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Unsteady Forced Oscillations of a Fluid Bounded by Rigid Bottom

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ABSTRACT

An unsteady flow of a second order thermo-viscous incompressible forced oscillations of a fluid bounded by rigid bottom is studied in this paper. The solutions for the velocity and temperature of the fluid have been obtained analytically with appropriate boundary conditions and the results are discussed with help of illustrations. In this work it is mentioned that the external forces which generated are perpendicular to the flow direction and this is the special feature of the thermo-viscous fluid.

Key Words: Thermo-viscous fluid, thermo-stress-viscosity coefficient, prandantl number.

I. INTRODUCTION

The study of second order thermo-viscous fluids is an extensive study over several decades, such fluids are special class of non-Newtonian fluids. The applications of thermo-viscous fluids are found in the areas like Space and energy systems, Geo-physics, petroleum industry, Astrophysics, Bio-physics, Soil sciences, Biomechanics, Biomedicines, Biomedical Engineering, Agricultural sciences, Artificial dialysis and so on. Some practical problems involving such studies include the extraction and filtration of oil from wells, the percolation of water through solids, the drainage of water for irrigation, the consideration of aquifier by the ground water hydrologists, the reserve bed used for filtering drinking of water, the oil reservoirs treated by the reservoir of engineers and the seepage through slurries by the sanitary engineer, Most of the chemical processes are employed by Filters and filter beds etc. The study of thermo-viscous fluids is because of its natural occurrence and its importance in industrial geophysical and medical applications. The extraction of energy from rocks and geo-thermal regions, the solids filtration from liquids, the liquids flow through ion-exchange beds, cleaning of oil-spills are some of the areas in which thermo-viscous fluids are noticed.

In 1963, the concept of thermo-viscous fluids which reflect the interaction between thermal and mechanical responses in fluids in motion due to external influences was introduced by Koh and Eringin[6]. Kelly[5] in 1965 examined some simple shear flows of non- Newtonian second order thermo-viscous fluids . Nageswara Rao and Pattabhi Ramacharyulu[9] later studied some steady state problems of certain flows dealing with thermo-viscous fluids in 1979. Green and Naghdi[4] has given A new theory on thermo-viscous fluids. Some more problems of thermo-viscous flows studied by Anuradha[1] and Nagaratnam[8] in plane, cylindrical and spherical geometries in 2006. In the present paper we study the normal stress in y-direction is the Reiner-Rivlin effect attributed to the cross-viscosity coefficient μ_c. Further the transverse force in the z-direction can be attributed to the coupling parameter thermo-stress-viscosity coefficient α_8.

II. BASIC EQUATIONS

As proposed by Koh and Eringin (1963) the stress-tensor 't' and heat flux bivector 'h' are expressed as polynomial functions, viz., the rate of deformation tensor 'd':

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$$t = \alpha_1 I + \alpha_3 d + \alpha_5 d^2 + \alpha_6 b^2 + \alpha_8 (db - bd) \text{ and } h = \beta_1 b + \beta_3 (bd + db)$$

with

$$d_{ij} = (u_{i,j} + u_{j,i})/2$$

and thermal by gradient bivector 'b'

$$b_{ij} = \in_{ijk} \ \theta_k$$

where u_i is the i^{th} component of fluid velocity and θ is the fluid temperature. The constitutive parameters α_i, β_i being polynomials in terms of d and b in which the coefficients depend on fluid density (ρ) and the temperature (θ). The fluid is called Stokesian fluid if the stress tensor depends on the rate of deformation tensor 'd' and it is called Fourier-heat-conducting fluid when the heat flux bi-vector depends on the temperature gradient, the coefficients α_1 and α_3 may be identified as the fluid pressure and coefficient viscosity coefficient respectively and α_5 as that of cross-viscosity coefficient.

