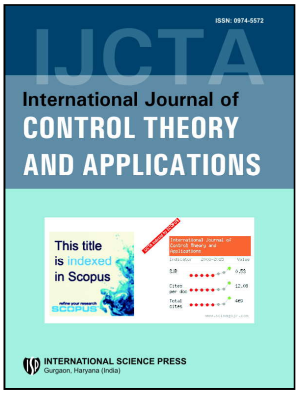

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Real Time Control of Inverted Pendulum using Optimized Fuzzy Logic Controller Based on Maximum Entropy

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Abstract: This paper proposes an algorithm to optimize a fuzzy variable based on fuzzy entropy function.Proposed algorithm uses predefined membership functionswhich are evenly distributed over the domain of a fuzzy variable. The predefined fuzzy sets are than displaced by the standard deviation obtained through real-time analysis of the system under study. Entropy for individual displaced fuzzy set is maximized which is subjected to: maximum combined entropy, thus forming the objective function which isoptimized using genetic algorithms.Being a benchmark control problem real time control of inverted pendulum angle stabilization has been chosen to evaluate the performance of the proposed algorithm. Theobtained results indicate an improvement in the stability and transient parameters for the proposed controller.

Keywords: Fuzzy entropy, membership functionoptimization, support optimization, genetic algorithms.

1. INTRODUCTION

Fuzzy logic controllers (FLC) are advantageous in control of systems where control signal needs to be generated with vague/noisy measurement data[1]. Fuzzy logic based systems are beneficial in a wide range of applications which has already been tested and proved by multiple sources[2][3]. Choosing right membership function has been amongst the most explored area for optimizing the performance of fuzzy logic controller. However majority of the literature is concentrated on choosing right shape of membership function and not on the support of membership function [4],[5],[6],[7]. For instance:[8] used the notion of s-function for defining membership function (MF) and maximizing fuzzy entropy corresponding to the MF. The algorithm proposed had been tested for image processing applications; however the same can be applied in other applications as well.[9] proposed tuning of a fuzzy type PID controller using particle swarm optimization. The proposed algorithm was implemented to control an electrical DC drive system; simulation and experimental results indicated better efficiency and robustness of the proposed controller.[10] proposed optimization of fuzzy system using cross-mutated operation in particle swarm optimization (PSO) technique. The robustness of algorithm is tested on two systems that is; I-economic load dipatch system and II- self-provisioningsystemsforcommunicationnetworkservices. Results indicate more efficient system and better robustness as compared to hybrid PSO technique.Numerous control strategies have been applied to control IP ; FLC being one of them. Some of the recent works include [11] which uses an adapvtive FLC based on feedback linearization utilizing fuzzy disturbance observer to ensure asymptotic stability.[12] proposed an optimized FLC for control of an IP which is based on minimization of an objective function based on mean square error. Performance comparison shows improved stability for the system. [13] proposed a method to analyze stability of FLCs using Lyapunov's direct method, the method was used to stabilize IP system.[14]evaluated the applicability of FLC to control non-linear by treating the system as a black box system; without considering system dynamics. The results indicate the effectiveness of FLC to control non-linear IP.

As a common approach, solving for a control theory problem the first step consists of acquiring dynamic models of the system under study, followinganalysis of these models a control strategy/law is designed to achieve the desired predefined control objectives. Generally the dynamical systems considered under industrial applications such as power industry, aerospace industry, robotic industry, etc. are highly nonlinear in nature[2]. Designing a control system for such class of systems is often a challenging task. Control of inverted pendulum (IP) is usually considered as a benchmark problem amongst researchers to demonstrate performance of new control strategies for non-linear systems.IP is a system with its centre of mass located at a position above its pivot point, formulated by Minorsky[15], the first solution of which was given by Roberge[16]. To achieve an inverted position the pivot point is mounted on a movable cart with restricted motion in horizontal axis; this system is also commonly known as "a cart and pole" arrangement. A normal pendulum's stable position is: downwards hanging position, an IP is intrinsically unstable; the carts motion provides the external torque needed to balance the pendulum in an upright position. This system can be implemented in various ways; one such system is demonstration of altitude control problems with control objective to retain an anticipated vertically oriented position at all times[17][18].

This paper presents application of a new optimization algorithm for designing FLC which is tetsted for real-time control of inverted pendulum and performance indices for proposed FLC are compared to non-optimized FLC and PID controllers. The paper is arranged as follows: in section 2 real-time digital pendulum model is discussed. Section 3 defines FLC developed for the system. In section 4 objective function is defined and optimization technique us discussed. In section 5 the algorithm is tested on a real-time system and results are discussed. Finally section 6 concludes the propsoed methodology.

