I J C T A, 9(1), 2016, pp. 1-20

Mathematical Analysis, Adaptive Control and Synchronization of a Ten-Term Novel Three-Scroll Chaotic System with Four Quadratic Nonlinearities

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Abstract: First, this paper announces a ten-term novel 3-D three-scroll chaotic system with four quadratic nonlinearities. The phase portraits of the novel chaotic system are displayed and the mathematical properties are discussed. The Lyapunov exponents of the novel chaotic system are obtained as $L_1 = 1.5015$, $L_2 = 0$ and $L_3 = -2.9367$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as $L_1 = 1.5015$ and Lyapunov dimension as $D_L = 2.5113$. Next, we derive new results for the adaptive control design of the novel chaotic system with unknown parameters. The adaptive controller is designed to achieve global exponential stability for the novel chaotic system with unknown parameters. Next, we derive new results for the adaptive synchronization design of the identical novel chaotic systems with unknown parameters. The adaptive control and synchronization results have been established using Lyapunov stability theory. Numerical simulations with MATLAB have been shown to validate and illustrate all the new results derived in this paper.

Keywords: Chaos, chaotic systems, novel chaotic system, three-scroll system, adaptive control, adaptive synchroization.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions. The sensitivity of a nonlinear chaotic system in response to small changes in the initial conditions is commonly called as *butterfly effect* [1] and this is one of the characterizing features of a chaotic system.

Chaos was historically discovered first by Henri Poincaré in 1890 when he was studying the n-body problem. Poincare discovered that the orbit of three or more interacting planets can depict unstable and unpredictable behaviour, and this is the first finding of a chaotic system. Subsequently, in 1963, Lorenz [2] discovered irregularity in a 3-D weather model and this is the first experimentally verified chaotic system.

The Lyapunov exponent of a dynamical system is a quantitative measure that characterizes the rate of separation of infinitesimally close trajectories of the system. Thus, a chaotic system is also defined mathematically as a dynamical system having at least one positive Lyapunov exponent.

In the last four decades, many chaotic systems have been found in the literature using modelling and other techniques. Some paradigms of chaotic systems can be listed as Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], etc. Many new 3-D chaotic systems have been discovered in the recent years such as Sundarapandian systems [12-13], Vaidyanathan systems [14-20], Vaidyanathan-Madhavan system [21], Vaidyanathan-Azar system [22], Vaidyanathan-Volos system [23-24], Pehlivan-Moroz system [25], Pham system [26], etc.

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Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering. Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [27-28], hyperchaotic Vaidyanathan-Azar system [29], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [30].

Chaos control and chaos synchronization are important research problems in the chaos literature, which have been studied extensively in the last four decades. There are several applications of chaos theory in a variety of fields such as lasers [31], oscillators [32-33], chemical reactors [34-35], biology [36-38], ecology [39-40], neural networks [41-43], robotics [44-45], memristors [46-48], fuzzy systems [49-50], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [51-52]. Some popular methods for chaos control are active control [53-57], adaptive control [58-59], sliding mode control [60-62], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. The synchronization of chaotic systems has applications in secure communications [63-65], cryptosystems [66-67], encryption [68-70], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [71-72] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [73-80], adaptive control method [81-107], sampled-data feedback control method [108-109], time-delay feedback approach [110], backstepping method [111-122], sliding mode control method [123-131], etc.

In this paper, we have proposed a novel 3-D three-scroll chaotic system with four quadratic nonlinearities. We have obtained the Lyapunov exponents of the novel three-scroll chaotic system as $L_1 = 1.5015$, $L_2 = 0$ and $L_3 = -2.9367$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is found as $L_1 = 1.5015$ and Lyapunov dimension as $D_L = 2.5113$. We have also derived new results for the adaptive control of the novel chaotic system and adaptive synchronization of identical novel chaotic systems with unknown parameters. The main adaptive results of this paper are proved using Lyapunov stability theory. MATLAB simulations have been provided in this paper to illustrate the phase portraits of the novel three-scroll chaotic system and the adaptive control results for the novel three-scroll chaotic system.

2. A THREE-SCROLL NOVEL CHAOTIC SYSTEM

In this section, we describe the equations and properties of a three-scroll novel 3-D polynomial chaotic system with four quadratic nonlinearities.

The proposed three-scroll novel chaotic system is modelled by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + c x_1 x_3 \\ \dot{x}_2 &= b x_1 + p x_2 - x_1 x_3 \\ \dot{x}_3 &= 2p - d x_1^2 + x_1 x_2 + 2 x_3 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b, c, d, p are constant, positive parameters of the system.

The system (1) exhibits a *three-scroll* chaotic attractor for the values

$$a = 40,\ b = 55,\ c = 0.16,\ d = 0.65,\ p = 12 \tag{2}$$

For numerical simulations, we take the initial state as $x_1(0) = 1.0,\ \ x_2(0) = 0.7$ and $x_3(0) = 1.2.$ Figure 1 shows the three-scroll chaotic attractor of the system (1). Figures 2-4 show the 2-D view of the chaotic attractor of the system (1) in $(x_1, x_2),\ \ (x_2, x_3)$ and (x_1, x_3) planes respectively.

