Explaining the Dark Matter without Inventing New Subatomic Particles and without Dramatically Changing Physical Laws

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ABSTRACT: From astrophysical observations it is clear that in the Universe there is over 5 times more of the unknown, dark matter than the known, ordinary matter. Existing hypotheses about the dark matter either introduce new, undiscovered subatomic particles or propose to dramatically change the known physical laws (such as the modified Newtonian dynamics or related proposals). A more natural explanation of the dark matter suggested in the present paper is based on the results of our previous paper (J. Phys. B: At. Mol. Opt. Phys. **34** (2001) 2235). There it was shown that for the ground state of hydrogenic atoms/ions (GSHA) for spherically symmetric potentials inside the nucleus, corresponding to realistic charge densities having the maximum at r = 0, it is possible to match the interior (inside the nucleus) solution of the Dirac equation, so that the singular solution should not be rejected for the GSHA. The existence of this alternative kind of hydrogen atoms was proven in the above paper both theoretically and by the analysis of atomic experiments. As a terminological reformulation of these results in the current paper, we presented the fact that hydrogen atoms can have two flavors: one flavor corresponding to the regular solution outside the proton, another – to the singular solution outside the proton, both solutions corresponding to the same energy. Since this means the additional degeneracy, then according to the fundamental theorem of quantum mechanics, there should be an additional conserved quantity, which we called isohydrogen spin (isohyspin). The singular flavor of hydrogen atoms (SFHA) does not have states required for the emission or absorption of the electromagnetic radiation – except for the hyperfine structure substates of the ground state. Therefore, the SFHA could represent the dark matter (or a nearly-dark matter) or a part of it. This idea seems to provide a more natural explanation of the dark matter – because it does not require introducing new, undiscovered subatomic particles or dramatically changing the known physical laws. We show that the SFHA could explain the puzzling results by Bowman et al (Nature **555** (2018) 67) concerning the observation of the 21 cm radio line from the early Universe – without introducing an unspecified dark matter. Therefore, further observational studies of the 21 cm radio line from the early Universe could provide a further proof that the dark matter or a part of it is the SFHA.

Kewwords: two flavors of hydrogen atoms; new symmetry of hydrogen atoms; explanation of the dark matter; explanation of the puzzle of 21 cm radio line; early Universe

1. INTRODUCTION

From astrophysical observations it is clear that in the Universe there is over 5 times more of the unknown, dark matter than the known, ordinary matter. Existing hypotheses about the dark matter either introduce new, undiscovered subatomic particles or propose to dramatically change the known physical laws (such as the modified Newtonian dynamics or related proposals) – see, e.g., reviews [1-3], book [4], and references therein. A more natural explanation of the dark matter suggested in the present paper proceeds from the results of our previous paper [5] dealing with singular solutions of the Dirac equation, as follows.

There existed a paradigm that, even with the allowance for the finite nuclear size, *singular* solutions of the Dirac equation for the Coulomb problem should be rejected for nuclear charges $Z < 1/\alpha$ ($\alpha = e^2/(\hbar c)$ being the fine structure constant) not only for the excited states, but even for the ground state of hydrogen atoms/hydrogenlike ions (hereafter, GSHA)*/. In paper [5] this paradigm was broken as follows.

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* *Here and below, by "singular" we mean the strongly-singular solution of the Dirac equation for the Coulomb field – in distinction to the commonly accepted "regular" solution that has a weak singularity at the origin.*

First, there was derived a general condition for matching a regular solution inside the nucleus with a singular exterior solution of the Dirac equation for *arbitrary* spherically-symmetric interaction potential V(r), which takes two different forms in the interior region r < R and in the exterior region r > R, where R is the nuclear size. For the models of the proton charge density, such as the spherical shell charge density or the uniform charge density (used in the literature before paper [5]), the corresponding interior solutions cannot be matched with the singular exterior solution of the Dirac equation for the Coulomb field even for the GSHA. However, these models were unrealistic: from experiments on the elastic scattering of electrons on protons [6-8] it is well known that the charge density inside proton neither has a peak at the periphery (as for the spherical shell charge density) nor is constant (as for the uniform charge density), but rather it has a maximum at r = 0.

Second, it was shown in paper [5] that for spherically symmetric interior potentials, corresponding to realistic charge densities having the maximum at r = 0, it is possible to match the interior solution with the singular exterior solution of the Dirac equation for the Coulomb field for the GSHA. This means that the singular solution of the Dirac equation for the Coulomb field should not be rejected for the GSHA.