2. REAL-TIME DIGITAL CONTROL OF INVERTED PENDULUM MODEL

The figure of digitalpendulum withcart system is as illustrated in Figure 1.

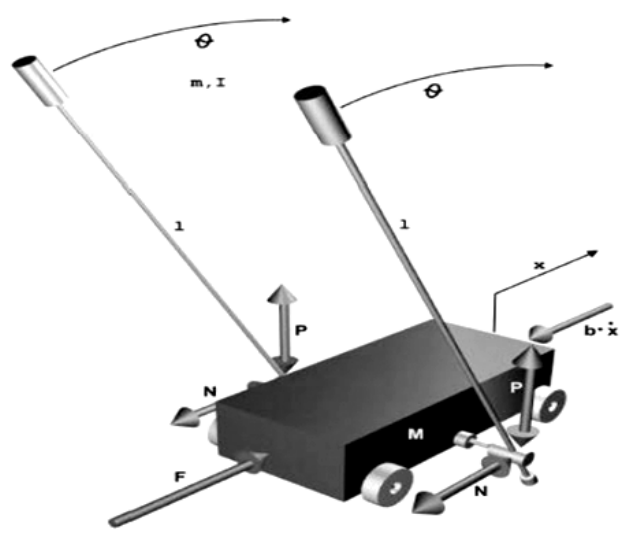

Figure 1: Cart driven inverted pendulum[19]

Mathematically forces acting on the system can be written as follows[19]:

$$F = (m + M)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \sin\theta \quad (1)$$

$$(I + ml^2)\ddot{\theta} - mglsin\theta + ml\ddot{x}cos\theta + d\dot{\theta} = 0 \quad (2)$$

To design a PID controller linear model of the system is required. The equations are thus linearized around the inverted position i.e. θ = 0. Here we get:

$$F = (m + M)\ddot{x} + b\dot{x} - ml\ddot{\theta} \quad (3)$$

$$(I + ml^2)\ddot{\theta} + mgl\theta - ml\ddot{x} + d\dot{\theta} = 0 \quad (4)$$

Table 1 presents the parameters for the real time model used for experiment. The values specified in last column indicate the specification for "Feedback instruments™ digital pendulum system: 33-936S"[19].

Table 1
Parameters for real-time model

Parameter	*Description*	*Value*
M	Cart mass	2.4 Kg
m	Pendulum mass	0.23 Kg
l	Pendulum length	0.4 m
g	acceleration due to gravity	9.81 m/s^2
F	force applied by the motor	Variable
x	distance moved by the cart	Variable
b	coefficient of friction between cart and rail	0.05 Ns/m
θ	angle of pendulum (with respect to positive y – axis which is considered as 0° angle)	Variable
$\dot{\theta}$	Pendulumangular velocity	Variable
$\ddot{\theta}$	pendulum angular acceleration	Variable

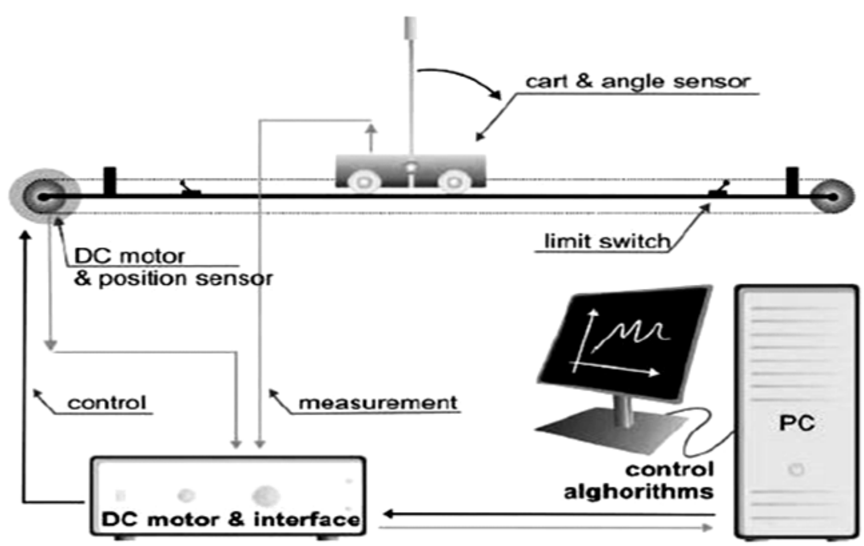

Figure 2: Digital Control for Inverted Pendulum system[19]

Figure 2 highlights the main components of the physical system being used, and figure 3 illustrates the controller block diagram for the same. The PC is equipped with a data acquisition card which acts as an interface between the digital PC and the analog pendulum system. The digital control voltage generated from PC via Matlab – Simulink® is translated to an analog level of ±5 volts by the DAQ (data acquisition) card, which is converted to ±24 volts for motor operation by DC motor interface. The position of the cart and angle of the pendulum are measured with the help of two encoders. One attached to the DC motor and latter to the cart-pendulum for respective angular measurements. These encoders have an analog to digital converter associated with them which converts the analog values from encoder to digital values and communicates to PC via DAQ card.

