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Generation of New M-J Sets using Sine Function for N-iterates

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Abstract: The study of fractals and M-J sets is one of the most popular topic in the field of complex dynamics and computer graphics. The world of fractals is very interesting to explore and there is so much to study in the fractal geometry. So many work has been done to generate different types of M-J sets based on iterative methods. In this paper we studied the complex dynamics of sine function and applied it Noor iteration to create new Modified Mandelbrot set and Modified Julia set.The symmetrical analysis of new MJ sets is also discussed. Thispapers gives mathematical analysis of $z \to Sin(z^n + c)$ for the different values of n.

1. INTRODUCTION

Fractal Geometry is a new science. It was a because of the advances in mathematical representation of equations using computers. Many researchers have worked on the iterative process like Picard, Mann, Ishikawa and the Noor iterative methods [1-4] and gives generalization of Mandelbrot and Julia sets for various complex relational maps in last four decades. The transcendental functions has always been a topic to explore and study in numerical and complex analysis. There has been various articles available using the transcendental function to create new M-J sets [7]. Mostly sine function is use to analyse different types of Mandelbrot and Julia set [7].In 2001, M. A. Noor [1, 2] stated and analyzed three-step iterative scheme known as Noor iteration. In their paper they give solutions for the inequalities present in Hilbert spaces. They use the techniques of updating the solution and the auxiliary principle. It has been already showed [3, 4] that the iterative methods with three steps perform better than 2-step (Ishikawa) and 1-step (Mann) iterative methods. In this paper we generated the MMS and MJS by using sine function and also calculated the fixed points for the corresponding values. The rest of the paper is divided into 5 section. In section 2 the basic preliminaries used for the analysis of new MJ sets are listed. In section 3 we generates new Mandelbrot set and new Julia sets for the quadratic, cubic and higher degree polynomials. Section 4 consist of fixed point representation of the results obtained in previous section. Section 5 is the conclusion of the obtained result and section 6 consist of list of the references used in the preparation of this article.

2. DEFINITIONS AND PRELIMINARIES

Definition 2.1: Noor Iteration:

The general Mann iterative process [5] is written as

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, n = 0,\ 1,\ 2$$

Where $\{x_n\} \subset [0,\ 1]$.

The formulae for the Ishikawa iteration process [10] is given as

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n$$
$$y_n = (1-\beta_n)x_n + \beta_n T x_n, n = 0,\ 1,\ 2$$

Where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in [0, 1].

The Noor iteration is a three step iterative process [2-4] and given as

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n$$
$$y_n = (1-\beta_n)x_n + \beta_n T z_n$$
$$z_n = (1-\gamma_n)x_n + \gamma_n T x_n, n = 0,\ 1,\ 2$$

Where α, β, γ are sequences of positive number in [0, 1]

Definition 2.2: Julia Set: The Julia set is one of the most important fractal in the history of fractal geometry. The Julia set is generated from the equation $f(z) = z^2 + c$ when we fix value of z and iterate the equation as required. Generally a Julia set is defined as set of all c points which lies in the boundary of basin of attraction [7].

Definition 2.3: Mandelbrot Set: The Mandelbrot set [8] M is also generated from the families of equation $f_c(z) = z^2 + c$. It is defined as the collection of point cfor which the orbit of the point is bounded [8], i.e.

$$M = \{c \in C : \{f_c^{(n)}(0)\}_{n=0}^{\infty} \text{ is bounded}\}$$

Definition 2.6: Escape Criterion for Mandelbrot Sets and Julia Set

For general function $G(z) = z^2 + c$, n =1,2,3,4... . We have

$$z_1 = (1-s)z + sG(z)$$

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$$z_1 = (1-s)z_{n-1} + sG(z_{n-1})$$

Thus the G.C.E is given as $\max\{|c|, (2/s)^{\frac{1}{n+1}}, (2/s')^{\frac{1}{n+1}}\}$.

Corollary 1: If $|c| > (2/s)^{\frac{1}{n-1}}$ and $|c| > (2/s')^{\frac{1}{n-1}}$ then the Modified Superior orbit $MSO(G_c, 0, s_n, s'_n)$ escapes to infinity.

Corollary 2: If $|z_k| > \max\{|c|, (2/s)^{\frac{1}{k-1}}, (2/s')^{\frac{1}{k-1}}\}$ for $k \geq 0$, then $|z_{k+1}| > \gamma |z_k|$ and $|z_n| \to \infty$, as $n \to \infty$. This gives us foundation for computing the Modified Mandelbrot set and Modified Superior Julia Sets for the functions.

