International Journal of Economic Research

ISSN : 0972-9380

available at http: www.serialsjournal.com

Volume 14 • Number 9 • 2017

An Inflationary Inventory Model for Deteriorating items under Two Storage Systems

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Abstract: In the present market scenario the demand of certain items does not remain constant with time and may increase/decrease or remain constant with time. In the most of the deteriorating items inventory model, demand rate has considered as a constant function, linear function or exponential function of time. Thus in this paper, we develop an inventory model with the concept of two storage system incorporating quadratic demand under inflation. The objective of this study is to derive retailer's optimal replenishment policy that minimizes the present worth of total relevant inventory cost per unit of time. In addition to this single warehouse system is also developed which may be considered as a particular case of the two storage system and the results have been compared with the help of numerical example. Sensitivity analysis is carried for the main model with the variation in the value of one parameter and keeping rest unchanged.

Key words: constant deterioration, quadratic demand, two storage system

1. INTRODUCTION

Most of the classical inventory models, it is assumed that the utility of inventory remains constant during their storage period. But in general, certain product, during the storage period, such as food, vegetables, fruits, medicine, blood, fish, alcohol, gasoline and radioactive chemicals deteriorates. The problem of deteriorating inventory has received considerable attention in recent years and to control and maintain the inventory of the deteriorating items to satisfy the demand of customers or retailers, order is important. Above listed researchers, have taken care of deteriorating items in their models and developed the models accordingly.

In addition with deterioration of inventory, in the present market scenario the inflation of cost is major problem affecting the demand so retailers during the period in which distributor offers discount, purchased items in bulk to get maximum profit and minimize the total inventory cost. During these time period, retailers need extra space to store the products purchased in bulk. Limited storage is also a major practical problem for real situation due to the lack of large storage space at the important market places, forcing retailers to own a small ware-house at important market places. However, to reduce the problem of storage retailers prefer to rent a house for a limited period. In case deteriorating items, specially equipped storage facility is required to reduce the amount of deterioration. To handle this situation the requirement of another storage space, providing the requite facilities become necessity.

In literature, own ware-house is abbreviated as OW and that of rented ware-house is as RW and generally it is assumed that the rented ware house provides better storage facilities as compared to own ware-house and due to this the rate of deterioration is smaller than OW which results more holding cost at RW therefore retailer prefer to vacant RW by supplying demanded items earlier to the customers and then from OW. Due to arising problem of storage facility the concept of two ware house inventory modelling come into existence. For two ware-houses, time dependent demand was considered by some authors, such as [13-18].

In traditional models, many researchers have considered the demand rate as constant, linear time dependent, stock dependent or exponentially increasing with time and the same trend of considering these type of demand is still continues but it is not always true that the demand occurs in same pattern i.e. either constant or increasing with time. The assumption of constant demand rate is usually valid at the mature stage of the life cycle of a product. Several researchers e.g.[1-8] etc. have developed inventory models for items stored in two ware-houses under a variety of modelling assumptions. In these all models the demand rate is considered to be constant over a time. However, in practice one would accept demand to vary with time. Donaldson[9] was the first to consider inventory model with time dependent demand and thereafter many researchers such as Goswami and Chaudhuri[10],Bhunia and Maiti[11,12] Banerjee and Agrawal[13] etc. were considered the time dependent demands for two-warehouse inventory systems.

Most of the above mentioned studies consider time varying demand rate either increases or decreases with time.Wu[14]developed an inventory model with ramp type demand rate, weibull deterioration distribution and partially backlogged shortages. Giri, Jalan and Chaudhary[15] used an exponential ramp type function to represent demand. Swati Agrawal & Snigdha Banarjee[16,17] developed a two ware-house inventory model with ramp type demand and partially backlogged shortages in which demand is general ramp type function of time is considered. In the recent years, many researches such as Hui-Ling Yang[18], Hui-Ling Yang and Chun-Tao Chang[19], H. L. Yang[20] developed inflationary inventory model with various combinations. R. Kumar and A. K. Vats[21] developed a deterministic inventory mode for single ware house incorporating demand rate as quadratic function of time under inflation. In the most of research papers, researchers have considered that deterioration rate in the both ware-houses are either constant or time dependent but it is not always true.