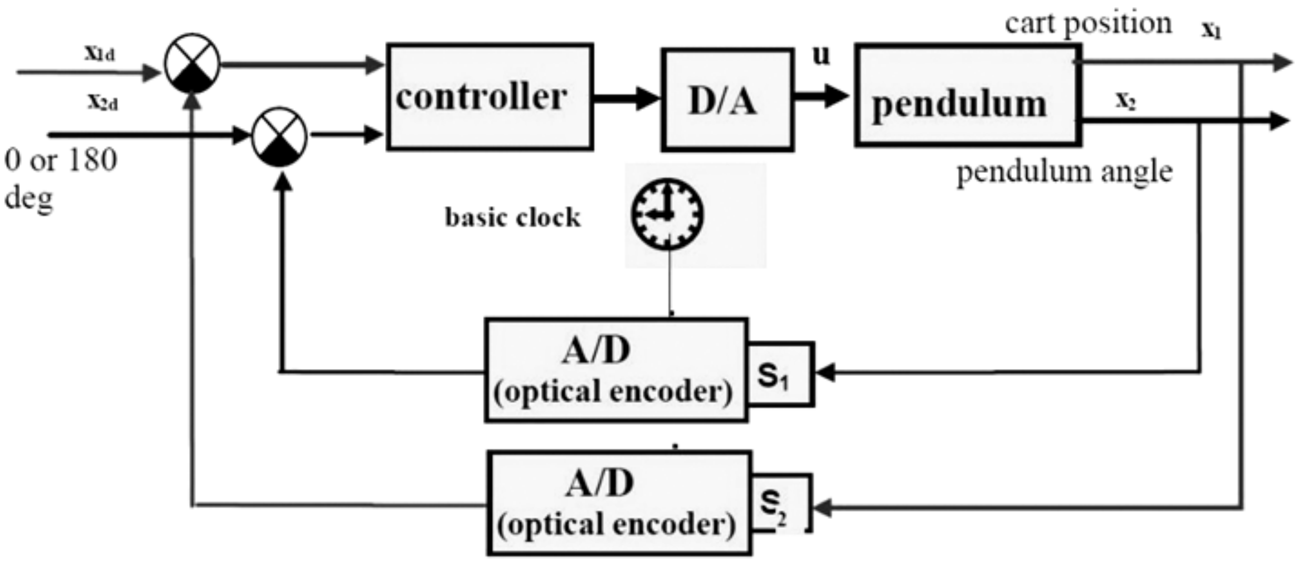

Figure 3: Digital control block diagram[19]

3. FUZZY LOGIC CONTROLLER

For designing a FLC precise mathematical model of the system is not mandatory, however expert knowledge of the system under study is required, as FLCs are primarily inspired from the decision making process of human beings. These are capable of inferring complex nonlinear relationships between input and output variables[20].FLC addresses the imprecision or vagueness in input–output descriptions of systems using FS.

3.1. Predefined membership functions

For control applications FS are commonly named with respect to their relevant position with reference to error. As for any control system the desired error is always 'zero', hence FS associated around 'zero error' is named as 'ZE'. Moving on to positive x-axis the FS which are associated with positive error are named as: 'PS', 'PM', and 'PB' respectively. Moving on to negative x-axis FS which are associated with negative error are named as:'NS', 'NM', AND 'NB' respectively[21]. Initially the MFs are distributed uniformly around the universe of discourse and triangular fuzzy MFs are chosen to lower the computation requirement of the system.