3. GENERATION OF MODIFIED MANDELBROT SETS AND MODIFIED JULIA SETS FOR SINE FUNCTION

3.1. Modified Mandelbrot Set for Noor iterates

We generated the Modified Mandelbrot sets using the software ultra-fractal. We present heresome Modified Mandelbrot sets for quadratic, cubic and Bi-quadratic function. The quadratic type of Mandelbrot and Julia set are obtained by using value of $n = 2$. The Cubic type of Mandelbrot and Julia set are obtained by using value of $n = 3$. The shape and symmetry of MMS and MJS are changing with each value of n. The relational map used to generate different Mandelbrot and Julia set is $z \to Sin(z^n + c)$.

3.2. Modified Julia set for Noor iterates

Here we created new Julia sets based on the N-iterates using sine function. The value of parameters are changed for every generation of Julia set. The result of the generated Julia sets is noticeable.

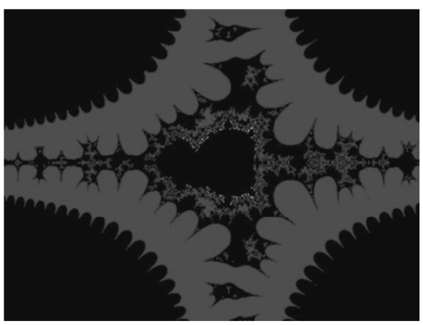

Figure 1: MMS for
$\alpha = 1, \beta = 1, \gamma = 1, n = 2$

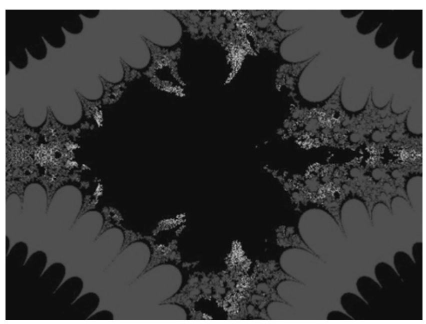

Figure 2: MMS for
$\alpha = 0.1, \beta = 0.5, \gamma = 0.7, n = 2$

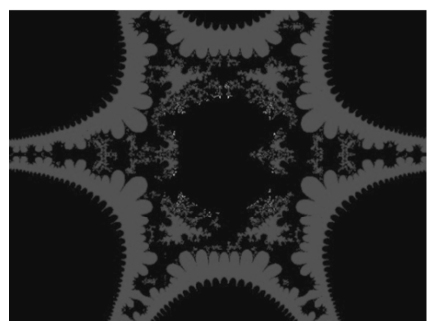