Motivated by above papers, we developed a deterministic deteriorating inventory model considering demand rate as quadratic function of time under inflation for two storage system in which shortages is not taken into account but rate of deterioration is taken different in two ware-houses i.e. in one ware-house as constant and in other as time dependent. The model is solved numerically by minimizing the total inventory cost for the total cycle length. The study shows that the optimal solution not only exists but is unique.

2. ASSUMPTIONS AND NOTATION

The mathematical model of the two-storage inventory problem is based on the following assumption and notations.

2.1. Assumptions

- Replenishment rate is infinite and instantaneous.
- Storage capacity of RW is considered to be unlimited.
- The lead time is zero or negligible and initial inventory level is zero.
- Shortages are not allowed.
- In OW deterioration rate is time dependent and constant in RW.RW offer better facility therefore $\theta(t) > \alpha$
- The holding cost is constant and higher in RW than OW and $(h_1 - h_2) > c\,(\theta(t) - \alpha)$
- The deteriorated units cannot be repaired or replaced during the period under review.
- Deterioration occurs as soon as items are received into inventory.
- Inventory system consider a single item and the demand rate f(t) is quadratic time dependent i. e if a is fixed fraction and b, c are that fraction of demand which vary with time such that a, b, and c are non-zero positive integers the $f(t) = a + bt + ct^2$

2.2. Notation

The following notation is used throughout the paper:

Demand (units/unit time) which quadratic time dependent given as

$$f(t) = \begin{cases} a + bt + ct^2 & if\ \ t > 0 \\ a & if\ \ t = 0 \end{cases}$$

W Finite capacity of OW.

α Constant deterioration rate in RW such that $0<\alpha<1$.

$\theta(t)$ Variable deterioration rate in OW given as $\theta(t) = \beta\, t$ where $0 < \beta < 1$.

C_o Ordering cost per order $\theta(t) > \alpha$

h_1 Holding cost per unit per unit time in OW

h_2 Holding cost per unit per unit time in RW such that$h_1 > h_2$

p Purchase cost per unit

M_{max} The maximum order quantity for a cycle length.

I_r Maximum inventory level at any time for r=1, 2, 3 etc.

T Length of the cycle.

r Inflation rate.

TC Transportation cost depending upon the distance from RW to retail shop given by $t_c\, q$ where q is distance t_c is transportation cost per unit per item.

R^1 Initial inventory level for S_1 system

R^2 Initial inventory level for S_2 system

Π^1 Present worth total inventory cost per unit of time for S_1-System.

Π^2 Present worth total inventory cost per unit of time for S_2-System.

3. MATHEMATICAL DEVELOPMENT OF MODEL

3.1. Two storage system (S_2 – system)

On the arrival of inventory in the beginning, a fixed amount of W units are kept at OW according to its capacity and the rest amount of inventory R^2 are kept into ranted warehouse. The evolution of stock level in the system is depicted in Figure 1. First the amount of inventory kept in RW is consumed to minimize the rent of RW and then demand of customers is fulfilled by supplying inventory from OW. In the RW, the level of inventory depleted due to combined effect of demand and constant deterioration and differential equation governing this situation is given as

$$\frac{dI_1(t)}{dt} = -f(t) - \alpha I_1(t)\ ; \qquad 0 \le t \le t_w \tag{1.1}$$

With boundary condition $I_1(t) = 0$. The solution of equation (1.1) is

$$I_1(t) = \left(\left\{a(t_w - t) + \frac{a\alpha}{2}(t_w^2 - t^2) + \frac{b}{2}(t_w^2 - t^2) + \frac{b\alpha}{3}(t_w^3 - t^3) + \frac{c}{3}(t_w^3 - t^3) + \frac{c\alpha}{4}(t_w^4 - t^4)\right\}\right) e^{-\alpha t} \tag{1.2}$$

When inventory is vanishes at RW, the demand of customers is fulfilled by supplying inventory from the OW and W units of OW in the interval $(0\ \ t_w)$ reduces due to varying deterioration rate only and during the interval $(t_w\ \ T)$, the inventory reduces due to combined effect of both variable deterioration and demand. The situations are governed by the following differential equations

$$\frac{dI_2(t)}{dt} = -\theta(t) I_2(t) \qquad 0 \le t \le t_w \tag{1.3}$$

$$\frac{dI_3(t)}{dt} = -f(t) - \theta(t) I_3(t) \qquad t_w \le t \le T \tag{1.4}$$