General equation for a normal triangular MF can be written as follows:

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ (x-a)/(b-1), & a < x \le b \\ (x-c)/(b-c), & b < x < c \\ 0, & x \ge c \end{cases} \tag{5}$$

Proposed FLC is a two input one output system; having (i) error in angle ($\Delta\theta$), and (ii) rate of change for error in angle ($d(\Delta\theta)/dt$) as input variables and (iii) control signal (u) for DC motor, acting as an output variable

FS for error in pendulum angle is given in figure 4. Here seven overlapping normal (i.e. μ_{max}=1) triangular FS are defined which are distributed uniformly across the universe of discourse.For example equation for set zero "ZE" can be written as:

$$\mu_{ZE}(x)=\begin{cases}0, & x\le-\varepsilon\\(x+\varepsilon)/\varepsilon, & -\varepsilon<x\le 0\\(\varepsilon-x)/\varepsilon, & 0<x<\varepsilon\\0, & x\ge\varepsilon\end{cases} \tag{6}$$

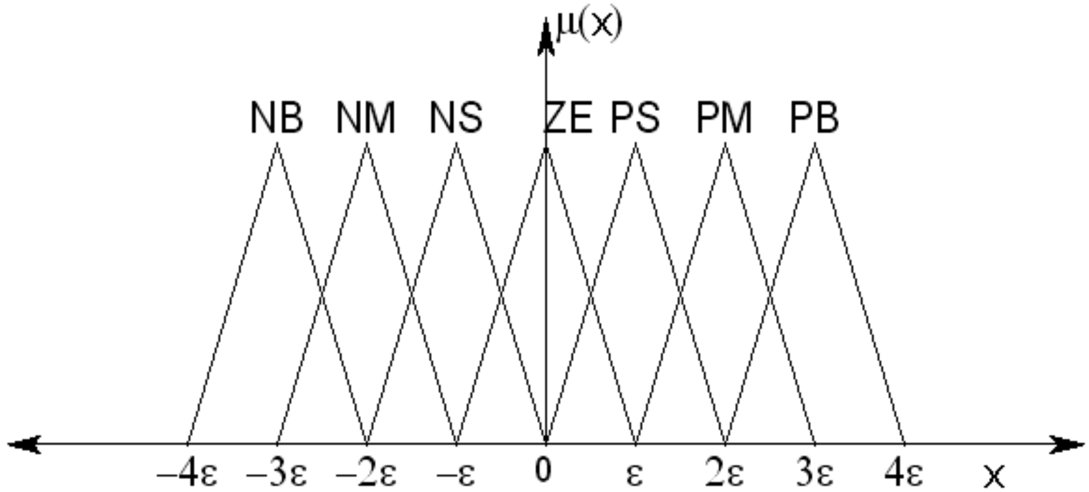

Figure 4: Predefined membership function

3.2. Rule base

The controller architecture designed for IPconsists of one proportional-derivative (PD) type FLC designed to control cart for driving pendulum to erected position. 49 rules are formed each for error in pendulum angle, rate of change for error in pendulum angle(input sets) and control force (output set).These rules are summarized in the table below.

Table 2
Rule base for pendulum angle controller using Mamdani FIS

Control force to DC motor		*Rate of change of error ($\dot{e}$)*						
		NB	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*
Error in angle (e)	NB	NB	NB	NB	NB	NM	NS	ZE
	NM	NB	NB	NB	NM	NS	ZE	PS
	NS	NB	NB	NM	NS	ZE	PS	PM
	ZE	NB	NM	NS	ZE	PS	PM	PB
	PS	NM	NS	ZE	PS	PM	PB	PB
	PM	NS	ZE	PS	PM	PB	PB	PB
	Pb	ZE	PS	PM	PB	PB	PB	PB

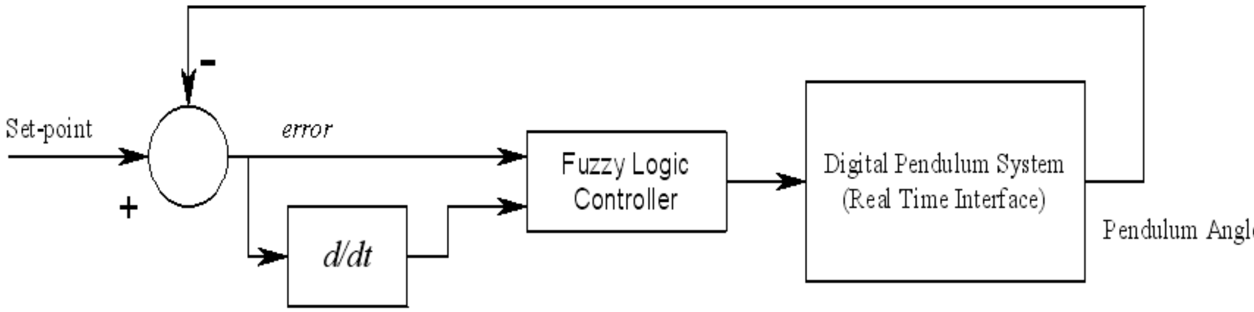

Figure 5: FLC architecture for digital pendulum

The architecture of fuzzy logic controller is illustrated in figure 5. The FLC used here is of PD type.