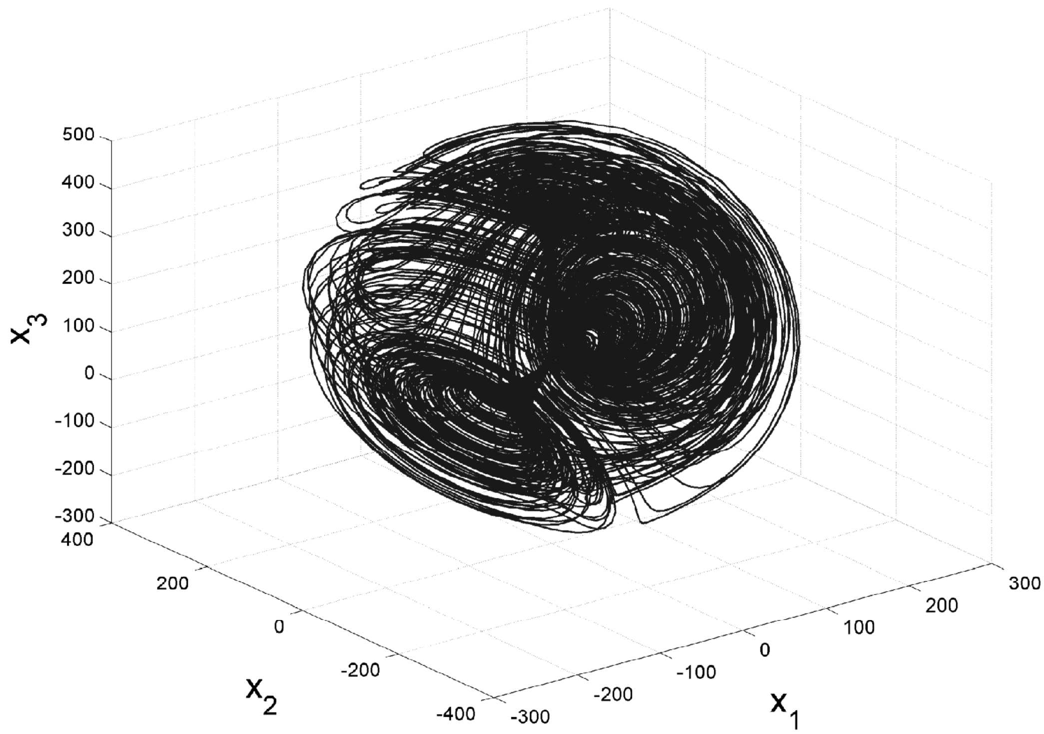

Figure 1: Strange attractor of the novel 3-scroll chaotic system

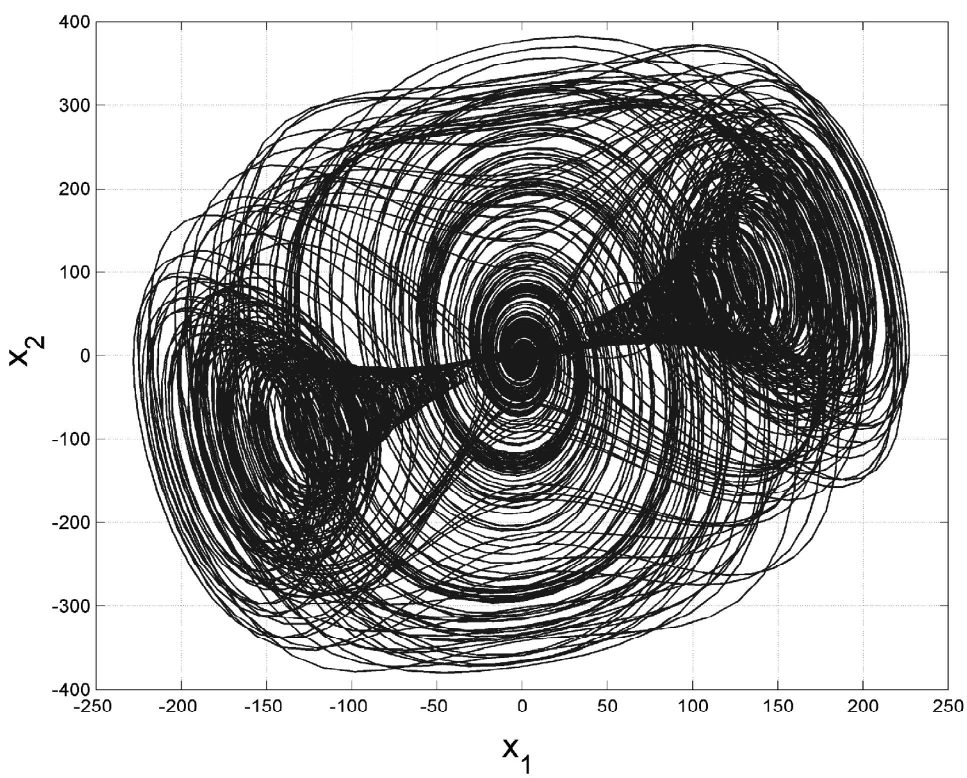

Figure 2: 2-D view of the novel 3-scroll chaotic system in (x_1, x_2) plane

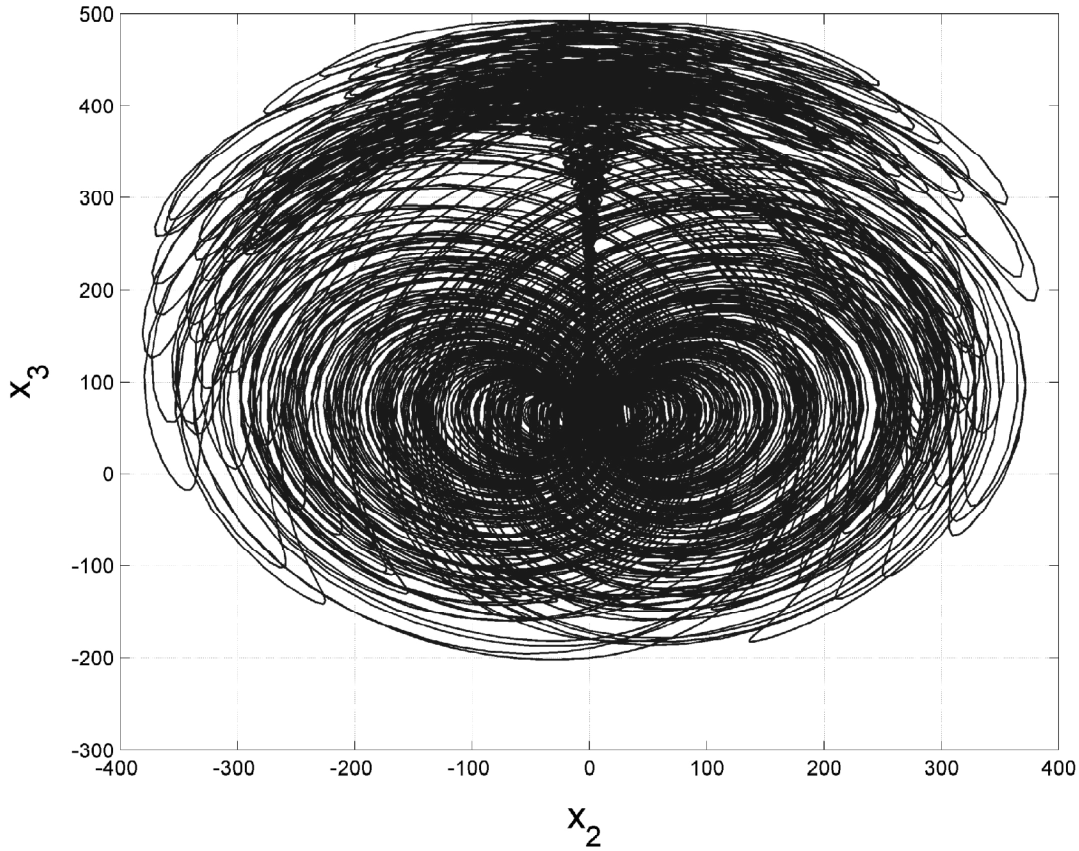

Figure 3: 2-D view of the novel 3-scroll chaotic system in (x_2, x_3) plane

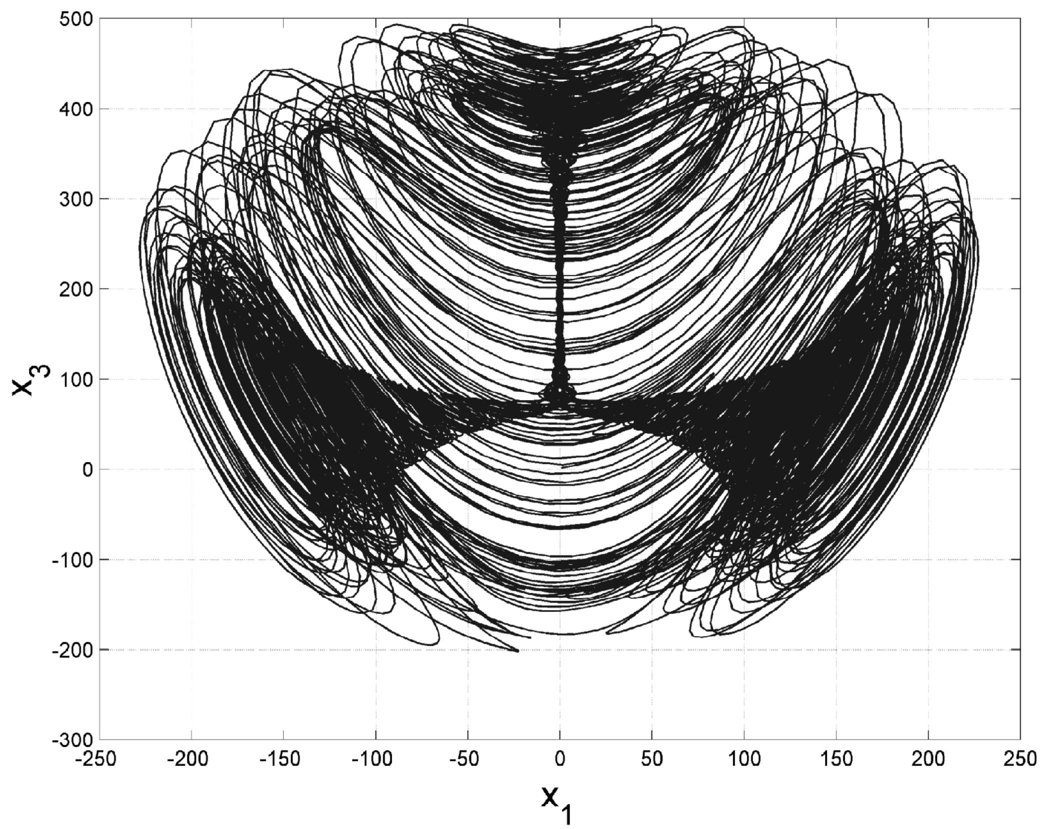

Figure 4: 2-D view of the novel 3-scroll chaotic system in (x_1, x_3) plane

3. PROPERTIES OF THE NOVEL THREE-SCROLL CHAOTIC SYSTEM

A. Symmetry

The novel 3-D chaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \to (-x_1, -x_2, x_3) \tag{3}$$

Since the transformation (3) persists for all values of the system parameters, the novel chaotic system (1) has rotation symmetry about the x_3– axis and that any non-trivial trajectory must have a twin trajectory.

B. Invariance

The x_3– axis ($x_1 = 0,\ x_2 = 0$) is invariant for the system (1). Hence, all orbits of the system (1) starting on the x_3– axis stay in the x_3– axis for all values of time. Also, this invariant motion is unstable.

C. Equilibrium Points

The equilibrium points of the novel chaotic system (1) are obtained by solving the following nonlinear system of equations

$$\begin{aligned} f_1(x_1, x_2, x_3) &= a(x_2 - x_1) + c x_1 x_3 = 0 \\ f_2(x_1, x_2, x_3) &= b x_1 + p x_2 - x_1 x_3 = 0 \\ f_3(x_1, x_2, x_3) &= 2p - d x_1^2 + x_1 x_2 + 2 x_3 = 0 \end{aligned} \tag{4}$$

We take the parameter values as in the chaotic case, *viz.*

