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Circular Rydberg States of Hydrogenlike Systems in Collinear Electric and Magnetic Fields of Arbitrary Strengths Immersed in a Plasma

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ABSTRACT: We consider classical circular Rydberg states of a hydrogenic atom/ion under collinear electric and magnetic fields of arbitrary strengths, the entire system being immersed into a plasma with the Debye screening. We show in detail how the screening decreases the value of the critical electric field required for the ionization at different values of the magnetic field. Our results should have a fundamental importance because hydrogenic atoms/ions under external fields remain a test-bench for atomic physics. Also our results could motivate experiments on the magnetic control of the "continuum lowering" in cold Rydberg plasmas, this being of practical importance.

***Keywords*:** circular Rydberg states; hydrogenic systems in collinear electric and magnetic fields; Debye screening; continuum lowering

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1. INTRODUCTION

In paper [1] there was a study of a hydrogen-like system with the stationary nucleus of charge Z at the origin subjected to collinear electric (**F**) and magnetic (**B**) fields, the z-axis being chosen along the direction of **F** ($F_z > 0$). The Circular Rydberg States (CRS) of the electron were considered: the orbit, whose plane is perpendicular to Oz, has radius ρ and its center is on Oz at some point z.

In paper [1] the author derived analytical expressions for the energy E of classical CRS of hydrogenlike systems in collinear electric (**F**) and magnetic (**B**) fields of arbitrary strengths. He also offered formulas for the dependence of the classical ionization threshold $F_c(B)$ and of the energy at this threshold $E_c(B)$ valid for the magnetic field B of an arbitrary strength. In addition, for two important particular cases – classical CRS in a magnetic field only and classical CRS in an electric field only – some new results were presented in paper [1] as well.

In the present study, the same configuration as in paper [1] is considered to be submerged into a plasma with the Debye screening. We show in detail how the Debye screening decreases the critical value of the electric field required for ionization at various values of the magnetic field. Both the electric and magnetic fields are considered to be of arbitrary strengths, as in paper [1].

2. RESULTS

For a hydrogen atom or a hydrogen-like ion, the allowance for the Debye screening is effected by replacing the Coulomb potential with the screened Coulomb potential:

$$\frac{Z}{r} \rightarrow \frac{Z}{r} e^{-r/a} \qquad (1)$$

where a is the parameter of the plasma called the screening length (atomic units are used in this study: $\hbar = e = m_e = 1$). In the cylindrical coordinates, the classical Hamiltonian from paper [1] can be rewritten in the form

$$H(\rho, z) = \frac{L^2}{2\rho^2} - \frac{Z}{\sqrt{\rho^2 + z^2}} e^{-\sqrt{\rho^2+z^2}/a} + Fz + \Omega L + \frac{\Omega^2 \rho^2}{2} \tag{2}$$

where $\Omega = B/(2c)$. Here L = const, it is the z-component of the angular momentum, and Ω is the Larmor frequency. With the scaled quantities

$$v = \frac{Z}{L^2}\rho, w = \frac{Z}{L^2}z, f = \frac{L^4}{Z^3}F, \omega = \frac{L^3}{Z^2}\Omega, h = \frac{L^2}{Z^2}H, \varepsilon = \frac{L^2}{Z^2}E, \lambda = \frac{L^2}{Za} \tag{3}$$

the expression for the Hamiltonian takes the following form:

$$h = \frac{1}{2v^2} - \frac{1}{\sqrt{w^2 + v^2}} e^{-\lambda\sqrt{w^2+v^2}} + fw + \omega + \frac{\omega^2 v^2}{2} \tag{4}$$

The conditions for the dynamic equilibrium are

$$\frac{\partial h}{\partial w} = 0, \frac{\partial h}{\partial v} = 0 \tag{5}$$

From the first equation in (5) we obtain

$$f = -\frac{w(1 + \lambda\sqrt{w^2 + v^2})e^{-\lambda\sqrt{w^2+v^2}}}{(w^2 + v^2)^{3/2}} \tag{6}$$

from where we see that for $f > 0$, $w < 0$. From the second equation in (5) we obtain

$$\frac{(1 + \lambda\sqrt{w^2 + v^2})e^{-\lambda\sqrt{w^2+v^2}}}{(w^2 + v^2)^{3/2}} + \omega^2 = \frac{1}{v^4} \tag{7}$$