4. OBJECTIVE FUNCTION AND OPTIMIZATION TECHNIQUE

The proposed optimization principle is based on maximization of entropy of MFdisplaced by standard deviation data obtained from the system.

Zadeh[22] formulated mathematical expression for probability measures of FS according to which fuzzy entropy can be written as:

$$H(A) = -\int_{-\infty}^{\infty}\{\mu_i \log \mu_i + (1-\mu_i)\log(1-\mu_i)\} \tag{7}$$

For a triangular MF fuzzy entropy can be calculated as:

$$H(\mu_A) = \int_a^b f\left(\frac{x-a}{b-a}\right)dx + \int_b^c f\left(\frac{x-c}{b-c}\right)dx \tag{8}$$

For instance entropy for set ZE can be calculated using following expression:

$$H(\mu_{ZE}) = -\left[\int_{-\varepsilon}^{0}\left(\frac{x+\varepsilon}{\varepsilon}\right)ln\left(\left(\frac{x+\varepsilon}{\varepsilon}\right)\right)dx + \int_{-\varepsilon}^{0}\left(-\frac{x}{\varepsilon}\right)ln\left(-\frac{x}{\varepsilon}\right)dx\right]$$
$$-\left[\int_{0}^{\varepsilon}\left(\frac{\varepsilon-x}{\varepsilon}\right)ln\left(\left(\frac{\varepsilon-x}{\varepsilon}\right)\right)dx + \int_{0}^{\varepsilon}\left(\frac{x}{\varepsilon}\right)ln\left(\frac{x}{\varepsilon}\right)dx\right] \tag{9}$$

The method discussed in figure 6 is used to obtain the optimized FS, the same algorithm is used for optimization of FS for error in angle (‘e’), rate of change forerror in angle (‘$\dot{e}$’), and control force (u)[23]:

Total entropy of fuzzy sets:

$$H(\mu) = H(\mu_{NB}) + H(\mu_{NM}) + H(\mu_{NS}) + H(\mu_{ZE}) + H(\mu_{PS}) + H(\mu_{PM}) + H(\mu_{PB}) \tag{10}$$

$$H(\mu) = \sum_{i=1}^{n} H(\mu_i) \tag{11}$$

Objective function:

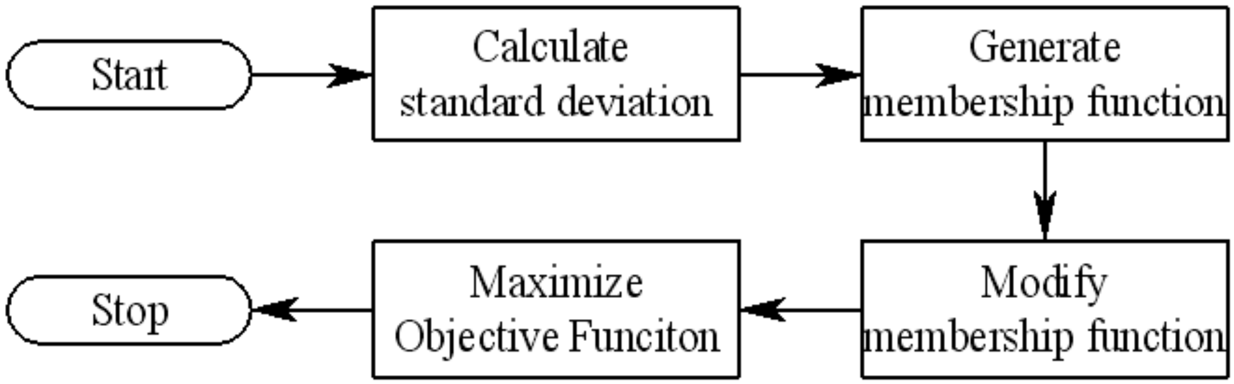

Figure 6: Process for optimization of predefined Fuzzy sets

$$\text{Maximize } H(A) = \int_a^b f\left(\frac{x-a}{b-a}\right)dx + \int_b^c f\left(\frac{x-c}{b-c}\right)dx \tag{12}$$