The motivation for paper [5] was to explain the long-standing mystery of the high-energy tail of the linear momentum distribution in the GSHA (below, for brevity we use simply the word "momentum" meaning "linear momentum"). In 1935 Fock [9] derived the following distribution of the momentum dw = f(p)dp for the bound electron in the GSHA, where the distribution distribution function f(p) is:

$$f(p) = 32p^2 p_0^5 / [\pi (p_0^2 + p^2)^4], \quad p_0 \equiv Zme^2/\hbar. \tag{1}$$

The quantity $p_0/Z \approx 1.992 \times 10^{-19}$ g×cm/s practically coincides with the atomic unit of the linear momentum; m is the reduced mass of an electron in a hydrogen atom or hydrogenlike ion. From Eq. (1) it follows that the high-energy tail of the momentum distribution (HTMD) has the form

$$f^{As}(p) \equiv f(p \gg p_0) \propto 1/p^6. \tag{2}$$

However, for hydrogen atoms the experiments seem to favor a HTMD ~ $1/p^k$, where k is *at least 1.5 times smaller than in Eq. (2)* – according to the detailed discussion in paper [5]. This conundrum is even more challenging because of the following. While the HTMD from Eq. (2) corresponds to relatively large values of the linear momentum p ≫ p_0, these values are still below the relativistic range of p. This is because for hydrogen atoms one has p_0/mc = α ≈ 1/137, so that there is a significant range of p, where it seems that the nonrelativistic quantum theory (used for deriving Eqs. (1), (2)) should remain valid.

In paper [5] it was shown that the engagement of the singular solution for the GSHA yields for the HTMD an effective power law $f^{As}(p) \propto 1/p^k$ with the value of k noticeably smaller than 6, thus resolving the above long-standing mystery. This fact constitutes the experimental evidence of the existence of this alternative kind of hydrogen atoms and of hydrogen atoms coming in two flavors.

In the present paper we offer further considerations resulting from this fact. We provide an alternative view on the dark matter – without resorting to new subatomic particles or dramatically changing the existing physical laws. Moreover, we explain the puzzling results by Bowman et al (Nature **555** (2018) 67) concerning the observation of the 21 cm radio line from the early Universe – without introducing an unspecified dark matter. We also shed a new "terminological light" on the results of paper [5], bringing to the readers attention a new additional symmetry and the corresponding new additional conserved quantity; however, it should be emphasized that our explanations of the dark matter in Sect. 3, as well as of the puzzling observation of the 21 cm radio line in Sect. 4, are based directly on the theoretical and experimental results of paper [5] regardless of the new terminology presented in Sect. 2.

2. TWO FLAVORS OF HYDROGEN ATOMS

Solutions of the Dirac equation for the electron in the Coulomb field are common eigenfunctions of four operators (as it is well-known – see, e.g., the textbook [10] – the Hamiltonian H, the projection J_z of the total angular momentum, the square J^2 of the total angular momentum, and the following operator:

$$K = \beta \, (2\mathbf{Ls} + 1). \tag{3}$$

Here β is the Dirac matrix of the rank four, whose nonzero elements are $\beta_{11} = \beta_{22} = 1$, $\beta_{33} = \beta_{44} = -1$; $\mathbf{L}$ and $\mathbf{s}$ are the operators of the orbital angular momentum and spin, respectively; $\mathbf{Ls}$ denotes the dot-product (also known as the scalar product) of the latter two operators. Eigenvalues of the operators K and J^2 are connected as follows: $k = \pm (j + 1/2)$.

Hydrogen atoms in the stationary states have the following well-known energies

$$E_{Nk} = mc^2 \{ 1 + \alpha^2 / [N + (k^2 - \alpha^2)^{1/2}]^2 \}^{-1/2}, \tag{4}$$

where N is the radial quantum number. For the ground state, the quantum numbers N and k have the following values

$$N = 0, \; k = -1, \tag{5}$$

so that

$$E_{0,-1} = mc^2 (1 - \alpha^2)^{1/2}. \tag{6}$$

For hydrogen atoms the radial part $R_{Nk}(r)$ of the coordinate wave functions has the following behavior at small r (see, e.g., the textbook [10]):

$$R_{Nk}(r) \propto 1/r^{1+s}, \; s = \pm (k^2 - \alpha^2)^{1/2}. \tag{7}$$

For the GSHA, Eq. (7) reduces to:

$$R_{0,-1}(r) \propto 1/r^q, \; q = 1 \pm (1 - \alpha^2)^{1/2}. \tag{8}$$

In paper [5] it was shown, that with the allowance for the finite proton size, both the regular exterior solution corresponding to $q = 1 - (1 - \alpha^2)^{1/2}$ and the singular exterior solution corresponding to $q = 1 + (1 - \alpha^2)^{1/2}$ are legitimate for the GSHA. Below is a logical continuation of this fundamental result.

