IJAMAA: *January-December 2015, Volume 10, Nos. 1-2, pp. 1-10*

RADIATION EFFECTS ON MHD FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE

P. CHANDRAKALA AND P. NARAYANA BHASKAR

ABSTRACT: MHD flow past an exponentially accelerated infinite vertical plate in the presence of thermal radiation with variable temperature is studied here. The fluid considered is a gray, absorbing-emitting radiation but a non-scattering medium. The dimensionless governing equations are solved using Laplace-transform technique. The velocity and temperature are studied for different physical parameters like magnetic field parameter, radiation parameter, thermal Grashof number, time and an accelerating parameter a. It is observed that for an increase in radiation parameter *N*, there is a fall in the velocity or temperature.

Keywords: Exponential, Accelerated vertical plate, Magnetic field, Radiation.

1. NOMENCLATURE

A – constant

a' – accelerating parameter

a – dimensionless accelerating parameter

B_0 – magnetic field strength

C_p – specific heat at constant pressure

g – acceleration due to gravity

Gr – thermal Grashof number

k – thermal conductivity of the fluid

k^* – mean absorption coefficient

M – magnetic field parameter.

N – radiation constant

Pr – Prandtl number

p – pressure

q_r – radiative heat flux in the y-direction

T – temperature of the fluid near the plate

T_w – temperature of the plate

T_∞ – temperature of the fluid far away from the plate
t' – time
t – dimensionless time
u' – velocity of the fluid in the x-direction
u_0 – velocity of the plate
u – dimensionless velocity
y – coordinate axis normal to the plate
y' – dimensionless coordinate axis normal to the plate

Greek symbols

α – thermal diffusivity
β – volumetric coefficient of thermal expansion
μ – coefficient of viscosity
ν – kinematic viscosity
ρ – density
σ – stefan-Boltzmann constant
τ – dimensionless skin-friction
θ – dimensionless temperature
η – similarity parameter
erfc – complementary error function

2. INTRODUCTION

Magneto convection plays an important in various industrial applications. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials. It is of importance in connection with many engineering problems, such as sustained plasma confinement for controlled thermonuclear fusion and electromagnetic casting of metals.

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry.

Sakiadis [5, 6] studied the growth of the two dimensional velocity boundary layer over a continuously moving horizontal plate emerging from a wide slot at uniform velocity. Soundalgekar [8] was the first to present an exact solution for the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the usual Laplace transform technique and the effects of heating or cooling of the plate on the flow field were discussed through *Gr*. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [7]. The Skin-friction for accelerated vertical plate has been studied analytically by Hossian and Shayo [3]. Raptis and Perdikis [4] have studied the effects of thermal radiation and free convection flow past a moving infinite vertical plate. Radiation effects on MHD flow past an impulsively started infinite vertical plate with variable temperature were studied by Chandrakala and Antony Raj [2]. The dimensionless governing equations were solved using Laplace-transform technique.

The object of the present paper is to study the thermal radiation effects on MHD flow past an exponentially accelerated infinite vertical plate with variable temperature. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

3. Mathematical Formulation

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature in the presence of thermal radiation and magnetic field is considered. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. It is also assumed that the radiation heat flux in the x-direction is negligible as compared to that in the y-direction. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the plate temperature is made to raise linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation is assumed to be negligible. Then under usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

where the Rosseland approximation (Brewster [1]) is used, which leads to

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y'}. \tag{3}$$

With the following initial and boundary conditions

$$
\begin{array}{llllll}
 & u' = 0, & T = T_\infty & \text{for all} & y', t' \le 0 & \\
t' > 0: & u' = u_0 \exp(at), & T = T_\infty + (T_w - T_\infty) A t' & \text{at} & y' = 0 & (4) \\
 & u' \to 0, & T \to T_\infty & \text{as} & y' \to \infty &
\end{array}
$$

Where $A = \frac{u_0^2}{\nu}$.

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 . \tag{5}$$

Using (3) and (5), Eq. (2) gives

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} . \tag{6}$$

On introducing the following non-dimensional quantities

$$
\begin{gathered}
u = \frac{u'}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y' u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad a = \frac{a'\nu}{u_0^2}, \\
Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \quad \Pr = \frac{\mu C_p}{k}, \quad N = \frac{k * k}{4\sigma B_0^2}, \quad M = \frac{\sigma B_0^2 \nu}{Q u_0^2}
\end{gathered} \tag{7}
$$

in equations (1) to (6), leads to

$$\frac{\partial u}{\partial t} = Gr\,\theta + \frac{\partial^2 u}{\partial y^2} - Mu \tag{8}$$

$$3N \Pr \frac{\partial \theta}{\partial t} = (3N + 4) \frac{\partial^2 \theta}{\partial y^2} . \tag{9}$$