Subject to maximum $H(\mu) = \sum_{i=1}^{n} H(\mu_i)$

For instance objective function for set "ZE" can be written as follows:

$$Maximize\ H(\mu_{z*}) = -\left[\int_{-\varepsilon\mp\sigma}^{0}\left(\frac{x+\varepsilon\mp\sigma}{\varepsilon\mp\sigma}\right)ln\left(\frac{x+\varepsilon\mp\sigma}{\varepsilon\mp\sigma}\right)dx + \int_{-\varepsilon\mp\sigma}^{0}\left(-\frac{x}{\varepsilon\mp\varepsilon}\right)ln\left(-\frac{x}{\varepsilon\mp\varepsilon}\right)dx\right] -$$

$$\left[\int_{0}^{\varepsilon\pm\sigma}\left(\frac{\varepsilon\pm\sigma-x}{\varepsilon+\sigma}\right)ln\left(\frac{\varepsilon\pm\sigma-x}{\varepsilon+\sigma}\right)dx + \int_{0}^{\varepsilon\pm\sigma}\left(\frac{x}{\varepsilon+\sigma}\right)ln\left(\frac{x}{\varepsilon+\sigma}\right)dx\right] \tag{13}$$

Subject to maximum $H(\mu) = \sum_{i=1}^{n} H(\mu_i)$

4.1. Genetic Algorithm based Optimization of fuzzy set

Predefined FS are defined as discussed in section 3.1.as the next step, standard deviation (σ) for the variables is obtained experimentally using PID control. For instance σ for "error in pendulum angle" is found as follows:

$$\sigma = 1.6752$$

The optimization process for FS "error in angle" is discussed as an example. Displaced FS "ZE" is illustrated in Figure 7 Displaced FS "ZE".

The optimization task includes: Obtaining optimized value of $\mu^*(x)$ for each fuzzy set satisfying the constrained objective function as described in equation (12).Genetic Algorithm (GA) based optimization technique is used for optimization of objective function.GA optimization techniques [24] are inspired form the phenomena of natural evolution i.e. survival of the fittest, which is followed by: reproduction, crossover and mutation. A population of size N is initially created, with each entity being a possible solution to the problem. Form this generation of N entities a new generation is generated through genetic operations. Applicability of GA technique is primarily affected by following two factors: (i) crossover rate, and (ii) mutation rate[25].

Flowchart for the above process is depicted in figure 8.[26][27][28]:

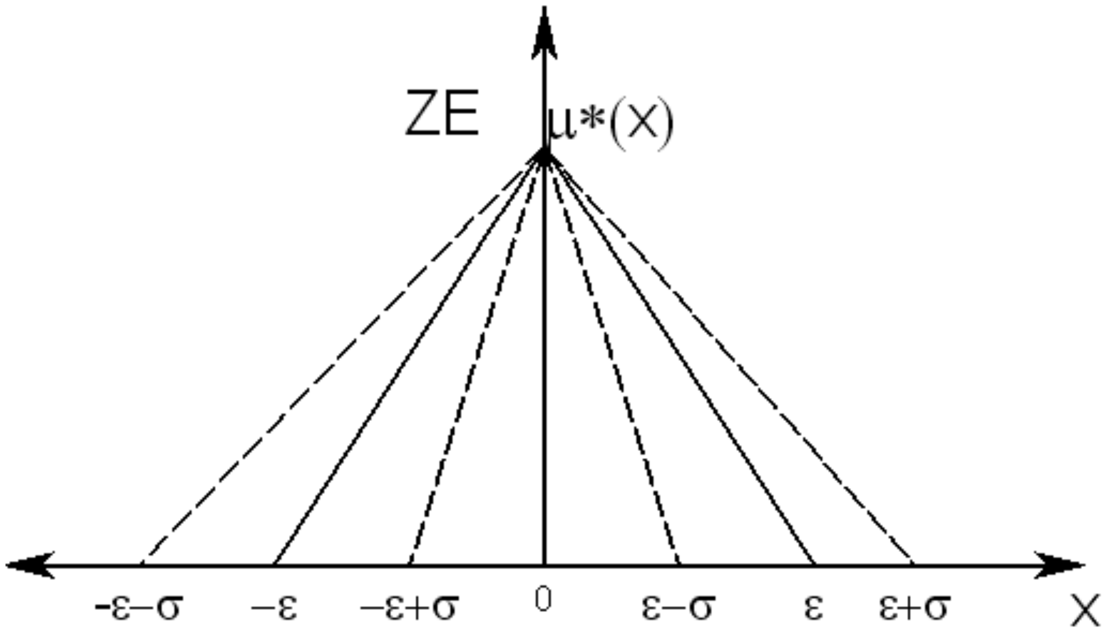

Figure 7: Displaced FS "ZE"