With boundary conditions $I_2(0) = W$ and $I_3(T) = 0$. The solution of (1.3) & (1.4) are resp.

$$I_2(0) = W e^{-\frac{\beta t^2}{2}} \tag{1.5}$$

$$I_3(t) = \left(\left\{a(T - t) + \frac{a\beta}{6}(T^3 - t^3) + \frac{b}{2}(T^2 - t^2) + \frac{b\beta}{8}(T^4 - t^4) + \frac{c}{3}(T^3 - t^3)\frac{c\beta}{10}(T^4 - t^4)\right\}\right) e^{-\frac{\beta t^2}{2}} \tag{1.6}$$

Since, initially inventory level in RW is $I_1(0) = R^2 - W$, therefore we obtain

$$R^2 = W + \left(\left\{a(t_w) + \frac{a\alpha}{2}\left(t_w^2\right) + \frac{b}{2}\left(t_w^2\right) + \frac{b\alpha}{3}\left(t_w^3\right) + \frac{c}{3}\left(t_w^3\right) + \frac{c\alpha}{4}\left(t_w^4\right)\right\}\right) > W \tag{1.7}$$

Remark-1: If $t_w = 0$ the above inequality does not satisfied and the inventory level at RW will be zero. This situation arises when the initial inventory level is less than or equal to W and corresponds to Case-2 of the - system, discussed later in section (4.0).

Continuity in OW at $t = t_w$ gives $I_2(t_w) = I_3(t_w)$ therefore

$$W = \left(\left\{a(T - t_w) + \frac{a\beta}{6}(T^3 - t_w^3) + \frac{b}{2}(T^2 - t_w^2) + \frac{b\beta}{8}(T^4 - t_w^4) + \frac{c}{3}(T^3 - t_w^3)\frac{c\beta}{10}(T^4 - t_w^3)\right\}\right) \tag{1.8}$$

Thus the total present worth inventory cost during the cycle length, consist of the following costs elements

- Ordering cost C_o
- Inventory holding cost in RW
- Inventory holding cost in OW
- Purchase cost
- Transportation cost

Inventory holding cost in RW is $h_1\left(\int_0^{t_w} e^{-rt} I_1(t)dt\right)$

Inventory holding cost in OW is $h_2\left(\int_0^{t_w} e^{-rt} I_2(t)dt + \int_{t_w}^{T} e^{-rt} I_3(t)dt\right)$

Purchase cost is $p\,R^2$

Transportation cost is $q\,t_c$

Hence the total cost per unit of time during cycle length is given by

$$\Pi^2(t_w, T) = \frac{1}{T}\left[C_o e^{-rt} + h_1\left(\int_0^{t_w} e^{-rt} I_1(t)dt\right) + h_2\left(\int_0^{t_w} e^{-rt} I_2(t)dt + \int_{t_w}^{T} e^{-rt} I_3(t)dt\right) + pR^2 + q\,t_c\right] \tag{1.9}$$