$$a = 40,\ b = 55,\ c = 0.16,\ d = 0.65,\ p = 12 \tag{5}$$

Solving the equations (4) using the values (5), we obtain the unique equilibrium point:

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ -12 \end{bmatrix} \tag{6}$$

The Jacobian matrix of the novel chaotic system (1) is obtained as

$$J(x) = \begin{bmatrix} -a + c x_3 & a & c x_1 \\ b - x_3 & p & -x_1 \\ -2d x_1 + x_2 & x_1 & 2 \end{bmatrix} \tag{7}$$

The Jacobian matrix at the equilibrium E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} -41.92 & 40 & 0 \\ 67 & 12 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{8}$$

which has the eigenvalues

$$\lambda_1 = -73.3282,\ \ \lambda_2 = 43.4082,\ \ \lambda_3 = 2 \tag{9}$$

This shows that the equilibrium E_0 is a saddle-point, which is unstable.

D. Lyapunov Exponents

We take the parameter values of the system (1) as

$$a = 40, \ b = 55, \ c = 0.16, \ d = 0.65, \ p = 12 \tag{10}$$

The Lyapunov exponents of the system (1) are numerically obtained with MATLAB as

$$\begin{cases} L_1 & = & 1.5015 \\ L_2 & = & 0 \\ L_3 & = & -2.9367 \end{cases} \tag{11}$$

Eq. (11) shows that the system (1) is chaotic, since it has a positive Lyapunov exponent.

Also, the maximal Lyapunov exponent (MLE) of the system (1) is obtained as $L_1 = 1.5015$.

Since $L_1 + L_2 + L_3 = -1.4352 < 0$, it is immediate that (1) is a dissipative chaotic system.

The dynamics of the Lyapunov exponents is depicted in Figure 5.