The flow of incompressible thermo-viscous fluids in general satisfies the usual following conservation laws (equations):

Law of conservation of mass (equation of continuity):

$$v_{i,i} = 0$$

Law of conservation of momentum(equation of momentum):

$$\rho\left[\frac{\partial v_i}{\partial t} + v_k v_{i,k}\right] = \rho F_k + t_{ji,i}$$

and the energy equation(Law of Conservation of energy)

$$\rho c \dot{\theta} = t_{ij} d_{ij} - q_{i,i} + \rho\gamma$$

where

where F_k, the k^{th} Component of external force per unit mass, c is the Specific heat, γ is thermal energy source per unit mass, q_i is the i^{th} Component of heat flux bi-vector t_{ij} is the components of stress tensor and d_{ij} is the components of rate of deformation tensor.

III. MATHEMATICAL FORMULATION AND SOLUTION

Let the second order thermo-viscous incompressible fluid of finite depth h bounded by the rigid bottom $y = 0$ be influenced by the external force $F_0 \operatorname{Re}.\exp(i\sigma t)$ in the X-direction. The fluid, otherwise at rest, responds to the fluctuations of the bottom. The periods of oscillation of the fluid response and the temperature distribution are assumed to be oscillatory with the same frequency. Consider the Cartesian coordinate system O(X,Y,Z) with the origin on oscillating plate, the X-axis is considered in the direction of the fluid flow and Y-axis is perpendicular to the plates.

Consider the unsteady flow be characterized by the velocity field [$u(y,t)$, 0, 0] and temperature field (y,t).This choice of assumption of velocity usually satisfies the law of conservation of mass i.e. continuity equation and in the absence of pressure and temperature gradients and in the absence of the internal heat energy source, the equation of momentum reduces to

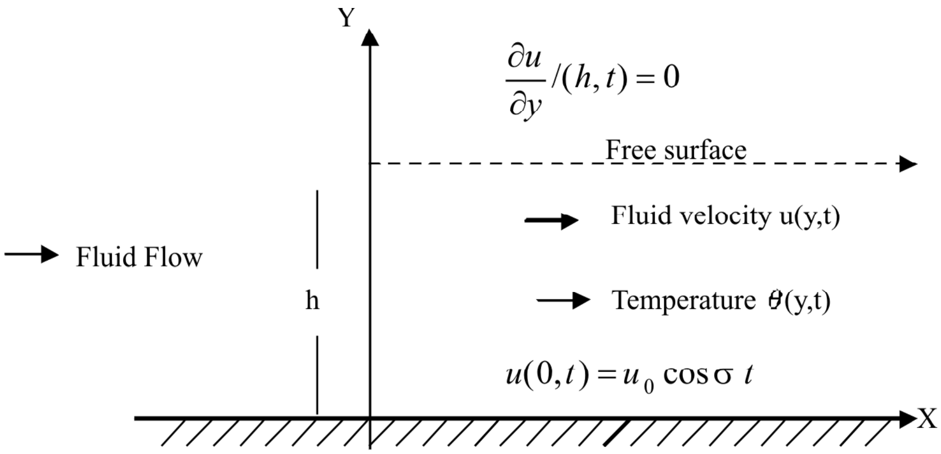

Figure 1: Physical Model of the Problem

in the X-direction:

$$\rho\frac{\partial u}{\partial t}=\mu\frac{\partial^2 u}{\partial y^2}+\rho F_0 e^{i\sigma t} \tag{1}$$

in the Y-direction :

$$0=\mu_c\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^2+\rho F_y \tag{2}$$

in the Z- direction :

$$0=\alpha_8\frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y}\right)+\rho F_z \tag{3}$$

and the energy equation

$$\rho c\frac{\partial \theta}{\partial t}=\mu\left(\frac{\partial u}{\partial y}\right)^2+k\frac{\partial^2\theta}{\partial y^2} \tag{4}$$

with the no slip condition on the bottom $u(0,t)=0$ (5)

and the free surface condition

$$\frac{\partial u}{\partial y}/(h,t)=0 \tag{6}$$

The boundary conditions for temperature are

$$\theta(0,t)=\text{Re}\,\theta_0\exp(i\sigma t) \tag{7}$$

$$\theta(h,t)=0 \tag{8}$$

Let the velocity distribution be assumed in the form

$$u(y,t)=\text{Re}.f(y)\exp(i\sigma t) \tag{9}$$

Substituting (9) in (1) and using the boundary conditions (5) and (6), the velocity distribution is obtained as