The initial and boundary conditions in a non-dimensional form are

$$
\begin{array}{llllll}
 & u = 0, & \theta = 0 & \text{for all} & y, t \le 0 & \\
t > 0: & u = \exp(at), & \theta = t & \text{at} & y = 0 & (10) \\
 & u \to 0, & \theta \to 0 & \text{for all} & y \to \infty &
\end{array}
$$

The dimensionless governing equations (8) and (9), subject to the boundary conditions (10), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1+2\eta^2 b)\, erfc\,(\eta\sqrt{b}) - \frac{2}{\sqrt{\pi}}\,\eta\sqrt{b}\, e^{-\eta^2 b}\right] \tag{11}$$

$$\begin{aligned}
u = {} & \frac{e^{at}}{2}\,[\bar{e}^{2\eta\sqrt{(a+M)\,t}}\; erfc\,(\eta - \sqrt{(a+M)\,t}) + e^{2\eta\sqrt{(a+M)\,t}}\; erfc\,(\eta + \sqrt{(a+M)\,t})] \\
& + \frac{Gr\,(1+ct)}{2\,(1-b)\,c^2}\,[\bar{e}^{2\eta\sqrt{Mt}}\; erfc\,(\eta - \sqrt{Mt}) + e^{2\eta\sqrt{Mt}}\; erfc\,(\eta + \sqrt{Mt})] \\
& - \frac{Gr\,\eta\sqrt{t}}{2\,(1-b)\,c\sqrt{M}}\,[\bar{e}^{2\eta\sqrt{Mt}}\; erfc\,(\eta - \sqrt{Mt}) - e^{2\eta\sqrt{Mt}}\; erfc\,(\eta + \sqrt{Mt})] \\
& - \frac{Gr\,e^{ct}}{2\,(1-b)\,c^2}\,[\bar{e}^{2\eta\sqrt{bct}}\; erfc\,(\eta - \sqrt{bct}) + e^{2\eta\sqrt{bct}}\; erfc\,(\eta + \sqrt{bct})] \\
& - \frac{Gr}{(1-b)\,c^2}\, erfc\,(\eta\sqrt{b}) - \frac{Gr}{(1-b)\,c}\left[(1+2\eta^2 b)\, erfc\,(\eta\sqrt{b}) - \frac{2}{\sqrt{\pi}}\,\eta\sqrt{b}\, e^{-\eta^2 b}\right] \\
& + \frac{Gr\,e^{ct}}{2\,(1-b)\,c^2}\,[\bar{e}^{2\eta\sqrt{bct}}\; erfc\,(\eta\sqrt{b} - \sqrt{ct}) + e^{2\eta\sqrt{bct}}\; erfc\,(\eta\sqrt{b} + \sqrt{ct})]
\end{aligned} \tag{12}$$

where $\eta = \dfrac{y}{2\sqrt{t}}$ and $b = \dfrac{3N\,\mathrm{Pr}}{3N+4}$, $c = \dfrac{M}{b-1}$.

4. RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different parameters *M*, *N*, a, *Gr* and *t* upon the nature of the flow and transport. The numerical values of the velocity and temperature are computed for different physical parameters like *M*, *N*, *a*, Prandtl number, thermal Grashof number and time.

The velocity profiles for different values of the magnetic field parameter are shown in Fig. 1. It is observed that the velocity decreases in the presence of magnetic field than its absence. This shows that the increase in the magnetic field parameter leads to fall in the velocity. This agrees with expectations, since the magnetic field exerts a retarding force on the free convective flow.

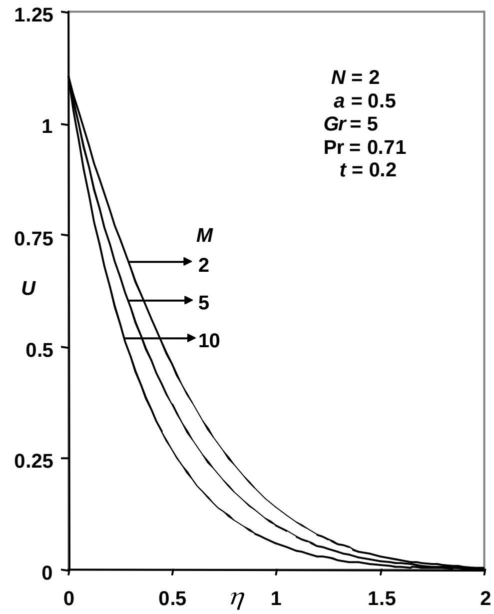

Figure 1: Velocity Profiles for Different *M*

In Fig. 2, the velocity profiles are shown for different values of the radiation parameter. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