F. Lyapunov Dimension

The Lyapunov dimension of the chaotic system (1) is determined as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.5113 \tag{12}$$

which is high value. This shows that the chaotic behaviour of the system (1) is very complex.

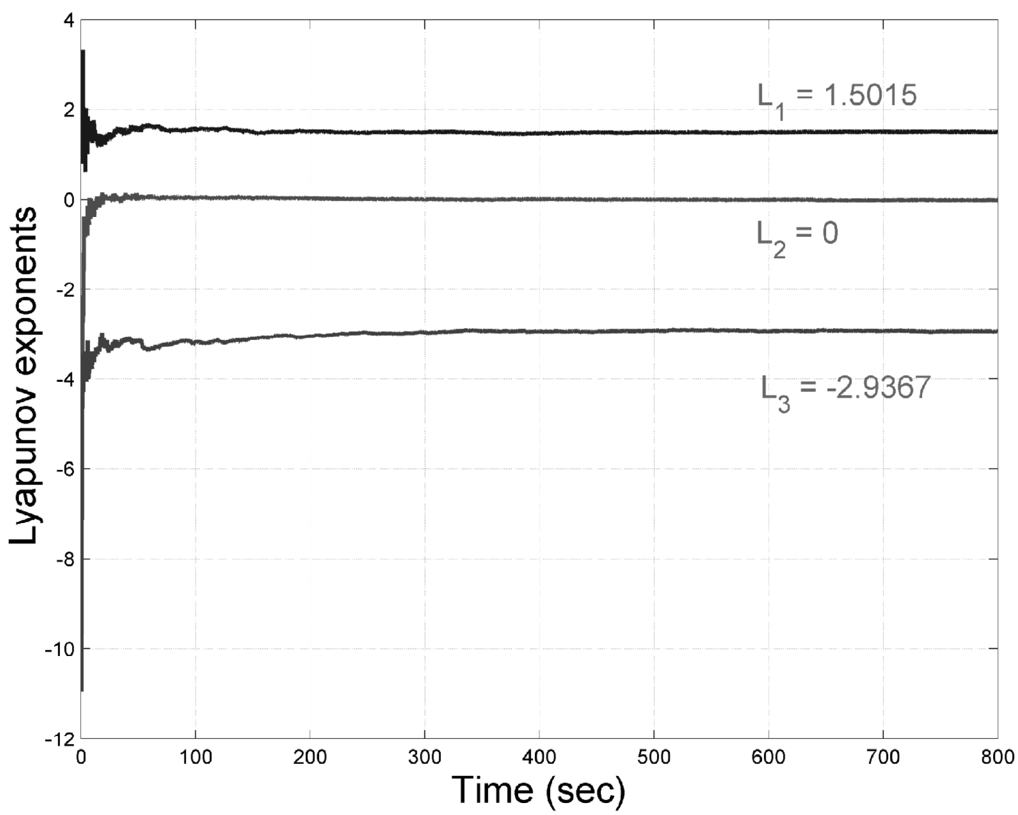

Figure 5: Dynamics of the Lyapunov exponents of the novel chaotic system

4. ADAPTIVE CONTROL OF THE THREE-SCROLL CHAOTIC SYSTEM

In this section, we design new results for the adaptive controller to stabilize the three-scroll novel chaotic system with unknown parameters for all initial conditions.

Thus, we consider the controlled novel 3-D chaotic system

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + cx_1x_3 + u_1 \\ \dot{x}_2 &= bx_1 + px_2 - x_1x_3 + u_2 \\ \dot{x}_3 &= 2p - dx_1^2 + x_1x_2 + 2x_3 + u_3\end{aligned} \tag{13}$$

where x_1, x_2, x_3 are state variables, a, b, c, d, p are constant, unknown, parameters of the system and u_1, u_2, u_3 are adaptive controls to be designed.

We aim to solve the adaptive control problem by considering the adaptive feedback control law

$$\begin{aligned}u_1 &= -\hat{a}(t)(x_2 - x_1) - \hat{c}(t)x_1x_3 - k_1x_1 \\ u_2 &= -\hat{b}(t)x_1 - \hat{p}(t)x_2 + x_1x_3 - k_2x_2 \\ u_3 &= -2\hat{p}(t) + \hat{d}(t)x_1^2 - x_1x_2 - 2x_3 - k_3x_3\end{aligned} \tag{14}$$

where $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$, $\hat{d}(t)$, $\hat{p}(t)$ are estimates for the unknown system parameters a, b, c, d, p, respectively, and k_1, k_2, k_3 are positive gain constants.

The closed-loop system is obtained by substituting (14) into (13) as

$$\begin{aligned}\dot{x}_1 &= (a - \hat{a}(t))(x_2 - x_1) + (c - \hat{c}(t))x_1x_3 - k_1x_1 \\ \dot{x}_2 &= (b - \hat{b}(t))x_1 + (p - \hat{p}(t)x_2 - k_2x_2 \\ \dot{x}_3 &= 2(p - \hat{p}(t)) - (d - \hat{d}(t))x_1^2 - k_3x_3\end{aligned} \tag{15}$$

To simplify (15), we define the parameter estimation error as

$$\begin{aligned}e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \\ e_c(t) &= c - \hat{c}(t) \\ e_d(t) &= d - \hat{d}(t) \\ e_p(t) &= p - \hat{p}(t)\end{aligned} \tag{16}$$

Substituting (16) into (15), we obtain

$$\begin{aligned}\dot{x}_1 &= e_a(x_2 - x_1) + e_cx_1x_3 - k_1x_1 \\ \dot{x}_2 &= e_bx_1 + e_px_2 - k_2x_2 \\ \dot{x}_3 &= 2e_p - e_dx_1^2 - k_3x_3\end{aligned} \tag{17}$$

Differentiating the parameter estimation error (16) with respect to t, we get

$$\begin{aligned} \dot{e}_a(t) &= -\dot{\hat{a}}(t) \\ \dot{e}_b(t) &= -\dot{\hat{b}}(t) \\ \dot{e}_c(t) &= -\dot{\hat{c}}(t) \\ \dot{e}_d(t) &= -\dot{\hat{d}}(t) \\ \dot{e}_p(t) &= -\dot{\hat{p}}(t) \end{aligned} \tag{18}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(x_1, x_2, x_3, e_a, e_b, e_c, e_d, e_p) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_p^2\right), \tag{19}$$

which is positive definite on R^8.

