\sum_{j=1}^{n}[b_{kj}^L, b_{kj}^U]x_j + [\beta_k^L, \beta_k^U]}, \quad k=1,2,\ldots\ldots K \quad (1)$$

$$\text{subject to } \sum_{j=1}^{n}[c_{ij}^L, c_{ij}^U]x_j \le [e_i^L, e_i^U], (i = 1,2,..,m;\ j = 1,2,...,n) \quad (2)$$

where $x_j(\ge 0) \in R$ and $[a_{kj}^L, a_{kj}^U]$, $[b_{kj}^L, b_{kj}^U]$, $[c_{ij}^L, c_{ij}^U]$(k=1,2,…,K; i=1,2,…,m; j=1, 2,…,n) represent the vectors of interval, $[\alpha_k^L, \alpha_k^U]$ and $[\beta_k^L, \beta_k^U]$ are constant intervals, L and U stands for the lower and upper bounds of the respective intervals.

To avoid infeasibility, it is customary to assume that,

$$\sum_{j=1}^{n}[b_{kj}^L, b_{kj}^U]x_j + [\beta_k^L, \beta_k^U] > 0, \forall X \in S.$$

Now, determination of target intervals of goal achievement is presented in the Section 2.1.

DETERMINATION OF TARGET INTERVALS

The k-th objective $\mathrm{Z_k(X)}$ in (1) can be explicitly expressed as [17]:

$$Z_k(\mathbf{X}) = \frac{[\sum_{j=1}^{n} a_{kj}^L x_j + \alpha_k^L, \sum_{j=1}^{n} a_{kj}^U x_j + \alpha_k^U]}{[\sum_{j=1}^{n} b_{kj}^L x_j + \beta_k^L, \sum_{j=1}^{n} b_{kj}^U x_j + \beta_k^U]}, \quad k=1,2,\ldots,K \quad (3)$$

Using the interval arithmetic rule [16], the objective in (3) can be presented as:

$$Z_k(\mathbf{X}) = [\frac{\sum_{j=1}^{n} a_{kj}^L x_j + \alpha_k^L}{\sum_{j=1}^{n} b_{kj}^U x_j + \beta_k^U}, \frac{\sum_{j=1}^{n} a_{kj}^U x_j + \alpha_k^U}{\sum_{j=1}^{n} b_{kj}^L x_j + \beta_k^L}], \quad k=1,2,\ldots,K$$

$$=[T_{kL}(X),T_{kU}(X)],\text{ (say), } k=1,2,\ldots,K \quad (4)$$

To determine the target intervals, the best and the worst solutions of the defined interval valued objectives are to be obtained first. For this it is necessary to transfer the interval-valued system constraints into crisp system.

Using the approach introduced by Tong [25] and further studied by Sengupta et al. [23] for the inequality constraints involving interval coefficients, the crisp equivalent system constraints of the *i*-th interval constraints in (2) can be written as:

$$\sum_{j=1}^{n}(c_{ij}^{L})x_j \le e_i^{L} ,$$

$$\sum_{j=1}^{n}(c_{ij}^{L}+c_{ij}^{U})x_j+\alpha\sum_{j=1}^{n}(c_{ij}^{U}-c_{ij}^{L})x_j \le (e_i^{L}+e_i^{U})-\alpha(e_i^{U}-e_i^{L}),$$

$$\forall i=1,2,\ldots,m \quad (5)$$

where $x_j \ge 0,\ \forall j;\alpha \in [0,1]$, and α can be determined according to the needs and desires of the DM in the decision making environment.

Let, the individual best and worst solutions of the *k*-th objective be $(X_k^b;T_{kU}^*)$ and $(X_k^w;T_{kL}^*)$, respectively,

where $T_{kU}^* = \underset{\mathbf{X}\in S}{Max}\, T_{kU}(X)$, and $T_{kL}^* = \underset{\mathbf{X}\in S}{Min}\, T_{kL}(X)$

Now, from the viewpoint of achieving the objective values within the best and worst decisions, the target intervals can be considered as $[t_k^L,t_k^U]$, where $T_{kL}^* \le t_k^L < t_k^U \le T_{kU}^*,\ k=1,2,\ldots,K$.