From (6) and (7) a more simple relation can be derived:

$$\omega^2 - \frac{f}{w} = \frac{1}{v^4} \tag{8}$$

which is independent of λ. From here we obtain the value $w(v, f, \omega)$ corresponding to the equilibrium:

$$w(v, f, \omega) = \frac{f}{\omega^2 - \frac{1}{v^4}} \tag{9}$$

Then we substitute this equilibrium value of w into (6) and obtain an implicit equation for $f(v, \omega)$, which can be presented in the following form:

$$e^{-\lambda k}(1 + \lambda k) = -(\omega^2 - \frac{1}{v^4})k^3 \tag{10}$$

where

$$k = \sqrt{v^2 + \left(\frac{f}{\omega^2 - \frac{1}{v^4}}\right)^2} \tag{11}$$

We numerically solve this equation and obtain the value $f(v, \omega, \lambda)$. Then we substitute this value into w in (9), getting $w(v, \omega, \lambda)$ and then substitute the resulting values of f and w into (4), obtaining the value of energy $\varepsilon(v, \omega, \lambda)$ corresponding to the equilibrium. Thus, we obtain the parametric dependence $\varepsilon(f)$ with the parameter v for the given values of ω and λ.

Figures 1 and 2 show the dependence $\varepsilon(f)$ at $\lambda = 0.3$ for $\omega = 1$ and $\omega = -1$.

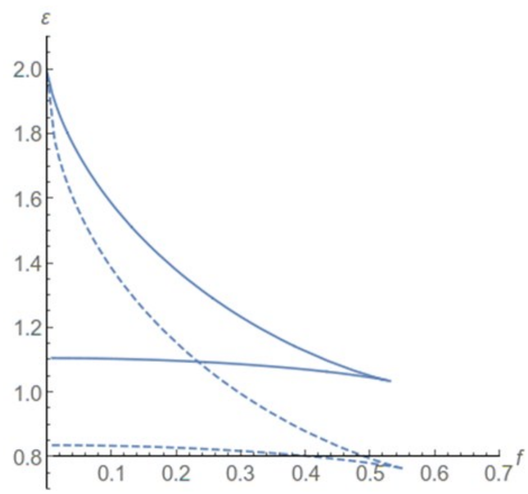

Fig. 1. The plot of the parametric dependence $\varepsilon(f)$ at $\lambda = 0.3$ for $\omega = 1$ (solid curves) compared to the same plot at $\lambda = 0$ (dashed curves).

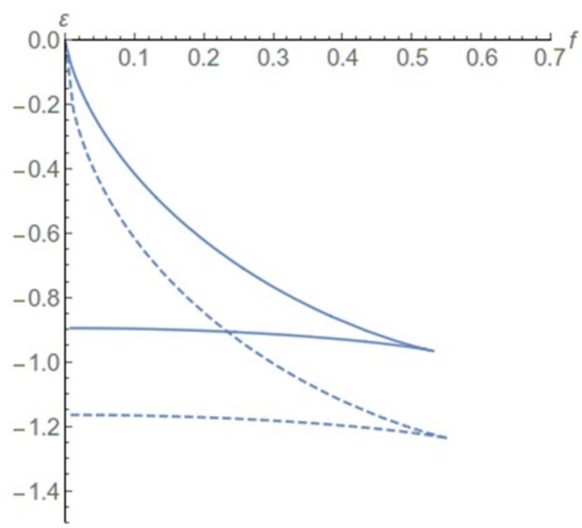

Fig. 2. The plot of the parametric dependence $\varepsilon(f)$ at $\lambda = 0.3$ for $\omega = -1$ (solid curves) compared to the same plot at $\lambda = 0$ (dashed curves).

The plots obtained for $\lambda = 0$ coincide with the corresponding plots in [1]. From the plots it is seen that the screening decreases the critical value of f required for ionization, because the screening lowers the continuum. To observe this effect more clearly, we made the similar plot, shown in Figure 3, for the case of $\omega = 0$ to remove the stabilization effect of the magnetic field.

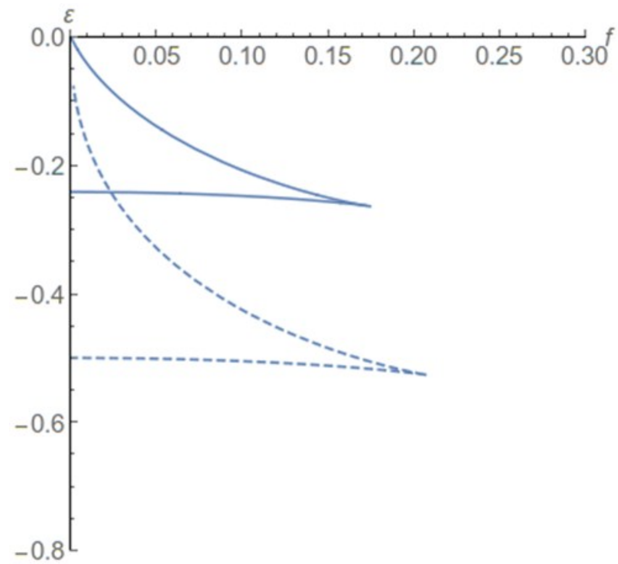

Fig. 3. The plot of the parametric dependence ε(*f*) at λ = 0.3 for ω = 0 (solid curves) compared to the same plot at λ = 0 (dashed curves).