Differentiating V along the trajectories of (17) and (18), we obtain

$$\begin{aligned} \dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[x_1x_2 - \dot{\hat{b}}\right] \\ + e_c\left[x_1^2x_3 - \dot{\hat{c}}\right] + e_d\left[-x_1^2x_3 - \dot{\hat{d}}\right] + e_p\left[x_2^2 + 2x_3 - \dot{\hat{p}}\right] \end{aligned} \tag{20}$$

In view of (20), we define an update law for the parameter estimates as

$$\begin{aligned} \dot{\hat{a}} &= x_1(x_2 - x_1) \\ \dot{\hat{b}} &= x_1x_2 \\ \dot{\hat{c}} &= x_1^2x_3 \\ \dot{\hat{d}} &= -x_1^2x_3 \\ \dot{\hat{p}} &= x_2^2 + 2x_3 \end{aligned} \tag{21}$$

Theorem 1. The novel chaotic system (13) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (14) and the parameter update law (21), where k_i, $(i = 1, 2, 3)$ are positive constants.

Proof. The result is proved using Lyapunov stability theory [132]. We consider the quadratic Lyapunov function V defined by (19), which is a positive definite function on R^8.

Substituting the parameter update law (21) into (20), we obtain $\dot{V}$ as

$$\dot{V} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 \tag{22}$$

which is a negative semi-definite function on R^8.

Therefore, it can be concluded that the state vector $x(t)$ and the parameter estimation error are globally bounded, i.e.

$$\begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & e_a(t) & e_b(t) & e_c(t) & e_d(t) & e_p(t) \end{bmatrix}^T \in L_\infty. \tag{23}$$

We define

$$k = \min\{k_1, k_2, k_3\}. \tag{24}$$

Then it follows from (21) that

$$\dot{V} \le -k\|x\|^2 \text{ or } k\|x\|^2 \le -\dot{V}. \tag{25}$$

Integrating the inequality (25) from 0 to t, we get

$$k\int_o^t \|x(\tau)\|^2 \, d\tau \ \le \ -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{26}$$

From (26), it follows that $x(t) \in L_2$.

Using (17), we can conclude that $\dot{x}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof.

Numerical Results

For the novel chaotic system (13), the parameter values are taken as in the chaotic case, *viz.*

$$a = 40,\ b = 55,\ c = 0.16,\ d = 0.65,\ p = 12 \tag{27}$$

We take the feedback gains as $k_i = 6$ for $i = 1, 2, 3$.

The initial values of the chaotic system (13) are taken as

$$x_1(0) = 3.2,\ x_2(0) = 5.7,\ x_3(0) = 1.8 \tag{28}$$

The initial values of the parameter estimates are taken as

$$\hat{a}(0) = 6,\ \hat{b}(0) = 5.1,\ \hat{c}(0) = 2.5,\ \hat{d}(0) = 3.4,\ \hat{p}(0) = 4.2 \tag{29}$$

Figure 6 depicts the time-history of the controlled novel chaotic system.

5. ADAPTIVE SYNCHRONIZATION OF THE IDENTICAL NOVEL THREE-SCROLL CHAOTIC SYSTEMS

In this section, we derive new results for the adaptive synchronization of the identical novel three-scroll chaotic systems with unknown parameters.

As the master system, we take the novel 3-D three-scroll chaotic system

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + cx_1x_3 \\ \dot{x}_2 &= bx_1 + px_2 - x_1x_3 \\ \dot{x}_3 &= 2p - dx_1^2 + x_1x_2 + 2x_3 \end{aligned} \tag{30}$$

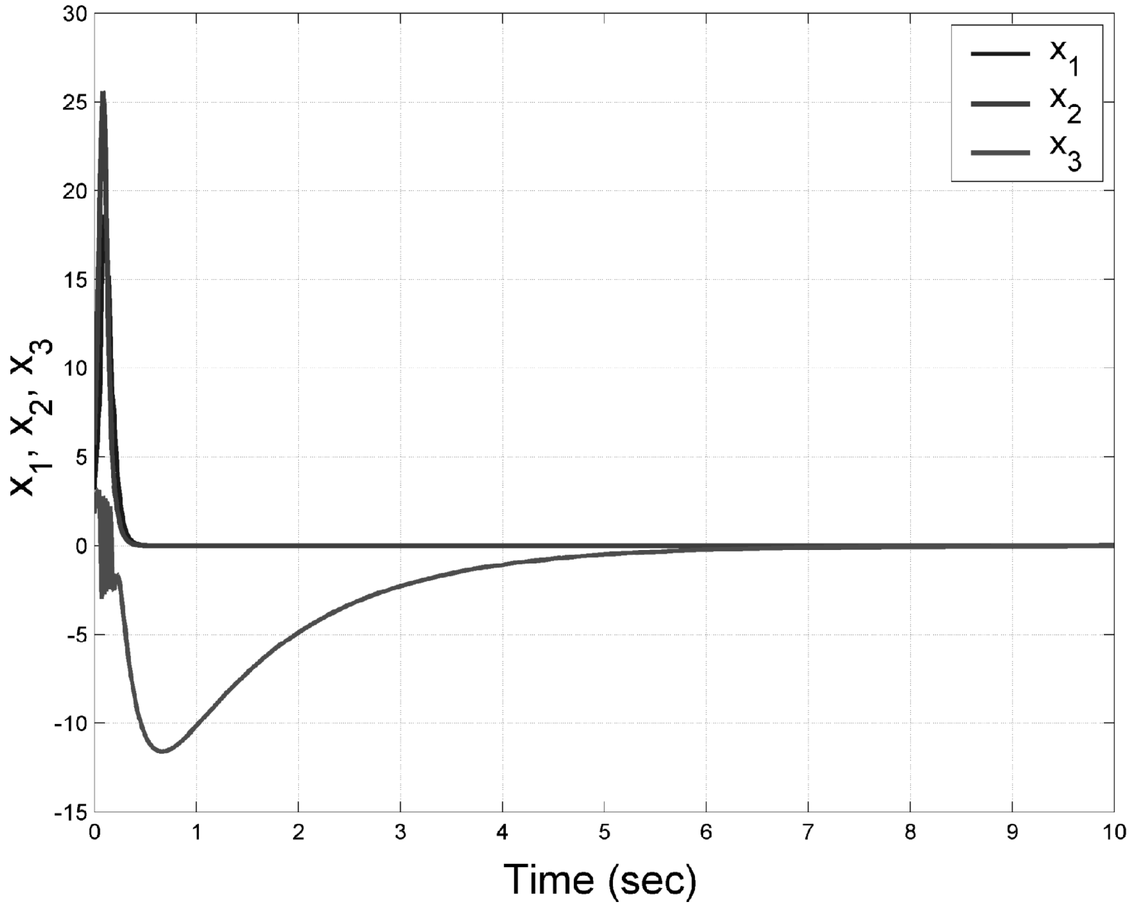

Figure 6: Time history of the controlled three-scroll chaotic system

where x_1, x_2, x_3 are state variables and a, b, c, d, p are constant, unknown, parameters of the system.