Then, incorporating the target intervals, interval valued objectives in (4) can be expressed as [21]:

$$Z_k(X):[T_{kL}(X),T_{kU}(X)]=[t_k^L,t_k^U],k=1,2,\ldots,K. \quad (6)$$

2.2. GOAL PROGRAMMING FORMULATION

In the GP framework [9], the interval-valued objectives are transformed into crisp objective goals by introducing under-and over-deviational variables to each of them. In the proposed problem, the objectives goals can be constructed from the expression in (6) as:

$$T_{kU}(X)+d_{kL}^{-}-d_{kL}^{+}=t_k^L, \quad (7)$$

$$T_{kL}(X)+d_{kU}^{-}-d_{kU}^{+}=t_k^U, \quad (8)$$

where $d_{kL}^{-}, d_{kL}^{+}, d_{kU}^{-}, d_{kU}^{-} \geq 0$ represent under-and over-deviational variables associated with respective goals and they satisfy the relations,

$d_{kL}^{-}.d_{kL}^{+} = 0$ and $d_{kU}^{-}.d_{kU}^{+} = 0$, $\qquad k = 1,2,\ldots..,K.$

It is to be observed that the goals in (7) and (8) are fractional in form. The computational complexity arises [21] due to this fractional goals. To overcome this complexity different approaches have been studied in the area of fractional programming [15]. In this paper, the iterative approach introduced in [18] extension of Dinkelbach approach [6] is adopted in the solution process of the problem.

2.3. LINEARIZATION OF THE RATIO GOALS

The fractional form of T_{kU} (X) in the k-th goal expression in (7) can be presented as follows:

Let, $$T_{kU}(X) = \frac{\sum_{j=1}^{n} a_{kj}^{U} x_j + \alpha_k^U}{\sum_{j=1}^{n} b_{kj}^{L} x_j + \beta_k^L} = \frac{L_k(X)}{H_k(X)} \; ; k=1,2,\ldots,K,$$

where $L_k(X)$ and $H_k(X)$ are linear functions of x_j; $j = 1,2,\ldots,n$.

Then the goal expression can be presented as

$$\frac{L_k(X)}{H_k(X)} + d_{kL}^{-} - d_{kL}^{+} = t_{kL} \; ; k = 1,2,\ldots,K. \qquad (9)$$

Now, optimization of the *k*-th functional goal expression in (9) is equivalent to optimize the functional form:

$(L_k(X) - \lambda_k(X).H_k(X))$, where λ_k is a real number.

Then the linear form of (9) is obtained as

$$(L_k(X) - \lambda_k(X).H_k(X)) + d_{kL}^{-} - d_{kL}^{+} = t_{kL} \, , \; k = 1,2,\ldots,K. \qquad (10)$$

Now, the fractional form of T_{kL}(X) in the k-th goal expression in (7) can be presented as follows:

Now, let $$T_{kL}(X) = \frac{L_{K+k}(X)}{H_{K+k}(X)}, \qquad k = 1,2,\ldots, K.$$

Proceeding analogous way, the linear parametric form of the goal expression in (8) can be presented as:

$$(L_k(X) - \lambda_k(X).H_k(X)) + d^-_{k-K,U} - d^+_{k-K,U} = t_{k-K,U},$$
$$k=K+1,K+2,\ldots,2K. \qquad (11)$$

Now, it is to be followed that in the solution process, the proposed approach is iterative in nature in the process of solving the problem.

A version of the iterative solution procedure in [19] is presented in the following steps.

2.3.1 THE ALGORITHMIC STEPS

Step 1: Rename λ_k by λ_{ki} to represent it at the i-th solution stage.