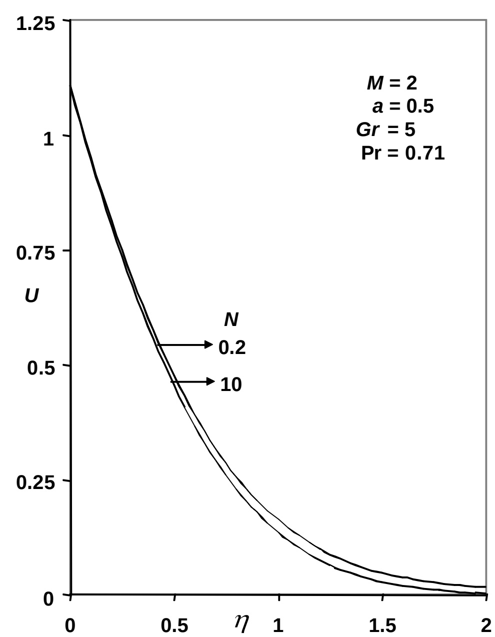

Figure 2: Velocity Profiles for Different *N*

The velocity profiles for different values of ($a = 0.2, 0.5, 0.8$) and time ($t = 0.2, 0.4, 0.6$) are shown in the Fig. 3. It is observed that the velocity increases with increasing values of a or time.

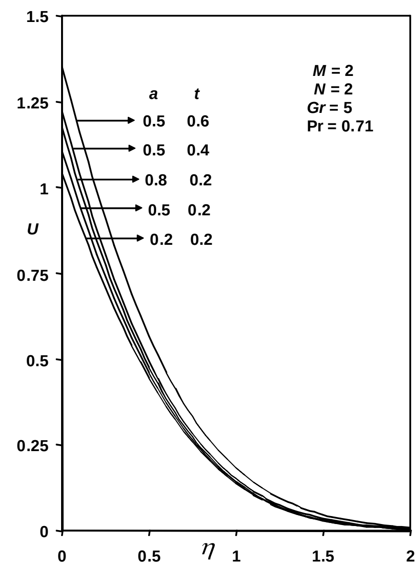

Figure 3: Velocity Profiles for Different a and *t*

The velocity profiles for different thermal Grashof number ($Gr = 2, 10$) are shown in the Fig. 4. It is observed that velocity increases with increasing values of Gr.

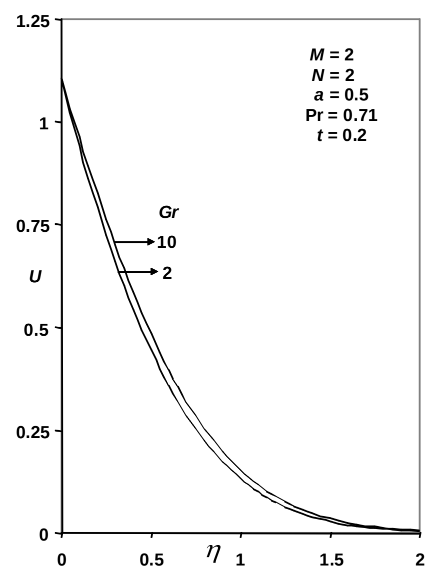

Figure 4: Velocity Profiles for Different *Gr*

The temperature profiles for different values of the radiation parameter ($N = 0.2, 2, 30$) are shown in the Fig. 5. It is observed that the temperature decreases in the presence of thermal radiation.

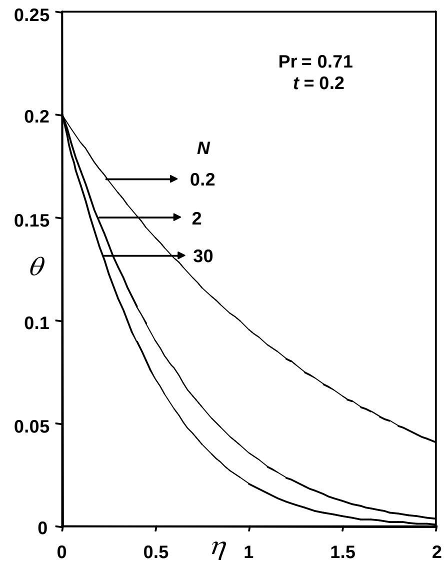

Figure 5: Temperature Profiles for Different *N*

The temperature profiles are calculated for different values of the time ($t = 0.2, 0.4, 0.6$) are shown in Fig. 6. It is observed that temperature increases with increasing values of time.