Figure 3: MMS for
$\alpha = 0.7, \beta = 0.5, \gamma = 0.7, n = 3$

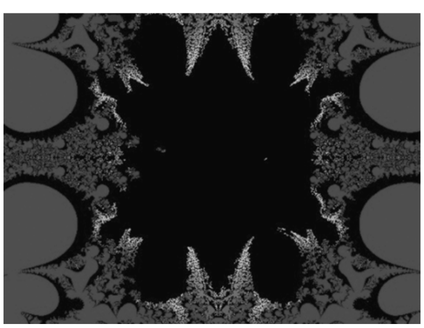

Figure 4: MMS for
$\alpha = 0.7, \beta = 0.5, \gamma = 0.7, n = 3$

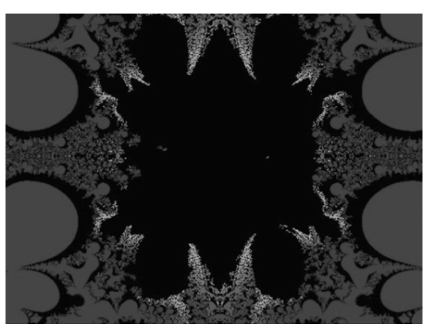

Figure 5: MMS for
$\alpha = 0.1, \beta = 0.5, \gamma = 0.3, n = 3$

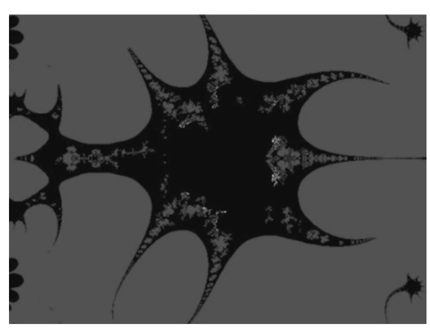

Figure 6: MMS for
$\alpha = 0.4, \beta = 0.9, \gamma = 0.3, n = 4$

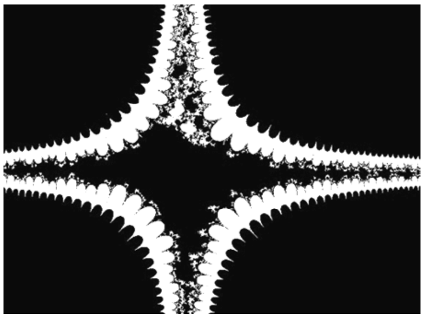

Figure 7: MJS for
$\alpha = 0.3, \beta = 0.7, \gamma = 0$, n = 2

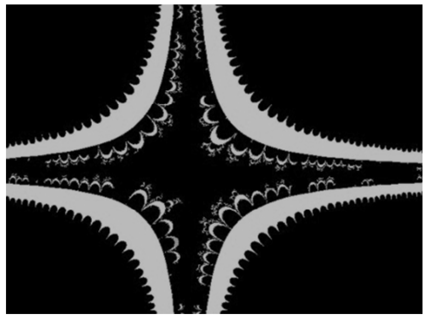

Figure 8: MJS for
$\alpha = 0.9, \beta = 0.3, \gamma = 0.1$, n = 2,

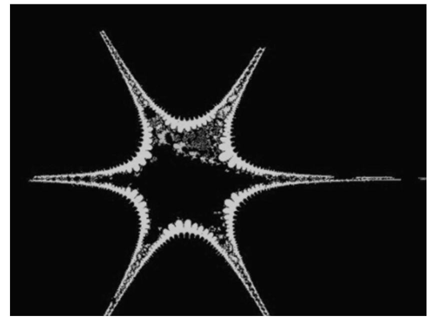

Figure 9: MJS for
$\alpha = 0.3, \beta = 0..7, \gamma = 0.01$, n = 3

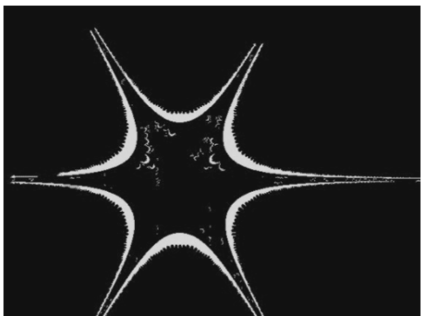

Figure 10: MJS for
$\alpha = 0.1, \beta = 0.3, \gamma = 0.4$, n = 3

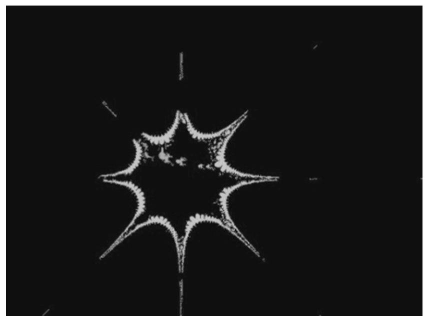

Figure 11: MJS for
$\alpha = 0.3, \beta = 0.7, \gamma = 0.1$, n = 4

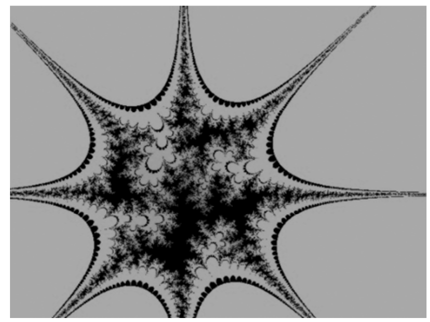

Figure 12: MJS for
$\alpha = 0.9, \beta = .001, \gamma = .001$, n = 4

4. FIXED POINT REPERESENTATION

Here we get the fixed point value after 15 iterations.