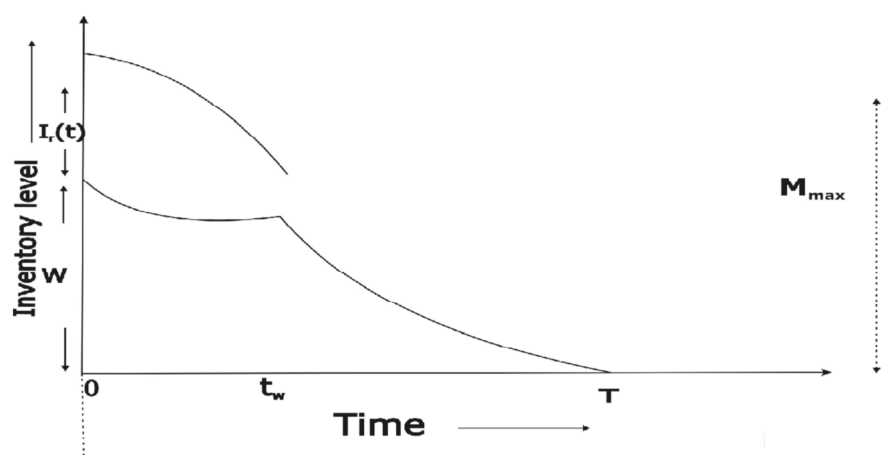

Figure 1

3.2. Optimality condition for system

The optimal problem can be formulated as

Minimize: $\Pi^2 (t_w, T)$

Subject to: $(t_w > 0, T > 0)$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial \Pi^2 (t_w, T)}{\partial t_w} = 0; \quad \frac{\partial \Pi^2 (t_w, T)}{\partial T} = 0;$$

$$\text{Provided } \left(\frac{\partial^2 \Pi^2 (t_w, T)}{\partial t_w^2}\right)\left(\frac{\partial^2 \Pi (t_w, T)}{\partial T^2}\right) - \frac{\partial^2 \Pi^2 (t_w, T)}{\partial t_w \partial T} > 0 \ \ att = t_w^* \ and \ T = T^* \tag{1.10}$$

Solving equation (1.10) respectively for t_w, and T, we can obtain t_w^* and with these values we can find the total minimum inventory cost from equation (1.10) for the model.

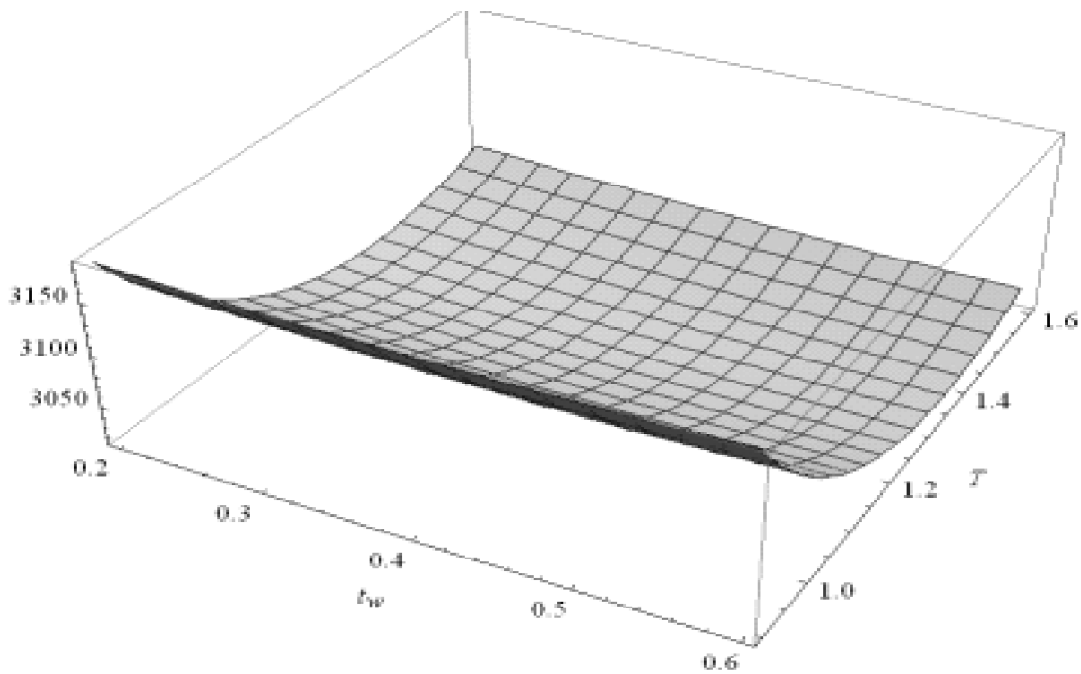

Figure 2: Representing convexity of the two ware-house model

4. SINGLE WARE-HOUSE (SYSTEM)

In this system, we consider the two cases one with unlimited capacity of a ware-house i.e. a rented ware-house (RW) and other one with limited capacity i.e. own ware-house (OW) separately and shall briefly present the analysis. As stated in remark-1 this may be treated as a particular case of S_2 – system by relaxing the condition $t_w > 0$.

4.1. RW Case

At, the amount of inventory ordered enters into the system. R^2 Units are stored in RW.

