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Effect of Thermal Diffusion and Periodic Permeability on MHD Convective Heat and Mass Transfer through a Porous Medium in a Vertical Channel

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Abstract: We analyse the mixed convective flow of a viscous incompressible conducting fluid through a porous medium confined in the vertical channel whose walls are maintained at constant temperature and concentration. The dissipative effects are taken in to the energy equation. A uniform magnetic filed of strength H_0 is applied normal to the walls. Also we consider the heat generating source in the energy equation. The momentum, energy and diffusion equations are non dimensionalised to obtain the governing parameters viz, Grashoff number *G*, Reynolds number *R*, Darcy Parameter *D*, Prandtl number *P*, Heat source Parameter a, Schmidt number *Sc*, buoyancy ratio *N* and Eckert number *Ec*. The coupled governing equations have been served using a perturbation technique with *Ec* (<<1). The velocity, temperature are obtained analytically and the behaviour for different variations in the parameter is discussed computationally. The skin friction and the local rate of heat transfer are calculated numerically for variations in the said parameters.

Keywords: Heat Transfer, Porous Medium, Darcy Law, Concentration.

1. Introduction

Thermal and solution transport by fluid flowing through a porous matrix is a phenomenon of great interest from both the theory and application point of view. Heat transfer in the case of homogenous fluid-saturated porous media has been studied with relation to different applications like dynamics of hot underground springs, terrestrial like dynamics of hot underground springs, terrestrial heat flow through aquifer, hot fluid and ignition front displacements in reservoir engineering, heat exchange displacements in reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials and heat exchange with fluidized beds. Mass transfer in isothermal conditions has been studied with applications to problems of mixing of fresh and salt water in aquifers, miscible displacements in oil reservoirs, spreading of solutes in fluidized beds and crystal washers, salt leaching in soils, etc. Prevention of salt dissolution into the lake waters neat the sea shores has become a serious problem of research. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Kahir [2], Lai and Kulacki [4]. Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous medium has been analyzed by Lai [3]. The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa *et al.*, [1].

The combined effects of thermal and mass diffusion in channel flows has been studied in the recent time by a few authors, notably, Nelson and Wood [5, 6] and others [7, 8, 9]. Nelson and Wood [5] have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature and concentration boundary conditions. For long channel (low Reynolds number) the numerical solutions approach the fully developed flow analytical solutions.

2. Formulation

We consider the flow of a viscous, electrically conducting, incompressible fluid through a highly porous medium confined in a vertical channel bounded by two flat plates. A uniform magnetic field of strength H_0 is applied transverse to the walls. Assuming the magnetic Reynolds number to be small we neglect the induced magnetic field in comparison to the applied field. We choose a rectangular Cartesian coordinate system $O(x, y, z)$ with the plates in the x-y plane. The z-axis is taken normal to the plane of the plates .The walls are maintained at constant temperature T_0 & T_1 and constant concentrations C_0 & C_1. The permeability of the porous medium is assumed to be of the form

$$K(y) = \frac{K_0}{1 + \varepsilon \cos(\pi y/d)} \tag{1}$$

where K_0 is the mean permeability of a medium, d is the wave length of the permeability distribution and $\varepsilon\,(< 1)$ is the amplitude of the permeability variation. Since the fluid extends to infinity in y-direction it follows from the equation of continuity that

$$\frac{\partial u}{\partial x} => u = u(y, z) \tag{2}$$

All the field properties are assumed constant except that the influence of the density variation with the temperature and concentration is considered only in the body force term. The viscous dissipation and Darcy dissipation are taken into account. We take Soret effect into account in the diffusion equation. A linear density variation is assumed with ρ_0, T_0 & C_0 being the density, temperature and concentration of the fluid in the equilibrium, state.

In view of this the governing equations are

$$\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\left(\frac{\mu}{k(y)}+\sigma\mu_e^2 H_0^2\right)u-\rho g-\frac{\partial p}{\partial x}=0 \tag{3}$$

$$\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\mu}{k_1}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]+\left(\frac{\mu}{k(y)}+\sigma\mu_e^2 H_0^2\right)u^2=0 \tag{4}$$

$$\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}=-\left(\frac{k_{11}}{D_1}\right)\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

The boundary conditions are

$$\begin{aligned} &u=0 \text{ on } z=0 \quad \text{and} \quad z=d \\ &T=T_0, \quad C=C_0 \quad \text{on} \quad z=0 \\ &T=T_1, \quad C=C_1 \quad \text{on} \quad z=d \end{aligned} \tag{6}$$