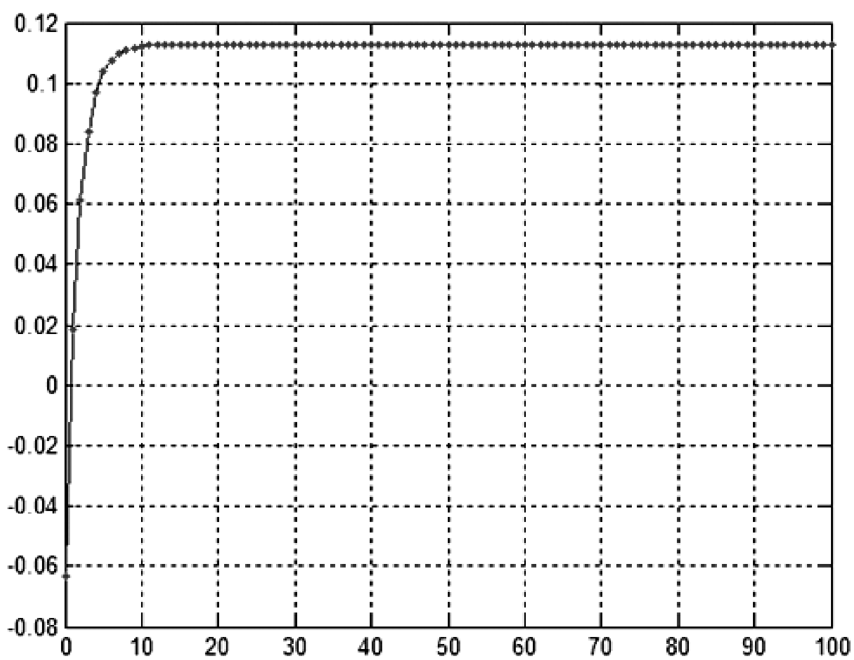

Figure 13: $\alpha = \beta = \gamma = 1$

Here we getthe fixed point value after 9 iterations.

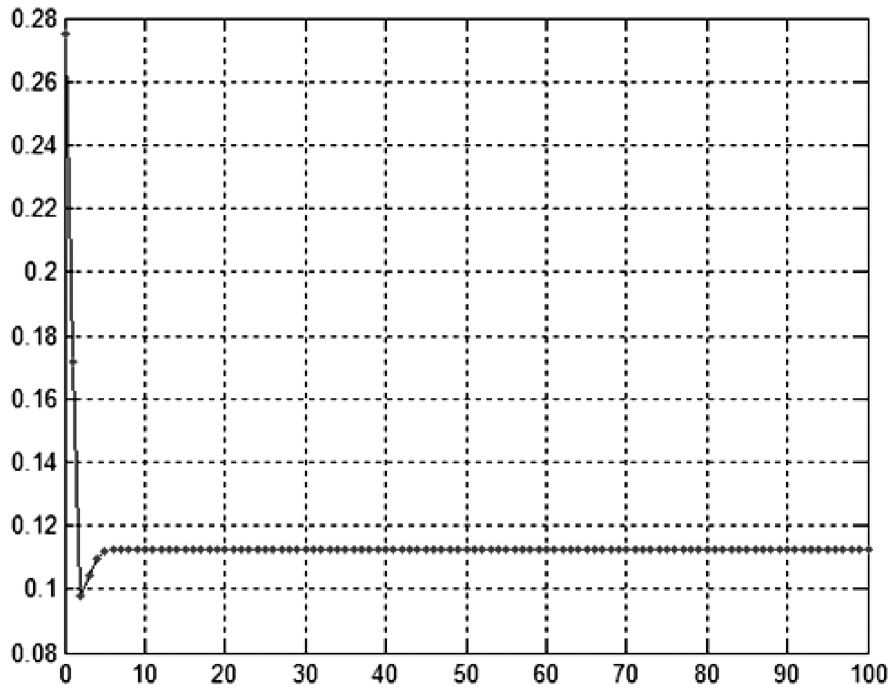

Figure 14: for $\alpha = 0.1, \beta = 0.5, \gamma$=0.7

Here we get the fixed point valued after 13 iterations.