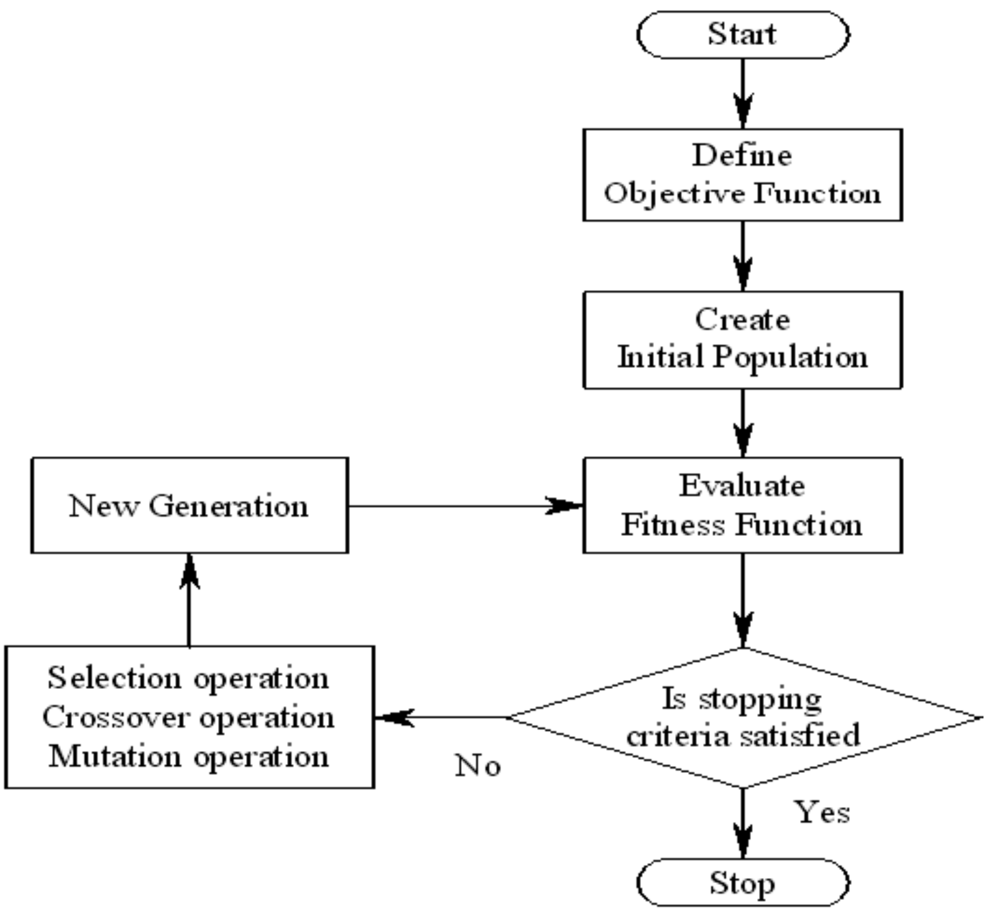

Figure 8: Flowchart for GA based optimization.

The parameters for finding the optimized FS are specified in table 3.Using the GA based optimization of objective function the resultant optimized FS is used to replace predefined MFs and is thus used for designing optimized FLC. The same is principle is used to obtain optimized MFs for rate of change for error in angle, and control signal for DC motor.

Table 3
Parameters chosen for optimization settings of GA

Name	*Value (Type)*
No. of generations	250
Population size	150
Selection type	Uniform
Crossover type	Arithmetic
Mutation type	Uniform
Termination method	Maximum generation

5. REAL-TIME DIGITAL PENDULUM SWING UP CONTROL

5.1. PID controller

In swing up stabilization the initial angle of the pendulum isθ=180^o (the natural equilibrium of a simple pendulum). The PID control algorithm consists of two controllers with only one being active at a time. One is designed for swinging up of the pendulum pole and other for stabilisation of pendulum as it reaches the inverted position. The control algorithm for pendulum swing up is designed to regulate the force applied to the cart in such a way that