Introducing the following non-dimensional quantities

$$(y^*, z^*)=\frac{(y,z)}{d}, \quad u=\left(\frac{\mu}{\rho_0 d}\right)u^*, \quad q^*=\left(\frac{T-T_0}{T_1-T_0}\right), \quad C^*=\left(\frac{C-C_0}{C_1-C_0}\right)$$

the equations (3)-(5) reduce to (dropping the a striks)

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}-D^{-1}(1+\varepsilon\cos\pi y)\,u+G(\theta+Nc)=0 \tag{7}$$

$$\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}+Ec.\Pr\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]+Ec.\Pr(D^{-1}+M^2)\,u^2=0 \tag{8}$$

$$\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}=-\left(\frac{ScSo}{N}\right)\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right) \tag{9}$$

where $G=\dfrac{\beta g(T-T_0)\,d^3}{\nu^2}$ (Grashof number), $D^{-1}=\dfrac{d^2}{k_0}$ (Darcy number)

$P=\dfrac{\mu C_p}{k_1}$ (Prandtl number) , $E_c=\dfrac{\nu^2}{d^2(T-T_0)C_p}$ (Eckert number)

$$M^2 = \frac{\sigma\mu_e^2 H_{02} L^2}{\nu^2} \text{ (Hartmann number)}, \quad Sc = \frac{\nu}{D_1} \text{ (Schmidt number)}$$

$$N = \frac{\beta^* \Delta C}{\beta \Delta\theta} \text{ (Buoyancy Ratio)}, \quad S_0 = \frac{k_{11}\beta^*}{\beta} \text{ (Soret parameter)}$$

The corresponding boundary conditions are

$$\begin{aligned} &u = 0 \quad \text{on} \quad z = 0 \quad \text{and} \quad z = 1 \\ &\theta = 0, \quad C = 0 \quad \text{on} \quad z = 0 \\ &\theta = 1, \quad C = 1 \quad \text{on} \quad z = 1 \end{aligned} \tag{10}$$

Assuming Ec (<< 1) to be small, we take the asymptotic expansions of velocity, temperature and concentration as

$$\begin{aligned} u &= u_0 + Ecu_1 + O(Ec^2) \\ \theta &= \theta_0 + Ec\theta_1 + O(Ec^2) \\ C &= C_0 + EcC_1 + O(Ec^2) \end{aligned} \tag{11}$$

the zeroth order equations are

$$\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 u_0}{\partial z^2} - (D^{-1}(1 + \varepsilon\cos(\pi y)) + M^2)u_0 = -G(\theta_0 + NC_0) \tag{12}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = 0 \tag{13}$$

$$\frac{\partial^2 C_0}{\partial y^2} + \frac{\partial^2 C_0}{\partial z^2} = -\left(\frac{ScSo}{N}\right)\left(\frac{\partial^2 \theta_0}{\partial y^2} + \frac{\partial^2 \theta_0}{\partial z^2}\right) \tag{14}$$

The respective boundary conditions are

$$\begin{aligned} &u = 0 \;\; \text{on} \;\; z = 0 \quad \text{and} \quad z = 1 \\ &\theta_0 = 0, \quad C_0 = 0 \quad \text{on} \quad z = 0 \\ &\theta_0 = 1, \quad C_0 = 1 \quad \text{on} \quad z = 1 \end{aligned} \tag{15}$$

The equations to the first order are

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} - (D^{-1}(1 + \varepsilon\cos(\pi y)) - M^2)u_1 = -G(\theta_1 + NC_1) \tag{16}$$

$$\frac{\partial^2\theta_1}{\partial y^2}+\frac{\partial^2\theta_1}{\partial z^2}+P\left(\left(\frac{\partial u_0}{\partial y}\right)^2+\left(\frac{\partial u_0}{\partial z}\right)^2\right)+P(D^{-1}+M^2)u_0^2=0 \tag{17}$$

$$\frac{\partial^2 C_1}{\partial y^2}+\frac{\partial^2 C_1}{\partial z^2}=-\left(\frac{ScSo}{N}\right)\left(\frac{\partial^2\theta_1}{\partial y^2}+\frac{\partial^2\theta_1}{\partial z^2}\right) \tag{18}$$

with the boundary conditions

$$u_1=0,\quad \theta_1=0,\quad C_1=0 \quad \text{on} \quad z=0 \quad \text{and} \quad z=1 \tag{19}$$

Keeping in view the periodic variation of permeability we take

$$\begin{aligned}
u_0 &= u_{00}(z)+\varepsilon\cos(\pi y)\,u_{01}(z)\\
\theta_0 &= \theta_{00}(z)+\varepsilon\cos(\pi y)\,\theta_{01}(z)\\
C_0 &= C_{00}(z)+\varepsilon\cos(\pi y)\,C_{01}(z)\\
\theta_1 &= \theta_{10}(z)+\varepsilon\cos(\pi y)\,\theta_{11}(z)\\
C_1 &= C_{10}(z)+\varepsilon\cos(\pi y)\,C_{11}(z)
\end{aligned} \tag{20}$$