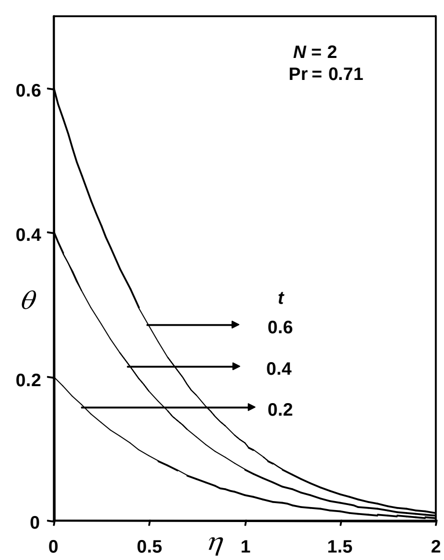

Figure 6: Temperature Profiles for Different *t*

From the velocity field, we now study the skin. It is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0}. \tag{13}$$

Hence, from the equations (12) and (13), the wall shear stress in the presence of magnetic field is as follows.

$$\tau = \frac{e^{at}}{\sqrt{\pi t}}[\sqrt{\pi(a+M)\,t}\; erf\;(\sqrt{(a+M\;t})+1]$$

$$+\frac{Gr(1+ct)}{\sqrt{\pi t}\;(1-b)\,c^2}[\sqrt{\pi M t}\; erf\;(\sqrt{Mt})+1]+\frac{Gr}{2(1-b)\,c\sqrt{M}}\; erf\;(\sqrt{Mt})$$

$$-\frac{Gr\,e^{ct}}{\sqrt{\pi t}\;(1-b)\,c^2}[\sqrt{\pi bct}\; erf\;(\sqrt{bct})+1]-\frac{Gr\sqrt{b}}{\sqrt{\pi t}\,(1-b)\,c^2}$$

$$-\frac{2Grt\sqrt{b}}{\sqrt{\pi t}\;(1-b)\,c}+\frac{Gr\,e^{ct}}{\sqrt{\pi t}\;(1-b)\,c^2}[\sqrt{\pi bct}\; erf\;(\sqrt{ct})+\sqrt{b}\,].$$

The numerical values of τ+ are presented in Table 1. It is observed that from this Table, skin-friction increases with increasing values of the magnetic field parameter, accelerating parameter and time. It is also observed that the skin-friction decreases with increasing values of the radiation parameter and Grashof number.

Table 1
Values of the Non-Dimensional Skin-Friction

M	*N*	*Gr*	*a*	*t*	*Pr = 0.71*
2	2	5	2	0.2	4.0068
5	2	5	2	0.2	5.1566
10	2	5	2	0.2	6.6396
2	0.2	5	2	0.2	4.1550
2	5	5	2	0.2	3.9633
2	2	2	2	0.2	4.1535
2	2	10	2	0.2	3.7623
2	2	5	0	0.2	1.9064
2	2	5	5	0.2	9.6987
2	2	5	2	0.2	5.2582

5. CONCLUSIONS

The theoretical solution of flow past an exponentially accelerated infinite vertical plate with variable temperature in the presence of magnetic field and thermal radiation is considered. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like magnetic field parameter, radiation parameter, thermal Grashof number, a and t are studied graphically. It is observed that the velocity decreases with increasing values of magnetic field parameter and radiation parameter but increases with increasing values of Gr, a and t.

REFERENCES

[1] M. Q. Brewster, Thermal Radiative Transfer and Properties, John Wiley and Sons Inc, New York, (1992).

[2] P. Chandrakala, and S. Antony Raj, Radiation Effects on MHD Flow Past an Impulsively Started Infinite Certical Plate with Variable Temperature, *Indian Journal of Mathematics*, **48**(2), (2006), 167-175.

[3] M. A. Hossian, and L. K. Shayo, The Skin Friction in the Unsteady Free Convection Flow Past an Accelerated Plate, *Astrophysics and Space Science*, **125**, (1986), 315-324.

[4] A. Raptis, and C. Perdikis, Radiation and Free Convection Flow Past a Moving Plate, *Int. J. App. Mech. and Engg.*, **4**, (1999), 817-821.

[5] B. C. Sakiadis, Boundary Layer Behaviour on Continuous Solid Surfaces, *AICHE Journal*, **7**, (1961), 26-28.

[6] B. C. Sakiadis, Boundary Layer Behaviour on Continuous Solid Surfaces II, *AICHE Journal*, **7**, (1961), 221-225.

[7] A. K. Singh, and Naveen Kumar, Free Convection Flow Past an Exponentially Accelerated Vertical Plate, *Astrophysics and Space Science*, **98**, (1984), 245-258.

[8] V. M. Soundalgekar, Free Convection Effects on Stokes Problem for an Infinite Vertical Plate, *Journal of Heat Transfer*, **99**, (1977), 449-501.

P. Chandrakala
Department of Mathematics,
Bharathi Women's College,
No. 34/2, Ramanujam, Garden Street,
Pattalam, Chennai-600 012, India.
E-mail: pckala05@yahoo.com

P. Narayana Bhaskar
CPCL, Chennai-600 068, India.