We can see that the screening decreases the critical value of the electric field by a greater value in the case of ω = 0.

Next, we follow the procedure given in [1] to find the dependence of the critical value of *f*, f_c, on the scaled magnetic field ω . We can use either $\partial \varepsilon/\partial v = 0$ or $\partial f/\partial v = 0$ to find the scaled radius of the orbit corresponding to f_c; we use the first equation here, which yields

$$\frac{4}{v^5}(k^2 - v^2) = (\omega^2 - \frac{1}{v^4})(v - k\frac{dk}{dv}|_c) \tag{12}$$

where *k* is the solution of (10), and $(dk/dv)|_c$ can be found by differentiating (10) with respect to *v*:

$$\frac{dk}{dv}|_c = -\frac{4k^3}{v^5(\lambda e^{-\lambda k} - \lambda k(1+\lambda k) + 3k^2(\omega^2 - \frac{1}{v^4}))} \tag{13}$$

Substituting (13) into (12) with *k* being the numerical solution of (10), we solve the resulting equation numerically for *v* to obtain the value $v_c(\omega, \lambda)$ corresponding to the ionization threshold. Substituting it further into the expression for the electric field $f(v, \omega, \lambda)$, we obtain the dependence $f_c(\omega, \lambda)$ of the critical electric field on the scaled magnetic field ω for a given value of the screening parameter λ.

Figure 4 shows the dependence of the critical electric field f_c on the scaled magnetic field ω for the values of λ = 0, 0.4 and 0.6. It is seen that the screening decreases the value of the critical electric field.

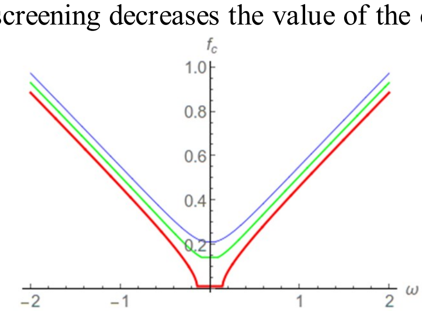

Fig. 4. The plot of the dependence $f_c(\omega)$ at λ = 0 (blue, thin top curve), λ = 0.4 (green, thicker middle curve) and λ = 0.6 (red, thick bottom curve).

We also made the plot of $|\omega|(f_c)$, shown below.

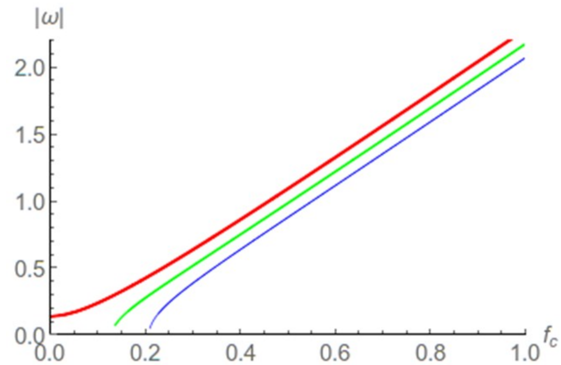

Fig. 5. The plot of the dependence $|\omega|(f_c)$ at $\lambda = 0$ (blue, thin bottom curve), $\lambda = 0.4$ (green, thicker middle curve) and $\lambda = 0.6$ (red, thick top curve).

The plot in Fig. 4, as well as our corresponding calculations, can be used for finding the critical electric field for the ionization at different values of the magnetic field and of the screening parameter. The plot in Fig. 5, as well as our corresponding calculations, can be used for finding the magnetic field at different values of the critical electric field for the ionization and of the screening parameter.

3. CONCLUSIONS

We considered classical CRS of a hydrogenic atom/ion under collinear electric and magnetic fields of arbitrary strengths, the entire system being immersed into a plasma with the Debye screening. We showed in detail how the screening decreases the value of the critical electric field required for the ionization at different values of the magnetic field.

Our results can be used for finding the critical electric field for the ionization at different values of the magnetic field and of the screening parameter. Alternatively, they can be used for finding the magnetic field at different values of the critical electric field for the ionization and of the screening parameter.

Our results should have a fundamental importance because hydrogenic atoms/ions under external fields remain a test-bed for atomic physics. As for the practical importance, it should be due to the fact that the results could motivate experiments on the magnetic control of the “continuum lowering” in cold Rydberg plasmas.

References

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