Substituting (20) in (12)-(19) and equating the coefficients of ε we get

$$\frac{\partial^2 u_{00}}{\partial z^2}-D^{-1}u_{00}=-G(\theta_{00}+NC_{00}) \tag{21}$$

$$\frac{\partial^2\theta_{00}}{\partial z^2}=0 \tag{22}$$

$$\frac{\partial^2 C_{00}}{\partial z^2}=-\left(\frac{ScSo}{N}\right)\frac{\partial^2\theta_{00}}{\partial z^2} \tag{23}$$

$$\frac{\partial^2 u_{10}}{\partial z^2}-(D^{-1}+M^2)u_{10}=-G(\theta_{10}+NC_{10}) \tag{24}$$

$$\frac{\partial^2\theta_{10}}{\partial z^2}=-P\left(\frac{\partial u_{00}}{\partial z}\right)^2-P(D^{-1}+M^2)u_{00}^2 \tag{25}$$

$$\frac{\partial^2 C_{10}}{\partial z^2}=-\left(\frac{ScSo}{N}\right)\frac{\partial^2\theta_{10}}{\partial z^2} \tag{26}$$

The respective boundary conditions are

$$u_{00} = 0, u_{10} = 0, \text{ on } z = 0 \quad \text{and} \quad z = 1$$
$$\theta_{00} = 0, \theta_{10} = 0, \quad C_{00} = 0, C_{10} = 0 \quad \text{on} \quad z = 0$$
$$\theta_{00} = 1, \theta_{10} = 0, \quad C_{00} = 1, C_{10} = 0 \quad \text{on} \quad z = 1 \tag{27}$$

and

$$\frac{\partial^2 u_{01}}{\partial z^2} - (\pi^2 + D^{-1} + M^2) u_{01} = (D^{-1} + M^2) u_{00} - G(\theta_{01} + NC_{01}) \tag{28}$$

$$\frac{\partial^2 C_{01}}{\partial z^2} - \pi^2 \theta_{01} = 0 \tag{29}$$

$$\frac{\partial^2 C_{01}}{\partial z^2} - \pi^2 C_{01} = -\left(\frac{ScSo}{N}\right)\left(\frac{\partial^2 \theta_{01}}{\partial z^2} - \pi^2 \theta_{01}\right) \tag{30}$$

$$\frac{\partial^2 u_{11}}{\partial z^2} - (\pi^2 + D^{-1} + M^2) u_{11} = (D^{-1} + M^2) u_{10} - G(\theta_{11} + NC_{11}) \tag{31}$$

$$\frac{\partial^2 \theta_{11}}{\partial z^2} - \pi^2 \theta_{11} = -2P\left(\frac{\partial u_{00}}{\partial z}\right)\left(\frac{\partial u_{01}}{\partial z}\right) - P(D^{-1} + M^2)(2u_{00} u_{01} + u_{00}^2) \tag{32}$$

$$\frac{\partial^2 C_{11}}{\partial z^2} - \pi^2 C_{11} = -\left(\frac{ScSo}{N}\right)\left(\frac{\partial^2 \theta_{11}}{\partial z^2} - \pi^2 \theta_{11}\right) \tag{33}$$

with boundary conditions

$$u_{01} = 0, u_{10} = 0, u_{11} = 0 \quad \text{on} \quad z = 0 \text{ and } z = 1$$
$$\theta_{01} = 0, \theta_{10} = 0, \theta_{11} = 0, \quad C_{01} = 0, C_{10} = 0, C_{11} = 0 \quad \text{on} \quad z = 0 \text{ and } 1 \tag{34}$$

3. Discussion

The aim of this analysis is to discuss the effect of variable permeability, soret and dissipative effect on the mixed convective heat and mass transfer of a viscous electrically conducting incompressible fluid through a vertical channel whose walls are maintained at constant temperature and concentrations. The velocity, temperature and concentration have been exhibited for different variations of the parameters M, S_c and C in Figs (1)-(6). Fig.1. shows that an increase in the Hartman number M

retards the magnitude of u in the entire fluid region, also the lesser molecular diffusivity higher the magnitude of u (Fig. 2). Fig. 3 shows that an enhancement in M leads to an increment in θ in the entire fluid region. From Fig. 4 it follows that lesser the molecular diffusivity greater the temperature in the fluid region. Fig. 5 shows that an increase in M retards the concentration, lesser the molecular diffusivity greater the concentration in the fluid region (Fig. 6).