In this case inventory level depleted due to combined effect of both demand and constant deterioration over time period (0 T). The situation is presented by Figure 3.

The total cost function per unit of time is found to be

$$\Pi^{1R}(t_w, T) = \frac{1}{T}\left[C_o e^{-rt} + h_2\left(\int_0^T e^{-rt} I_3(t)dt\right) + pW + qt_c\right] \tag{2.1}$$

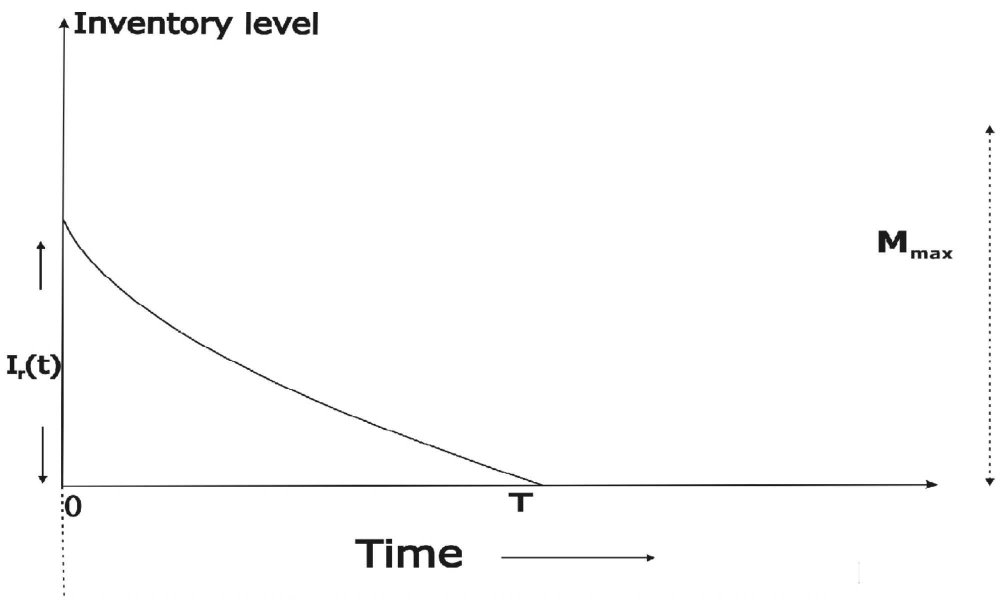

Figure 3: Representing Inventory-Time graph

4.2. OW Case

At, W units of inventory ordered enters into the system.

In this case inventory level depleted due to combined effect of both demand and constant deterioration over time period (0 T). The situation is presented by Figure 4.

The total cost function per unit of time is found to be

$$\Pi^{1W}(t_w, T) = \frac{1}{T}\left[C_o e^{-rt} + h_1\left(\int_0^T e^{-rt} I_1(t)dt\right) + p\,\mathrm{R}^1 + qt_c\right] \tag{2.2}$$