On the one hand, for hydrogen atoms both the regular ground state and the singular ground state correspond to the same energy given by Eq. (4) with the same quantum numbers $N = 0$, $k = -1$. In other words, the ground state of hydrogen atoms, in addition to the trivial double-degeneracy with respect to the z- projection m_j of the total angular momentum J, has an *additional double-degeneracy*.

On the other hand, there is a fundamental theorem of quantum mechanics concerning the cause of any additional degeneracy. Namely, it is caused by the existence of an additional conserved quantity (or quantities), whose operator commutes with the Hamiltonian, but does not commute with other conserved quantities or if it does, but the additional conserved quantity is a multi-component one, then its components do not commute with each other – see, e.g., the textbook [11]. The corresponding degenerate states differ by the eigenvalues (i.e., by additional quantum numbers) of the additional conserved quantity. For example, in the non-relativistic Coulomb problem, the higher than geometrical degeneracy (the geometrical one being due to the spherical symmetry of the problem) is caused by the existence of the Runge-Lenz vector $\mathbf{A}$, whose operator commutes with the Hamiltonian, but does not commute with some components of the angular momentum operator (and the components of the Runge-Lenz vector operator do not commute with each other) – see, e.g., the textbook [11]. For a given energy, the corresponding non-relativistic degenerate states differ not only by the eigenvalue of the operator L_z, but also by the eigenvalue of the operator A_z.

However, according to paper [5], the GSHA is double-degenerate despite these two states have the same values of all the known conserved quantities E, K, J^2, and J_z. This means that *there should be an additional, new conserved quantity, by eigenvalues of which these two ground states differ*. In other words, the situation is that hydrogen atoms have *two flavors*, differing by the eigenvalue of an additional, new conserved quantity: hydrogen atoms have *flavor symmetry*.

Speaking of flavors, let us recall by analogy that quarks have flavors: for example, there are up and down quarks. For representing this particular flavor symmetry, there was assigned an operator of the isotopic spin (isospin)

I – the operator having two eigenvalues for its z-projection: $I_z = 1/2$ assigned to the up quark and $I_z = -1/2$ assigned to the down quark.

Therefore, it seems reasonable to introduce here a new operator: the operator of *isohydrogen spin*, abbreviated as *isohyspin* and denoted as $I^{(h)}$. Similarly to the isospin, the z-projection of the isohyspin operator has two eigenvalues: $I^{(h)}_z = 1/2$ assigned to the regular flavor of hydrogen atoms and $I^{(h)}_z = -1/2$ assigned to the singular flavor of hydrogen atoms.

The isospin couples to the strong force (strong interaction). This is logical because it is related to intra-nuclear physics, where the strong interaction plays the dominant role. As a result, the strong force can transform the up quark into the down quark and vice versa.

In distinction, the isohyspin does not relate to intra-nuclear physics: so, it would be logical to state that the isohyspin does not couple to the strong force/interaction – since the isohyspin relates to a hydrogen atom as the whole. For the same reason, it would be logical to state that the isohyspin does not couple to the weak force/ interaction. Also there seems no ground to expect that the isohyspin would couple to the gravitational force/interaction. As for the electromagnetic force/interaction, the (ordinary) spin couples to the magnetic field, but the isospin does not couple to the electromagnetic force/interaction. Therefore, there seem to be no reason for the isohyspin to couple to the electromagnetic force/interaction either.

3. POSSIBLE EXPLANATION OF THE DARK MATTER AND OF THE LATEST ASTROPHYSICAL OBSERVATIONS OF THE 21 CM RADIO LINE

We suggest that *the dark matter or a part of it could be represented by the singular flavor of hydrogen atoms*. This is due to the fact that the singular flavor of hydrogen atoms does not have states required for the emission or absorption of the electromagnetic radiation – because such states cannot be described by the singular solution outside the proton. (The only possible radiative transitions for the singular flavor of hydrogen atoms are between the hyperfine structure substates of the ground state, the latter transition being known as the radio line of 21 cm wavelength.*/) This idea seems to provide a more natural explanation of the dark matter – because it does not require introducing new, undiscovered subatomic particles or dramatically changing the known physical laws.