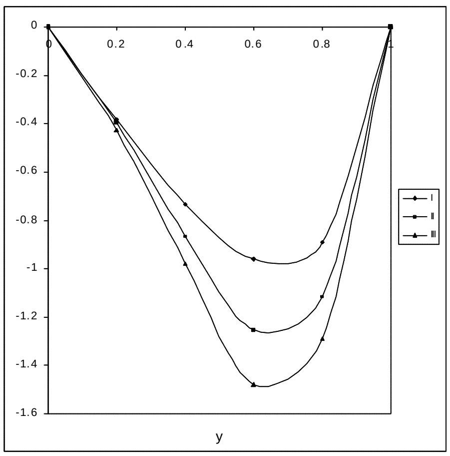

Figure 1: u with M

	I	II	III
M	2	4	6

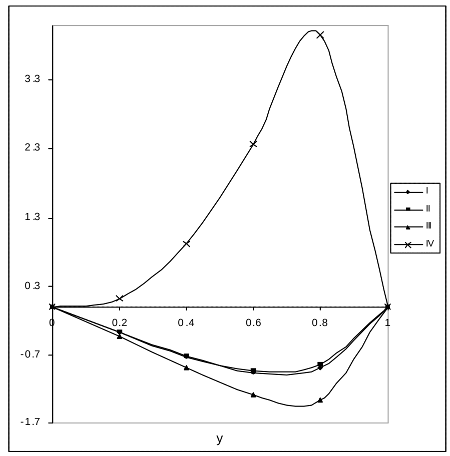

Figure 2: Variation of u with Sc

	I	II	III	IV
Sc	1.3	2.01	0.24	0.6

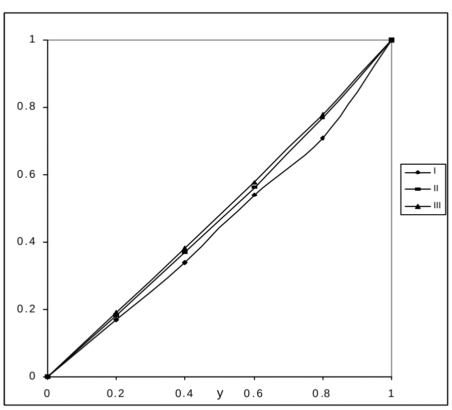

Figure 3: Variation of θ with M

	I	II	III
M	2	4	6

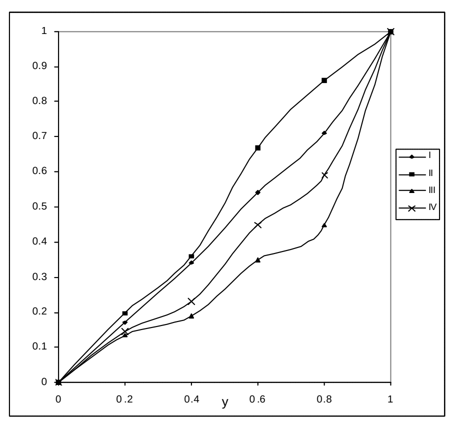

Figure 4: Variation of θ with Sc

	I	II	III	IV
Sc	1.3	2.01	0.24	0.6

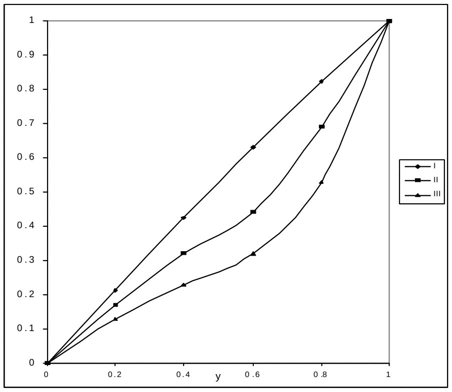

Figure 5: Variation of C with M

	I	II	III
M	2	4	6

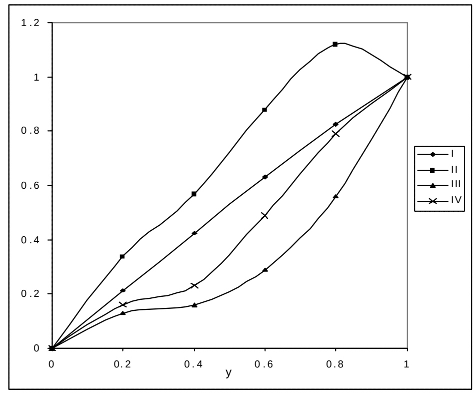

Figure 6: Variation of C with Sc

	I	II	III	IV
Sc	1.3	2.01	0.24	0.6

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