Step 2: Set $\lambda_{ki} = 0$ for $i = 1$ and $k = 1, 2,\ldots, K, K+1, K+2,\ldots,2K$ and solve the GP problems in (10) and (11).

Step 3: Let X_1 be the solution obtained in the Step 2. Then set $i = 2$ and determine $\lambda_{k2} = L_k(X_1)/H_k(X_1)$.

Step 4: Solve the problem in (10) and (11) with the defined λ_{k2}.

Step 5: Determine $D(\lambda_{ki}) = L_k(X) - \lambda_{ki}.H_k(X)$

Step 6: Define ε such that ε is a sufficiently small positive number.

Step 7: If $|D(\lambda_{ki})| \geq \varepsilon$ go to Step 8, otherwise go to Step 10.

Step 8: Set $i = i + 1$.

Step 9: Compute $\lambda_{k,i+1} = L_k(X_i)/H_k(X_i)$ and return to the Step 2.

Step 10: If $|D(\lambda_{ki})| < \varepsilon$, terminate the algorithm, and identify the solution

$$X_i = \begin{cases} X^*, \text{if } F(\lambda_{ki}) = 0 \\ X_i^*, \text{if } |F(\lambda_{ki})| > 0 \end{cases}$$

where X^* is the optimal solution and X_i^* is the approximate solution to the problem in the notion of satisficing philosophy in the conventional GP approach.

Note 1: Regarding convergence of the proposed algorithm, it is to be noted that the executable IP model involves a number of linear programs in the solution process. Since the solution space is bounded and only the linear programs are involved there in the solution search process, the algorithm always stops after a finite number of iterations.

3. GP MODEL FORMULATION

In a decision making situation, the aim of each of the DMs, is to achieve the goal values within the specified ranges by means of minimizing the possible regrets in terms of the deviational variables involved in the decision situation.

In the present decision situation, the goal achievement function is termed as the 'regret function', since the regret intervals defined for goal achievement within the specified target intervals are to be minimized to the extent possible in the decision making horizon.

Now, in the field of IP, both the aspects of GP, *minsum* GP [21] for minimizing the sum of the weighted unwanted deviational variables as well as *minmax* GP [9] for minimizing the maximum of the deviations, are simultaneously taken into account as a convex combination of them to reach a satisfactory decision within the specified target intervals of the goals.

The regret function appears as [10]:

Minimize $Z= \mu\left\{\sum_{k=1}^{K} w_k (d_{kL}^- + d_{kU}^+)\right\} + (1-\mu)\left\{\max_k (d_{kL}^- + d_{kU}^+)\right\}$

(12)

Taking $\max_k (d_{kL}^- + d_{kU}^+) = V$, the executable GP model of the problem takes the form:

Minimize $Z = \mu\sum_{k=1}^{K} w_k (d_{kL}^- + d_{kU}^+) + (1-\mu)V$,

and satisfy

$(N_k(X) - \lambda_k(X).D_k(X)) + d_{kL}^- - d_{kL}^+ = t_{kL}$, k=1, 2,…, K;

$(N_k(X) - \lambda_k(X).D_k(X)) + d_{k-K,U}^- - d_{k-K,U}^+ = t_{k-K,U}$; k=K+1, K+2,…,2K,

$d_{kL}^- + d_{kU}^+ \le V$, k =1,2,….,K,

together with the system constraints defined in (5), (13)

where $d_{kL}^-, d_{kL}^+ \ge 0$ with $d_{kU}^-.d_{kU}^+ = 0$ and Z represents the regret function for goal achievement and $w_k \ge 0$ with $\sum_{k=1}^{K} w_k = 1$ denote the numerical weights of importance of achieving the goals within the respective target intervals, and $0 \le \lambda \le 1$.

To demonstrate the feasibility of the approach one numerical example is solved.

4. NUMERICAL EXAMPLE

The following numerical example is considered to illustrate the proposed approach.