As the slave system, we take the controlled novel 3-D chaotic system

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + cy_1y_3 + u_1 \\ \dot{y}_2 &= by_1 + py_2 - y_1y_3 + u_2 \\ \dot{y}_3 &= 2p - dy_1^2 + y_1y_2 + 2y_3 + u_3 \end{aligned} \tag{31}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are adaptive controllers to be designed.

The synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{32}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + c(y_1y_3 - x_1x_3) + u_1 \\ \dot{e}_2 &= be_1 + pe_2 - y_1y_3 + x_1x_3 + u_2 \\ \dot{e}_3 &= 2e_3 - d(y_1^2 - x_1^2) + y_1y_2 - x_1x_2 + u_3 \end{aligned} \tag{33}$$

We consider the adaptive control law defined by

$$\begin{aligned} u_1 &= -\hat{a}(t)(e_2 - e_1) - \hat{c}(t)(y_1 y_3 - x_1 x_3) - k_1 e_1 \\ u_2 &= -\hat{b}(t) e_1 - \hat{p}(t) e_2 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 &= -2e_3 + \hat{d}(t)(y_1^2 - x_1^2) - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{aligned} \tag{34}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (34) into (33), we get the closed-loop error dynamics as

$$\begin{aligned} \dot{e}_1 &= (a - \hat{a}(t))(e_2 - e_1) + (c - \hat{c}(t))(y_1 y_3 - x_1 x_3) - k_1 e_1 \\ \dot{e}_2 &= (b - \hat{b}(t)) e_1 + (p - \hat{p}(t)) e_2 - k_2 e_2 \\ \dot{e}_3 &= -(d - \hat{d}(t))(y_1^2 - x_1^2) - k_3 e_3 \end{aligned} \tag{35}$$

To simplify the error dynamics (35), we define the parameter estimation error as

$$\begin{aligned} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \\ e_c(t) &= c - \hat{c}(t) \\ e_d(t) &= d - \hat{d}(t) \\ e_p(t) &= p - \hat{p}(t) \end{aligned} \tag{36}$$

Using (36), we can simplify the error dynamics (35) as

$$\begin{aligned} \dot{e}_1 &= e_a (e_2 - e_1) + e_c (y_1 y_3 - x_1 x_3) - k_1 e_1 \\ \dot{e}_2 &= e_b e_1 + e_p e_2 - k_2 e_2 \\ \dot{e}_3 &= -e_d (y_1^2 - x_1^2) - k_3 e_3 \end{aligned} \tag{37}$$

Differentiating the parameter estimation error (36) with respect to t, we get

$$\begin{aligned} \dot{e}_a(t) &= -\dot{\hat{a}}(t) \\ \dot{e}_b(t) &= -\dot{\hat{b}}(t) \\ \dot{e}_c(t) &= -\dot{\hat{c}}(t) \\ \dot{e}_d(t) &= -\dot{\hat{d}}(t) \\ \dot{e}_p(t) &= -\dot{\hat{p}}(t) \end{aligned} \tag{38}$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b, e_c, e_d, e_p) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_p^2\right), \tag{39}$$

which is positive definite on R^8.

Differentiating V along the trajectories of (37) and (38), we obtain

$$\begin{aligned}\dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_a\left[e_1(e_2-e_1)-\dot{\hat{a}}\right] + e_b\left[e_1e_2-\dot{\hat{b}}\right] \\ &+ e_c\left[e_1\left(y_1y_3 - x_1x_3\right)-\dot{\hat{c}}\right] + e_d\left[-e_3(y_1^2-x_1^2)-\dot{\hat{d}}\right] + e_p\left[e_2^2-\dot{\hat{p}}\right]\end{aligned} \tag{40}$$

In view of (40), we define an update law for the parameter estimates as

$$\begin{aligned}\dot{\hat{a}} &= e_1(e_2-e_1) \\ \dot{\hat{b}} &= e_1e_2 \\ \dot{\hat{c}} &= e_1\left(y_1y_3 - x_1x_3\right) \\ \dot{\hat{d}} &= -e_3(y_1^2-x_1^2) \\ \dot{\hat{p}} &= e_2^2\end{aligned} \tag{41}$$

Theorem 2. The identical novel chaotic systems (30) and (31) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (34) and the parameter update law (41), where k_i, (i = 1, 2, 3) are positive constants.

Proof. The result is proved using Lyapunov stability theory [132]. We consider the quadratic Lyapunov function V defined by (39), which is a positive definite function on R^8.

Substituting the parameter update law (41) into (40), we obtain $\dot{V}$ as

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \tag{42}$$

which is a negative semi-definite function on R^8.

Thus, it can be concluded that the synchronization vector $e(t)$ and the parameter estimation error are globally bounded, i.e.

$$\begin{bmatrix} e_1(t) & e_2(t) & e_3(t) & e_a(t) & e_b(t) & e_c(t) & e_d(t) & e_p(t)\end{bmatrix}^T \in L_\infty. \tag{43}$$

We define

$$k = \min\left\{k_1, k_2, k_3\right\}. \tag{44}$$

Then it follows from (42) that

$$\dot{V} \le -k\|e\|^2 \text{ or } k\|e\|^2 \le -\dot{V}. \tag{45}$$

Integrating the inequality (45) from 0 to t, we get

$$k\int_o^t \|e(\tau)\|^2 \, d\tau \le -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{46}$$

Therefore, we can conclude that $e(t) \in L_2$.

Using (37), we can conclude that $\dot{e}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof.

Numerical Results

For the novel chaotic systems, the parameter values are taken as in the chaotic case, *viz.*

$$a = 40,\ b = 55,\ c = 0.16,\ d = 0.65,\ p = 12 \tag{47}$$

We take the feedback gains as $k_i = 6$ for $i = 1, 2, 3$.