$$u(y,t)=\mathrm{Re}.\frac{F_0}{\sigma}i\left[\frac{\cosh m(h-y)}{\cosh(mh)}-1\right]e^{i\sigma t} \tag{10}$$

$$=\frac{F_0}{\sigma}[(1-P_1(y))\sin\sigma t-H_1(y)\cos\sigma] \tag{11}$$

Where $m=\delta(1+i)$

here $$P_1(y)=\frac{\cos\delta(2h-y)\cosh(\delta y)+\cos(\delta y)\cosh\delta(2h-y)}{\cosh 2\delta h+\cos 2\delta h}$$

$$Q_1(y)=\frac{\sin\delta(2h-y)\sinh\delta y-\sin\delta y\sinh\delta(2h-y)}{\cosh 2\delta h+\cos 2\delta h}$$

From the equations (4),(7), (8) and (10) we obtain the temperature distribution as

$$\theta(y,t)=\mathrm{Re}.\theta_0\frac{\sinh\sqrt{pr}\,\delta(1+i)(h-y)}{\sinh\sqrt{pr}\,\delta(1+i)h} \tag{12}$$

$$=\theta_0[P_2(y)\cos\sigma t-H_2(y)\sin\sigma t]$$

with $p_r=\dfrac{\mu c}{k}$ is the prandtl number.

$$P_2(y)=\frac{\cos(\delta\sqrt{p_r}\,y)\cosh(\delta\sqrt{p_r}\,(2h-y))-\cosh(\delta\sqrt{p_r}\,y)\cos(\delta\sqrt{p_r}\,(2h-y))}{\cosh 2\delta\sqrt{p_r}\,h-\cos 2\delta\sqrt{p_r}\,h}$$

$$Q_2(y)=\frac{\sinh(\delta\sqrt{p_r}\,y)\sin(\delta\sqrt{p_r}\,(2h-y))-\sin(\delta\sqrt{p_r}\,y)\sinh(\delta\sqrt{p_r}\,(2h-y))}{\cosh 2\delta\sqrt{p_r}\,h-\cos 2\delta\sqrt{p_r}\,h}$$

IV. DISCUSSION OF THE RESULTS

The present investigation is on an unsteady flow of a second order non-Newtonian thermo-viscous incompressible forced oscillations of a fluid bounded by rigid bottom . It is noticed from the equations (2) and (3) that, the forces generated in the transverse direction is a special feature of thermo-viscous fluid flows. The normal stress given by the equation (2) is the Reiner-Rivlin effect attributed to the cross-viscosity coefficient μ_c. Further the transverse force ρF_z given by the equation (3) can be attributed to the thermo-viscous nature nature of the fluid characterized by the coupling parameter thermo-stress-viscosity coefficient α_8 and this is independent of thermo-stress-coefficient α_6. These effects are not felt in the case of classical viscous fluids i.e. when the fluid is Newtonian-viscous and Fourier-heat conducting . It can also be noted that the fluid velocity is also fluctuating with the same period ($=\dfrac{2\pi}{\sigma}$) and an exponentially decreasing amplitude with the characteristic distance $\dfrac{1}{\delta}=\sqrt{\dfrac{\mu}{\rho\sigma}}$ and this can be taken as the thickness of the boundary layer.

The following Illustrations are generated with the help of MATLAB Code for fixing the values $\mu = 1$, $\upsilon = 1$ $p_r = 1$ and $c = 1$. The velocity profile variations with the various values of time parameter(t) are illustrated in figures 2 and 3. The temperature profile variations with the various values of time parameter(t) are illustrated in figures 4 and 5. Figure 2 shows that the velocity of the fluid is oscillating with the period 2π and Figure 4 shows that the velocity is fluctuating with the period π.

From the Figure 2, it is noted that the velocity fluctuations moving towards the centre from upwards as the values of the time parameter increases from $t = 0$ to $t = \frac{\pi}{2}$. It is also found that the velocity fluctuations moving towards the centre from downwards as the values of the time parameter increases from $t = \pi$ to $t = \frac{3\pi}{2}$. Figure 3 depicts that, as the frequency of oscillations (α) increases from 1 to 2 the velocity oscillations decreases and moves towards the centre for the values increases from $t = 0$ to $t = \frac{\pi}{4}$ and shifting towards the centre for the values increases from $t = \frac{\pi}{2}$ to $t = \frac{3\pi}{4}$.