Optimality condition for System

RW Case

The optimal problem can be formulated as

Minimize: $\Pi^{1R}(T)$

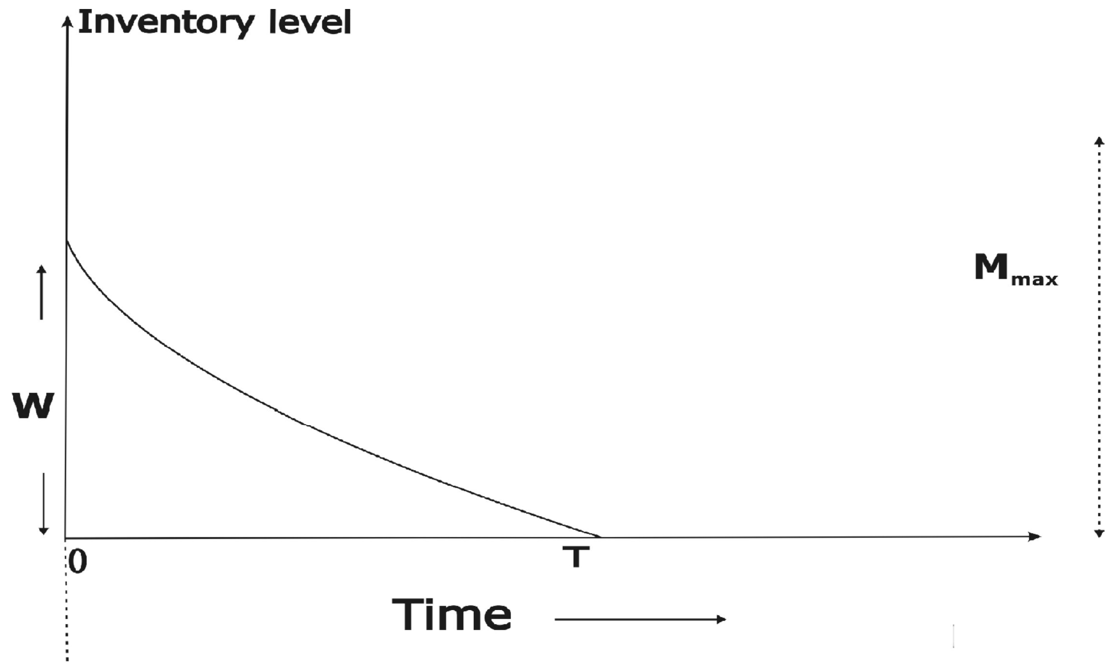

Figure 4: Representing Inventory-Time graph

Subject to: $(T > 0)$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial \Pi^{1R}(T)}{\partial T} = 0;$$

$$\text{Provided } \left[\frac{\partial^2 \Pi^{1R}(T)}{\partial T^2}\right] > 0 \text{ at } \quad t = T^*$$

Solving equation (1.1) for , we can obtain and with these values we can find the total minimum inventory cost from equation (2.1) for this model.

OW Case

The optimal problem can be formulated as

Minimize: $\Pi^{1W}(T)$

Subject to: $(T > 0)$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial \Pi^{1W}(T)}{\partial T} = 0;$$

$$\text{Provided } \left[\frac{\partial^2 \Pi^{1W}(T)}{\partial T^2}\right] > 0 \text{ at } \quad t = T^*$$

Solving equation (1.2) for T, we can obtain T^* and with these values we can find the total minimum inventory cost from equation (2.2) for this model.

Table 1

S_1 – System

Case	t^*_W	T	*Total inventory Cost*
RW	————	1.0628	2681.54
OW	————	1.2062	2536.07

S_2 – System

t^*_W	T	*Total inventory Cost*
0.5651	1.3824	3014.46

Table 2

S_1 – System

Case	t^*_W	T	*Total inventory Cost*
RW	————	4.1337	2681.54
OW	————	4.4896	1192.72

S_2 – System

t^*_W	T	*Total inventory Cost*
5.69469	16.5937	3837.91

Table 3

S_1 – System

Case	t^*_W	T	*Total inventory Cost*
RW	————	11.7464	1866.91
OW	————	5.23511	1033.35

S_2 – System

t^*_W	T	*Total inventory Cost*
0.658339	8.02558	1268.48

Table 4

S_1 – System

Case	t^*_W	T	*Total inventory Cost*
RW	————	1.4528	2997.13
OW	————	1.12107	2504.44

S_2 – System

t^*_W	T	*Total inventory Cost*
0.4849	1.6341	2874.11

Table 5
Sensitivity analysis for the case-1 of two ware-house model with change in the value of one parameter keeping rest unchanged

Initial value of parameter	*change value of parameter*	t^*_W	T	*% change in cycle length*	*Change in Total cost*	*% change in total cost*
30	15	0.6980	1.6670	65.2862	1947.72	-35.39
A	45	0.5757	1.6006	58.7055	3802.16	26.13
6	3	0.6518	1.8320	81.6596	2713.60	-9.98
B	9	0.5831	1.4818	46.9180	3020.32	0.19