The initial values of the master system (30) are taken as

$$x_1(0) = 1.2,\ x_2(0) = 0.7,\ x_3(0) = 0.2 \tag{48}$$

The initial values of the slave system (31) are taken as

$$y_1(0) = 1.8,\ y_2(0) = 1.6,\ y_3(0) = 1.4 \tag{49}$$

The initial values of the parameter estimates are taken as

$$\hat{a}(0) = 2,\ \hat{b}(0) = 9,\ \hat{c}(0) = 1,\ \hat{d}(0) = 2,\ \hat{p}(0) = 3 \tag{50}$$

Figures 7-9 depicts the complete synchronization of the identical novel chaotic systems.

Figure 10 depicts the time-history of the synchronization errors.

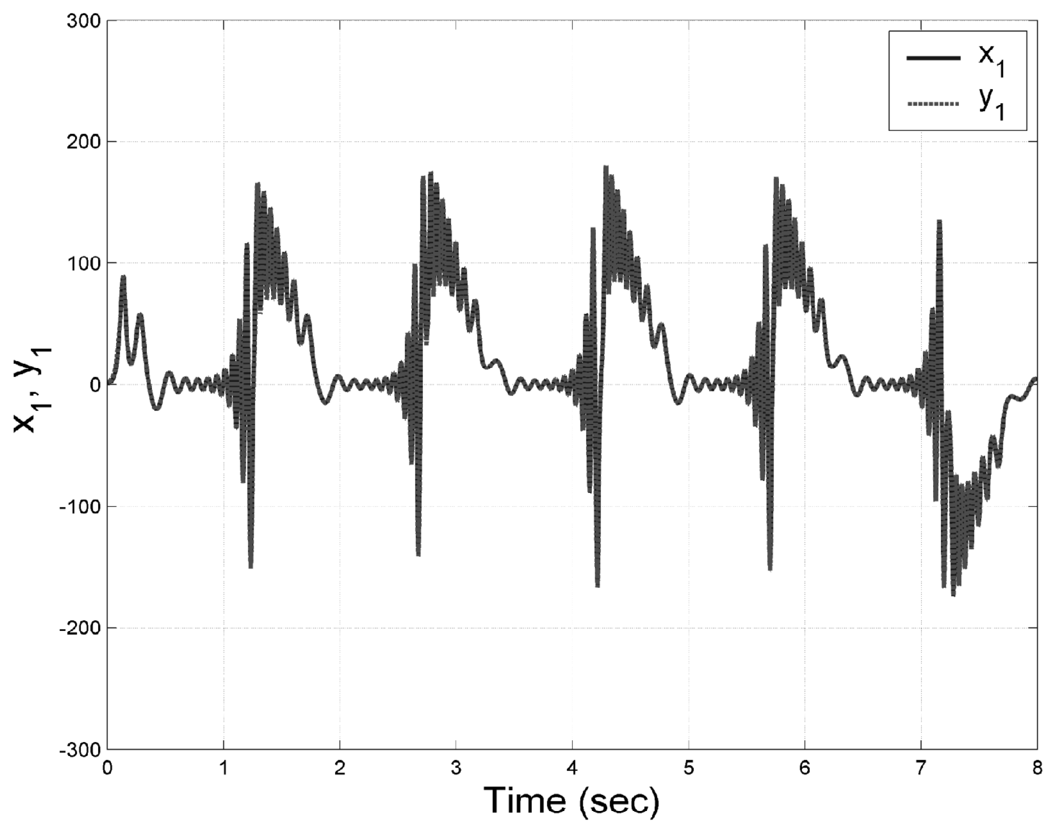

Figure 7: Complete synchronization of the states x_1 and y_1

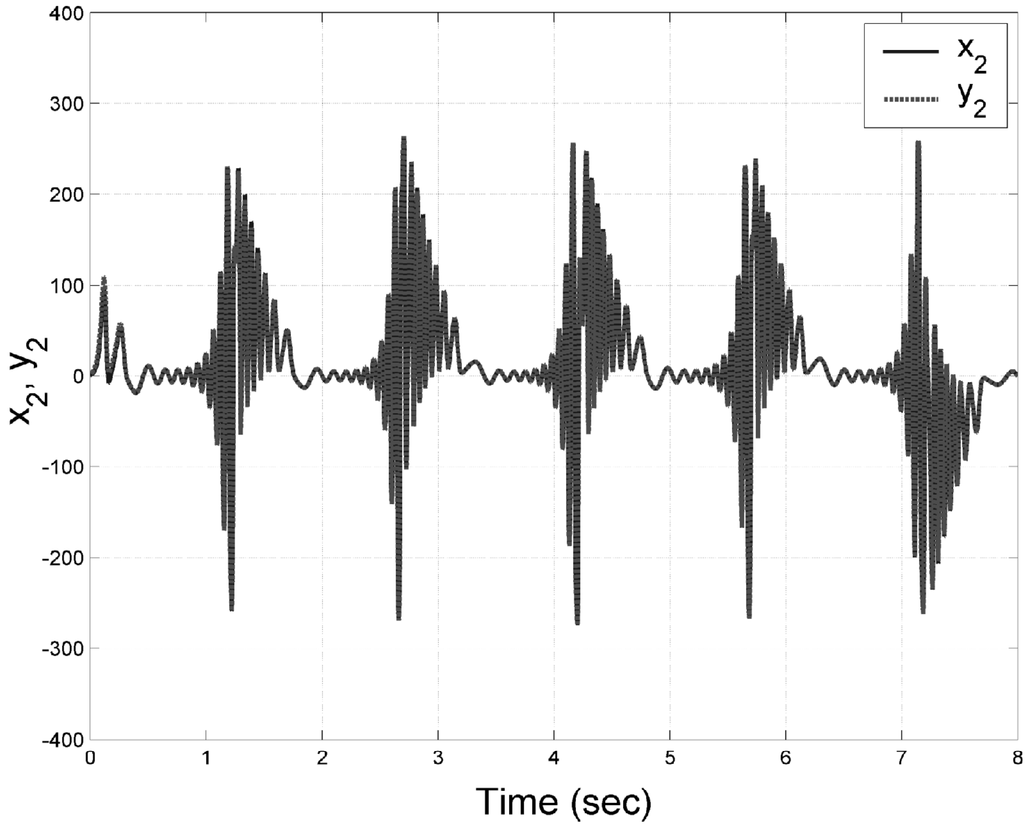

Figure 8: Complete synchronization of the states x_2 and y_2

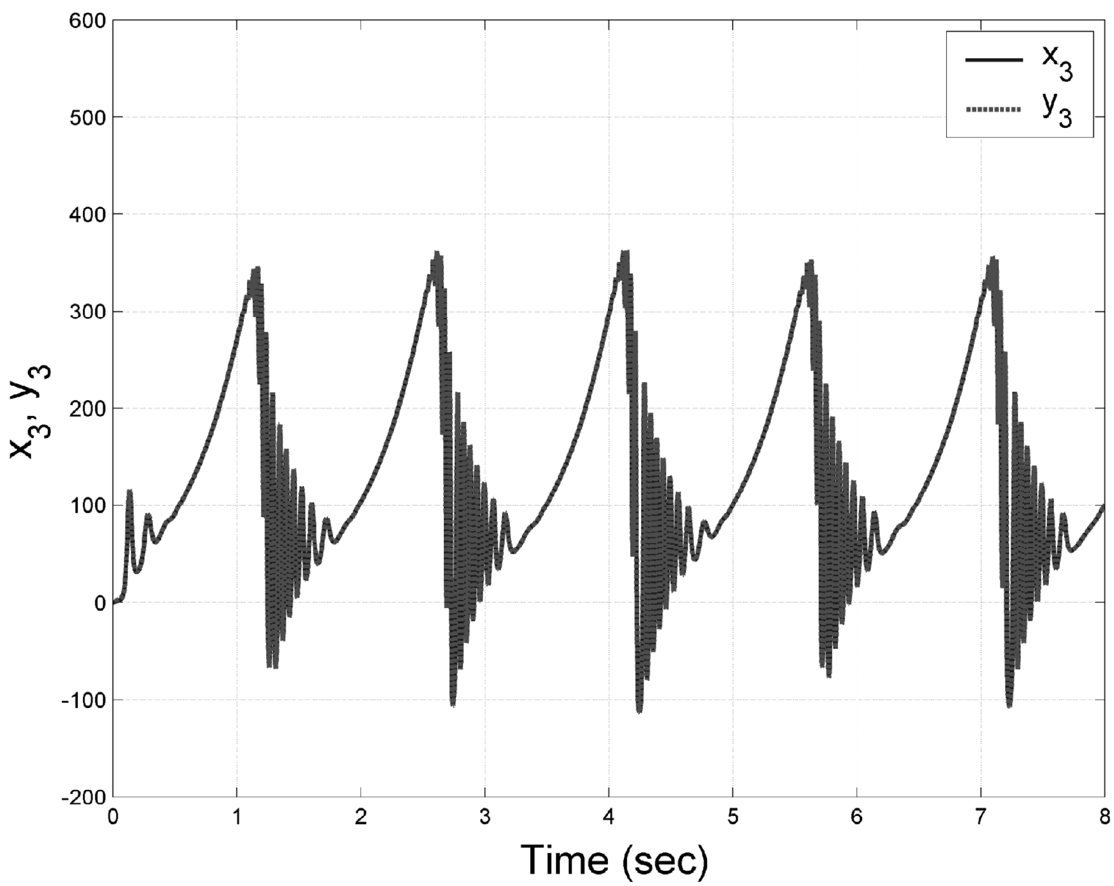

Figure 9: Complete synchronization of the states x_3 and y_3