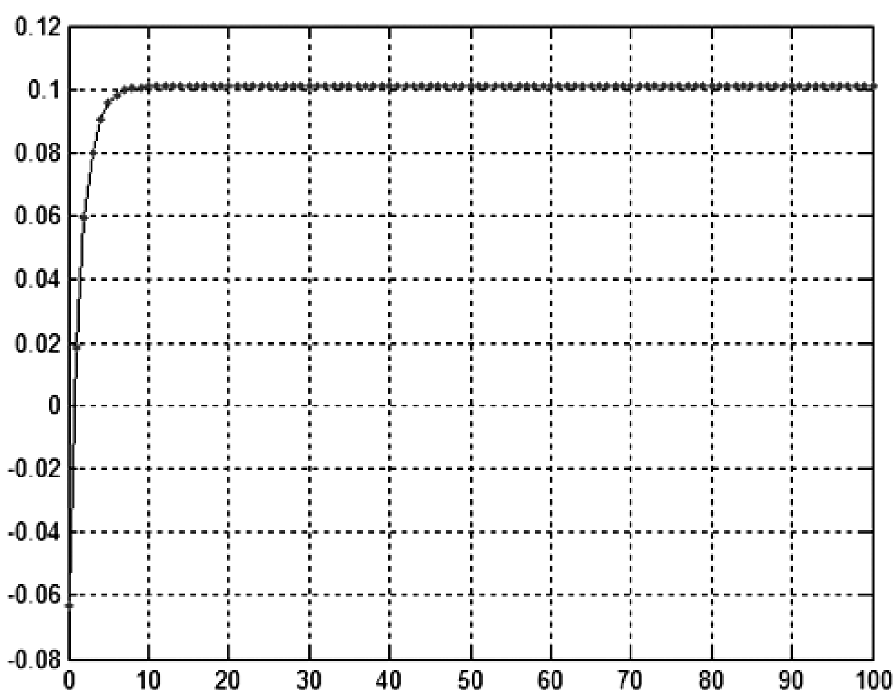

Figure 15: $\alpha = 0.3, \beta = 0..7, \gamma$=0.01

Here we get the fixed point value after the iteration process.

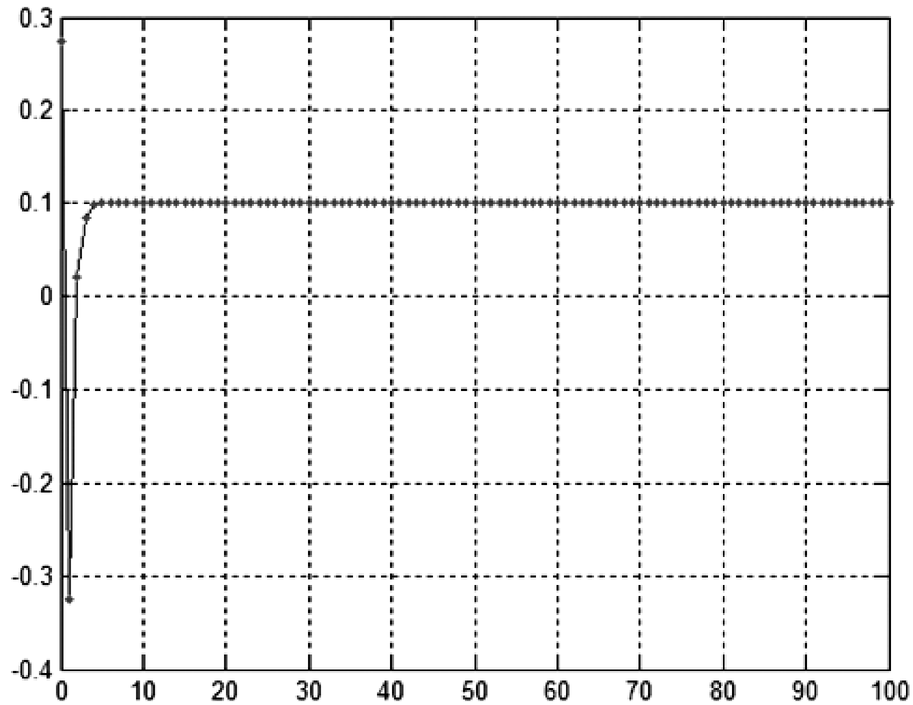

Figure 16: $\alpha = 0.1, \beta = 0.3, \gamma = 0.4$

Here we get the fixed point value after 13 iterations.