Find $X(x_1, x_2)$ so as to

Maximize Z1: $\dfrac{[3,4]x_1+[1,2]x_2+[2,2]}{[6,7]x_1+[2,4]x_2+[5,6]}$,

Maximize Z2: $\dfrac{[1,2]x_1+[5,11]x_2+[7,8]}{[4,5]x_1+[3,7]x_2+[3,3]}$,

Maximize Z3: $\dfrac{[2,4]x_1+[15,17]x_2+[4,4]}{[6,8]x_1+[3,5]x_2+[5,5]}$,

subject to

$$[0.5, 1.5]\,x_1+[0.75,1.75]x_2\le[4,8],$$
$$[0.25, 1.25]x_1+[1, 3]x_2\le[3, 5]\ ,\ x_1,x_2\ge 0 \qquad (14)$$

Using (5) and considering α = 0.5 (according to the needs and desires of the DM), interval valued system constraints in (14) are converted into crisp form as [23]:

$$0.5x_1+0.75x_2\le 4,\ 2.5x_1+x_2\le 10,$$
$$0.25x_1+x_2\le 3,\ 2x_1+3x_2\le 7. \qquad (15)$$

The objectives in standard form of interval can be presented as [17]:

$$Z_1(X)=[\frac{3x_1+x_2+2}{7x_1+4x_2+6},\frac{4x_1+2x_2+2}{6x_1+2x_2+5}],$$
$$Z_2(X)=[\frac{x_1+5x_2+7}{5x_1+7x_2+3},\frac{2x_1+11x_2+8}{4x_1+3x_2+3}],$$
$$Z_3(X)=[\frac{2x_1+15x_2+4}{8x_1+5x_2+5},\frac{4x_1+17x_2+4}{6x_1+3x_2+5}]. \qquad (16)$$

Then, following the proposed procedure the individual best and worst solutions of the first objective in (16) are obtained as:

$(X_1^b;T_{1U}^*)=(0, 2.33\,;\ 0.6897)$ and $(X_1^w;T_{1L}^*)=(0, 2.33\,;\ 0.2826)$ respectively.

The best and worst solutions of the second objective in (16) are obtained as: $(X_2^b;T_{2U}^*)=(0, 2,33\,;\ 3.3667)$ and $(X_2^w;T_{2L}^*)=(3.5, 0\,;\ 0.5122)$ respectively.

Similarly, the best and worst solutions of the third objective are obtained as:

$(X_3^b;T_{3U}^*)=(0, 2.33\,;\ 3.6389)$ and $(X_3^w;T_{3L}^*)=(3.5, 0\,;\ 0.333)$ respectively.

Now, the target intervals for the defined individual best and worst decisions of the objectives are successively considered as:

[0.30, 0.67], [0.65, 3.05] and [0.75, 3.15].

Then, interval valued objectives with their specified target intervals can be presented as:

$$[\frac{3x_1+x_2+2}{7x_1+4x_2+6}, \frac{4x_1+2x_2+2}{6x_1+2x_2+5}] = [0.30, 0.67],$$

$$[\frac{x_1+5x_2+7}{5x_1+7x_2+3}, \frac{2x_1+11x_2+8}{4x_1+3x_2+3}] = [0.065, 3.05],$$

$$[\frac{2x_1+15x_2+4}{8x_1+5x_2+5}, \frac{4x_1+17x_2+4}{6x_1+3x_2+5}] = [0.75, 3.15].$$

Using interval arithmetic technique [10, 11], equivalent objective goals can be defined as:

$$\frac{4x_1+2x_2+2}{6x_1+2x_2+5} + d_{1L}^- - d_{1L}^+ = 0.30, \qquad \frac{3x_1+x_2+2}{7x_1+4x_2+6} + d_{1U}^- - d_{1U}^+ = 0.67,$$
$$\frac{2x_1+11x_2+8}{4x_1+3x_2+3} + d_{2L}^- - d_{2L}^+ = 0.65, \qquad \frac{x_1+5x_2+7}{5x_1+7x_2+3} + d_{2U}^- - d_{2U}^+ = 3.05,$$
$$\frac{4x_1+17x_2+4}{6x_1+3x_2+5} + d_{3L}^- - d_{3L}^+ = 0.75, \qquad \frac{2x_1+15x_2+4}{8x_1+5x_2+5} + d_{3U}^- - d_{3U}^+ = 3.15. \tag{17}$$