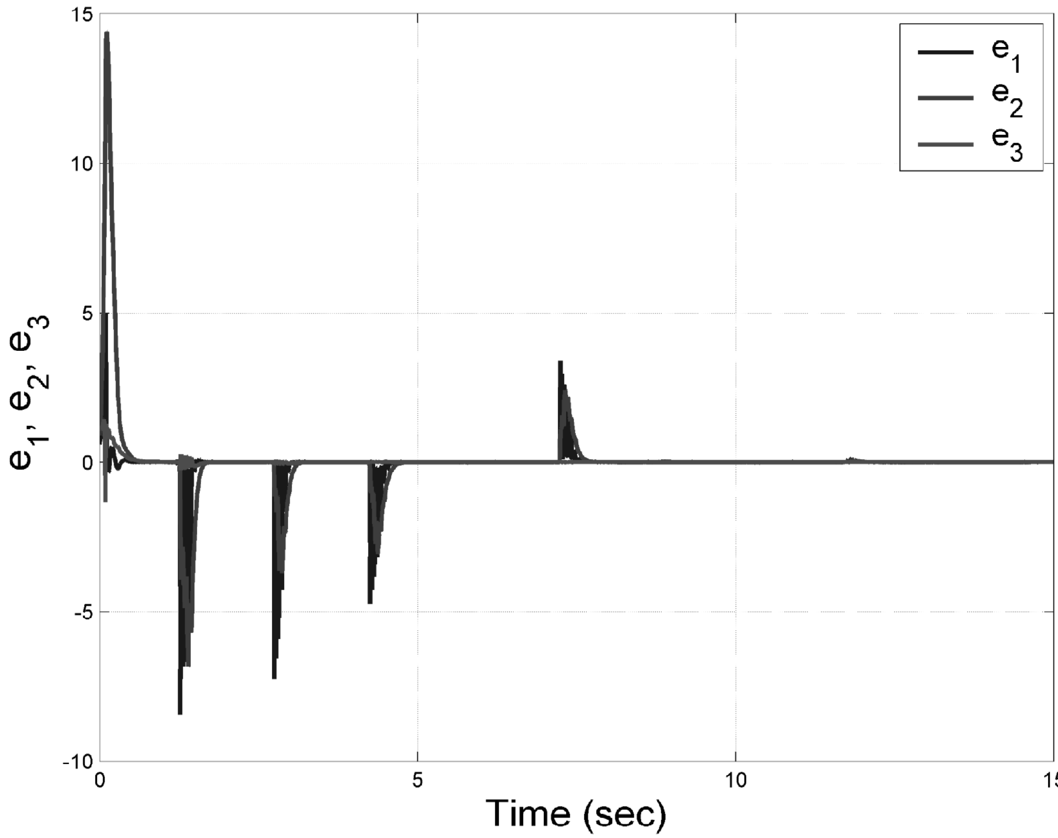

Figure 10: Time history of the chaos synchronization errors e_1, e_2, e_3

6. CONCLUSIONS

In this paper, we have derived a ten-term novel 3-D three-scroll chaotic system with four quadratic nonlinearities. We gave a qualitative analysis of the mathematical properties of the novel 3-D chaotic system. We determined the Lyapunov exponents and Lyapunov dimension of the three-scroll chaotic system. Next, we have derived adaptive control and synchronization results for the novel three-scroll chaotic system with unknown parameters, which have been established using Lyapunov stability theory. Numerical simulations with MATLAB were exhibited to demonstrate the phase portraits of the novel three-scroll chaotic system and the adaptive results derived in this paper.

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