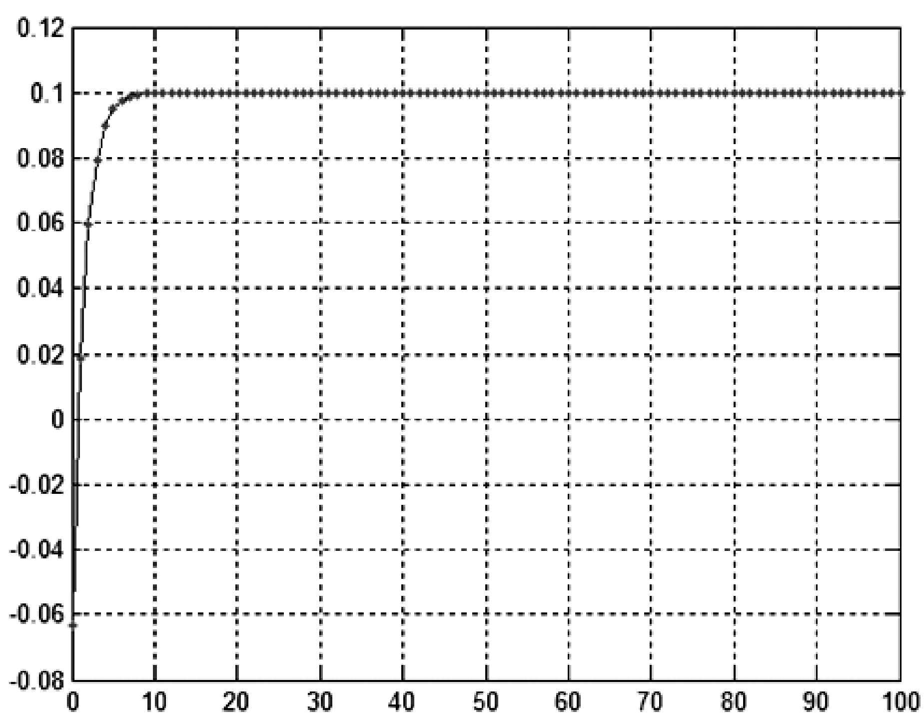

Figure 17: $\alpha = 0.3, \beta = 0.7, \gamma = 0.1$

Here we get the fixed point value after 4 iterations

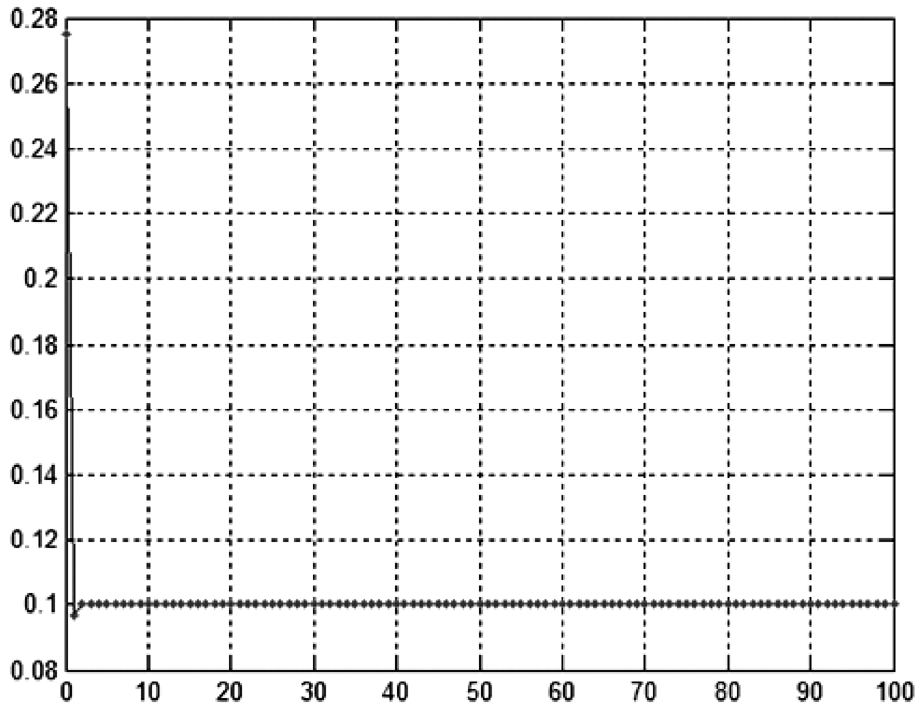

Figure 18: Orbit of F(z) for $\alpha = 0.9, \beta = .001, \gamma = .001$

5. CONCLUSION

This paper gives the generalization of sine complex map for the different complex families. From above observation we can conclude that

i. The Mandelbrot set for the quadratic polynomial is symmetrical along x-axis.

ii. For cubic polynomial it shows symmetry along both axis.

iii. For biquadratic polynomials its show symmetry only along x-axis

Using the observation of Julia sets for the sine function we can conclude that the Julia set have 4 petals when n = 2 and 6 petals when n = 3. So generally we can state that if we have a polynomial of degree n it will have 2n petals in resultant Julia set.

The fixed point representation of polynomials show that

i. The process take more iterations to converge if the value of parameters is near to unity.

ii. The process take less iteration to converge if the values of the parameters is near to zero.

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