Then, following the above procedure and iterative approach, GP formulation of the problem can be obtained as:

Find $X(x_1, x_2)$ so as to

Minimize Z =

$$\mu[w_1(d_{1L}^- + d_{1U}^+) + w_2(d_{2L}^- + d_{2U}^+) + w_3(d_{3L}^- + d_{3U}^+)] + (1-\mu)V$$

so as to satisfy

$$(4x_1+2x_2+2) - \lambda_1(6x_1+2x_2+5) + d_{1L}^- - d_{1L}^+ = 0.30,$$
$$(2x_1+11x_2+8) - \lambda_2(4x_1+3x_2+3) + d_{2L}^- - d_{2L}^+ = 0.65,$$
$$(4x_1+17x_2+4) - \lambda_3(6x_1+3x_2+5) + d_{3L}^- - d_{3L}^+ = 0.75,$$
$$(3x_1+x_2+2) - \lambda_4(7x_1+4x_2+6) + d_{1U}^- - d_{1U}^+ = 0.67,$$
$$(x_1+5x_2+7) - \lambda_5(5x_1+7x_2+3) + d_{2U}^- - d_{2U}^+ = 3.05,$$
$$(2x_1+15x_2+4) - \lambda_6(8x_1+5x_2+5) + d_{3U}^- - d_{3U}^+ = 3.15,$$

subject to the crisp system constraints in (15),

and $d_{1L}^- + d_{1U}^+ \le V$, $d_{2L}^- + d_{2U}^+ \le V$, (18) where $d_{kL}^{-}, d_{kL}^{+}, d_{kU}^{-}, d_{kU}^{+} \geq 0$ (k=1, 2) with

$d_{kL}^{-}.d_{kL}^{+} = 0, \; and \;\; d_{kU}^{-}.d_{kU}^{+} = 0$.

Considering equal weights, i.e. $w_1 = w_2 = w_3 = 1/3$ and, the problem is solved by using a linear GP methodology.

In the solution process, following the algorithmic steps as defined in the section 2.3.1, and using the *Software* LINGO (ver 12.0) iteratively, the optimal solution is achieved at the fourth iteration (considering $\alpha = 0.5$).

The resulting decision is

(x_1, x_2) = (0,0.12) with Z_1 = [0.33,0.43], Z_2 = [1.98,2.77] and Z_3 = [1.04, 2.99].

The result shows that a satisfactory solution within the specified target intervals is reached in the decision making environment.

Note 2: It is to be noted that, instead of employing the linearization approach, if the problem of the achievement of the fractional goals in (17) is directly considered and solved by using the conventional interval GP approach, the solution is obtained as:

(x_1, x_2) = (0, 2.33) with Z_1 = [0.30,0.71], Z_2 = [0.97,3.37] and Z_3 = [2.33, 2.47].

It can easily be followed from the above diagram that the achievement of all objective values under the proposed approach lies within the aspired range. Thus more acceptable decision is achieved here under the proposed approach than the conventional approach with respect to the achieving the goal values within their specified target intervals.

5. CONCLUSION

The main advantage of the proposed approach is that the computational complexity with the fractional goals does not arise here due to the efficient use iterative approach. The use of the proposed approach to real-world decision problems is an emerging area for study in future. The proposed approach may be extended to solve the hierarchical decentralized decision problem with interval parameter sets. However, it is hoped that the approach presented in this paper will open up a new vistas of research on interval programming for its actual implementation of real-world problem in inexact environment.

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