Moreover, the existence of the singular flavor of hydrogen atoms could explain a puzzling observational result published in 2018 in Nature by Bowman et al [12]. The authors observed the 21 cm line (redshifted from the rest frequency of 1,240 MHz to the frequency of 78 MHz) from the early Universe. They observed the absorption profile of this line: namely, as hydrogen atoms absorb photons from the cosmic microwave background (CMB). The underlying physical mechanism was the ultraviolet light from stars formed in the early Universe – the light that is expected to penetrate the primordial hydrogen gas and to alter the excitation of the hydrogen 21 cm hyperfine structure line.

The puzzling result from [12] was that the amplitude of the profile was more than a factor of two greater than the largest predictions. This could mean that the primordial hydrogen gas was much cooler than expected.

The intensity of the observable 21 cm line from the early Universe is given as the brightness temperature T_B, which is a linear combination of the CMB temperature T_{CMB} and the spin temperature T_S (the latter being the excitation temperature of the hyperfine transition). In its turn, the spin temperature T_S is, generally speaking, a linear combination of the following three contributions (see, e.g., paper [13] and review [14]): the CMB temperature T_{CMB}, the gas kinetic temperature T_K (due to the fact that there is the collisional excitation of the hyperfine transition), and the color temperature of the radiation field in the Lyman series T_{Ly}. The latter contribution is due to the Wouthuysen-Field effect. Physically, the Wouthuysen-Field effect is the transition between the hyperfine structure sublevels of the ground state facilitated by the absorption and the subsequent reemission of a photon of the Lyman series – mostly the Ly-alpha photon.

Our possible explanation of the puzzling observational result from [12] is the following. Hydrogen atoms of the singular flavor do not have states required for the absorption or emission of Lyman quanta. Therefore, for such

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*For this reason, rigorously speaking, the singular flavor of hydrogen atoms could be called the nearly-dark matter.

atoms the contribution, which would have been due to the Wouthuysen-Field effect, is absent. Therefore the spin temperature of the hydrogen atoms of the singular flavor is significantly smaller than for the hydrogen atoms in the regular flavor. The effective spin temperature of the mixture of the two flavors would be dominated by the spin temperature of the singular flavor since it could be expected that the latter flavor is the dominating part of the mixture – consistent with the fact that the dark matter is considered to be more abundant than the ordinary matter. The smaller effective spin temperature leads to the greater amplitude of the 21 cm absorption signal observed in [12]. This explanation seems to be more natural than resorting to a possible cooling of barions by unspecified dark matter particles, as in papers [15, 16].

3. CONCLUSIONS

As a terminological reformulation of the results from our previous paper [5], we presented the fact that hydrogen atoms can have two flavors (as it was proven in the above paper both theoretically and by the analysis of atomic experiments): one flavor corresponding to the regular solution outside the proton, another - to the singular solution outside the proton.

As a logical continuation of the results from our previous paper [5], we presented the fact that hydrogen atoms can have two flavors: one flavor corresponding to the regular solution outside the proton, another – to the singular solution outside the proton. Since this means the additional degeneracy, then according to the fundamental theorem of quantum mechanics, there should be an additional conserved quantity, which we called isohydrogen spin (isohyspin) $I^{(h)}$. The eigenvalues of the operator $I^{(h)}$ are $+1/2$ for the regular flavor and $-1/2$ for the singular flavor of hydrogen atoms. The isohyspin does not seem to couple to any of the four known fundamental forces/interactions.

The singular flavor of hydrogen atoms does not have states required for the emission or absorption of the electromagnetic radiation – except for the hyperfine structure substates of the ground state (the latter transition being known as the radio line of 21 cm wavelength). Therefore, *the singular flavor of hydrogen atoms could represent the dark matter (or a nearly-dark matter) or a part of it*. In this way, the dark matter or a part of it could be explained without resorting to new subatomic particles or without dramatically changing the physical laws.

As for the radiative transitions between the hyperfine structure substates of the ground state, the singular flavor of hydrogen atoms could explain the puzzling observational results by Bowman et al [12] – without introducing an unspecified dark matter. Therefore, further observational studies of the redshifted 21 cm radio line from the early Universe could provide a further proof that the dark matter or a part of it is the singular flavor of hydrogen atoms.

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