contd. table 5

2	1	0.6409	1.7832	76.8138	2814.79	-6.62
C	3	0.5951	1.5275	51.4522	2927.26	-2.89
0.04	0.02	0.5763	1.6321	61.8282	2880.12	-4.46
R	0.06	0.3561	1.6335	61.9700	2870.02	-4.77
4	2	1.0157	1.6314	61.7538	2863.84	-4.99
h_1	6	0.4912	1.6314	61.7598	2876.95	-4.41
3	1.5	0.4687	1.6862	67.1918	2847.45	-5.54
h_2	4.5	0.7268	1.5865	57.3058	2899.29	-3.82
60	30	0.6904	2.0241	100.7142	1745.24	-42.11
P	90	0.5639	1.7862	40.2321	3959.31	31.35
5	2.5	0.5753	1.4525	44.0115	2713.50	-4.77
t_c	7.5	0.6458	1.7862	77.1164	3021.61	0.24
500	500	0.5339	1.4542	44.1762	2717.31	-9.86
C_O	750	0.6848	1.7843	76.9269	3017.88	0.11
0.002	0.001	0.6199	1.6331	61.9224	2875.22	-4.62
α	0.003	0.6103	1.6330	61.9155	2875.65	-4.61
0.003	0.0015	0.6126	1.6370	62.3153	2873.92	-4.66
β	0.0045	0.6175	1.6291	61.5276	2876.95	-4.56
100	50	0.5753	1.4249	41.2747	2713.5	-9.99
Q	150	0.6458	1.7862	77.1162	3021.61	-0.24

From Table 5, the following observation has been obtained:

1) The cycle length of the model is directly proportional to the all parameters and increases when the value of parameters increases and highly sensitive to all but too much sensitive to the price of inventory.

2) The average inventory cost increases as the parameters a, p, t_c, C_O increases and is highly sensitive to the a,p and slightly sensitive to t_c, C_O.

3) The average inventory cost decreases as the parameters c, r, h_1, h_2, α, β, q increases and is moderately sensitive to the r, h_1, h_2, α, β and slightly sensitive to c and q.

4) The convexity of inventory-time graph depicted in Figure-2 shows that there exist a unique point where inventory cost total is minimum.

5. NUMERICAL EXAMPLES

To analyse the model, we consider four different parameter sets corresponding to the situations where (i) Δ_1–system is optimal and (ii) Δ_2 system is optimal.

Example-1: Consider the following set of values of parameters: $a = 30$. $b = 6$, $c = 2$, $C_o = 500$, $r = 0.04$, $h_1 = 4.0$, $h_2 = 3.0$, $p = 60$, $t_c = 5.0$, $\alpha = 0.002$, $\beta = 0.003$, $q = 100$. The optimal results obtained from the Δ_1–system has shown in Table-1. For these parameters of value the OW has optimal solution.

Example-2: Next we take the know value of W. Let us take $W = 50$, keeping the other set of values of parameters same as in example – 1. For the Δ_1–System the optimal values obtained are shown in Table 2. In this case we found that OW Case of S_1–system is very low expensive as compared to other systems.

Example-3: Next we take the know value of W and increase the inflation rate much higher as compared to example-1. Let us take $r = 0.15$, keeping the other set of values of parameters same as in example –1. For the S_1 –System and S_2–System, the optimal values obtained are shown in Table 3. In this case amount of inventory is > W and is much higher than W. We observed from the table that S_2– System which is less expensive than RW case of S_1–System.

Example-4: Next in example-3 increasing inflation rate and holding cost much higher than example-1 and taking $r = 0.10$, $h_1 = 8.0$, the optimal values obtained are shown in Table 4. From table, we found that S_2–System is more useful and less expensive than S_1–System when inflation rate and holding cost of RW increases and large amount of inventory is to be stored.

6. CONCLUSIONS

In this paper, deterministic inventory model under inflation and varying rate of deterioration is presented to determine the optimal replacement cycle for two storage system. The model assumes that the capacity of retailer's warehouse is limited. The optimization technique is used to derive the optimum replenishment policy i.e. to minimize the total relevant cost of the inventory system. Different numerical example is presented to illustrate and validate the model. When there is single ware house is assumed in the inventory system then the total relevant cost per unit time of the system are higher than the two warehouse model when more amount inventory has to be purchased and extra space is required to store. This model is is most useful for the instant deteriorating items under varying deterioration rate. This model is very practical in nature since total inventory cost per unit time is indirectly proportional to the inflation rate. Further this paper can be enriched by incorporating other types of probabilistic demand and another extension of this model can be done for a bulk release pattern. In practice now days displaying the stock and advertising also have effect on the demand rate and must be taken into consideration while extending the model.

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