Figure 4 and Figure 5 depicts that the temperature distributions moving towards centre for the values increases from $t = 0$ to $t = \frac{\pi}{2}$ and $t = 0$ to $t = \frac{\pi}{4}$. It is also notified that the temperature distributions shifting towards centre from down for the values increases from $t = \pi$ to $t = \frac{3\pi}{2}$ and $t = \frac{\pi}{2}$ to $t = \frac{3\pi}{4}$.

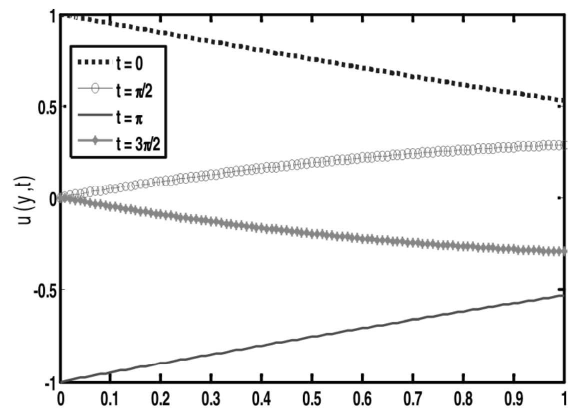

Figure 2: Values of y versus Velocity profiles with σ = 1

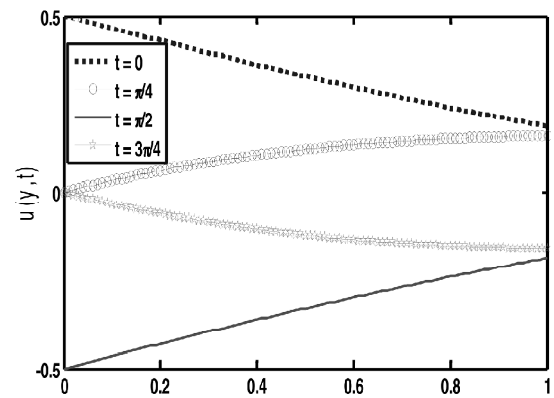

Figure 3: Values of y versus Velocity profiles with σ = 2

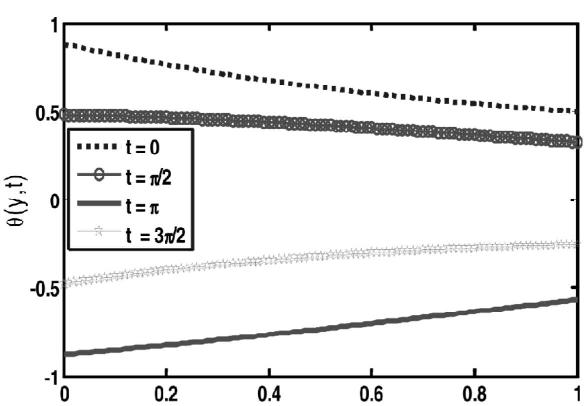

Figure 4: Values of y versus Temp profiles with σ = 1 and p_r = 1

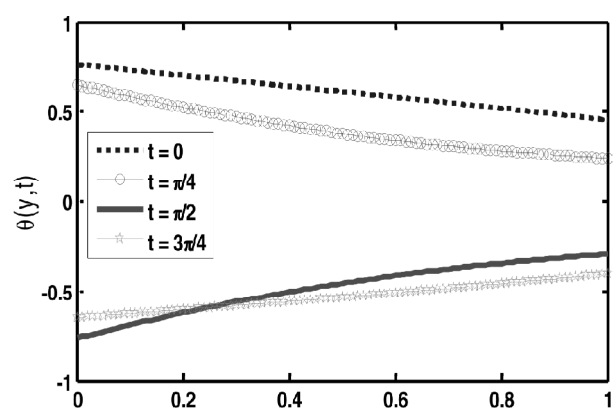

Figure 5: Values of y versus Temp profiles with σ = 2 and p_r = 1

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