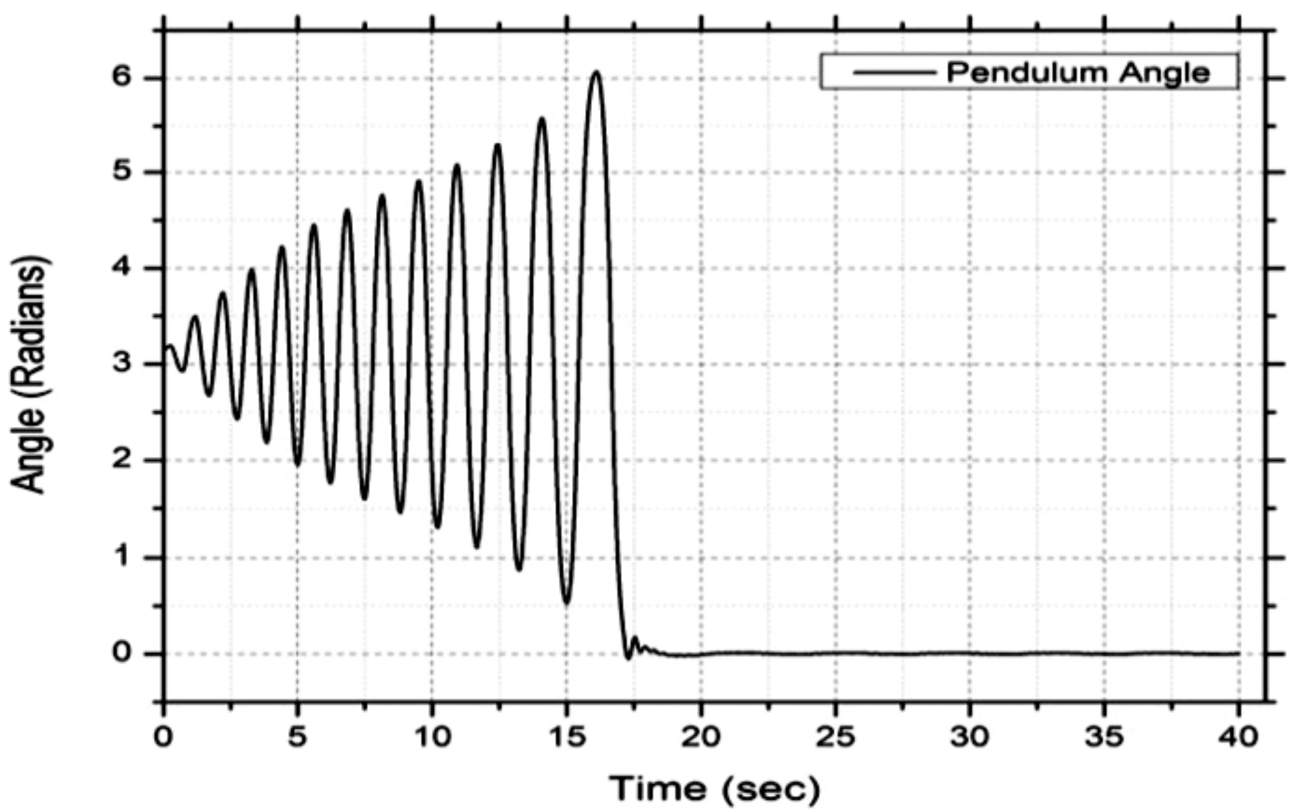

Figure 9: Pendulum angle for PID controller

pendulum starts to oscillate with successive increase in the oscillation magnitude. When the pendulum reaches inverted position the stabilisation algorithm than tries to maintain the inverted position with minimal control effort applied to the cart. PID controller for the system is illustrated in figure 8. Here the PID settings have been optimized for minimum ITSE once the values are achieved with Ziegler-Nichols method[19]. Pendulum angle stabilization using PID controller is given in figure 9. With this result the controller performance parameters can be evaluated as: (a) Settling time – t_s = 17.3 seconds (b) Peak value – M_p = 6.01 radians.

5.2. Fuzzy Logic Controllerand Optimized Fuzzy Logic Controller

The PID control is now replaced by fuzzy logic controller as illustrated in figure 5.Initially FLC is designed using predefined MFs using the method given in section 3.1.Pendulum angle stabilization control using FLC is given infigure 10.With this result the controller performance parameters can be evaluated as: (a) Settling time – t_s = 7.4 seconds (b) Peak value – M_p = 4.9 radians. This clearly indicates an improvement over PID control, with the settling time being reduced by 57.2% and peak value by 18%.

The predefined MFs used in FLC are replaced by optimized MFs obtained from the proposed algorithm and optimized FLC is used to control the digital pendulum, pendulum angle stabilization using optimized FLC is given in figure 11.With this result the controller performance parameters can be evaluated as: (a) Settling time – t_s= 4.6 seconds (b) Peak value – M_p = 5.52 radians. These values clearly indicate an improvement over PID control, with the settling time being reduced by 73.4% and peak value by 8.2%. However the comparison of these parameters over FLC indicates a reduction of settling time by 37.8% but an increase in peak value by 12.6%.

5.3. Comparative analysis of results obtained for pendulum stabilization

Comparison of performance (pendulum angle stabilization)in swing up mode for PID control, FLC and optimized FLC are given in figure 12.

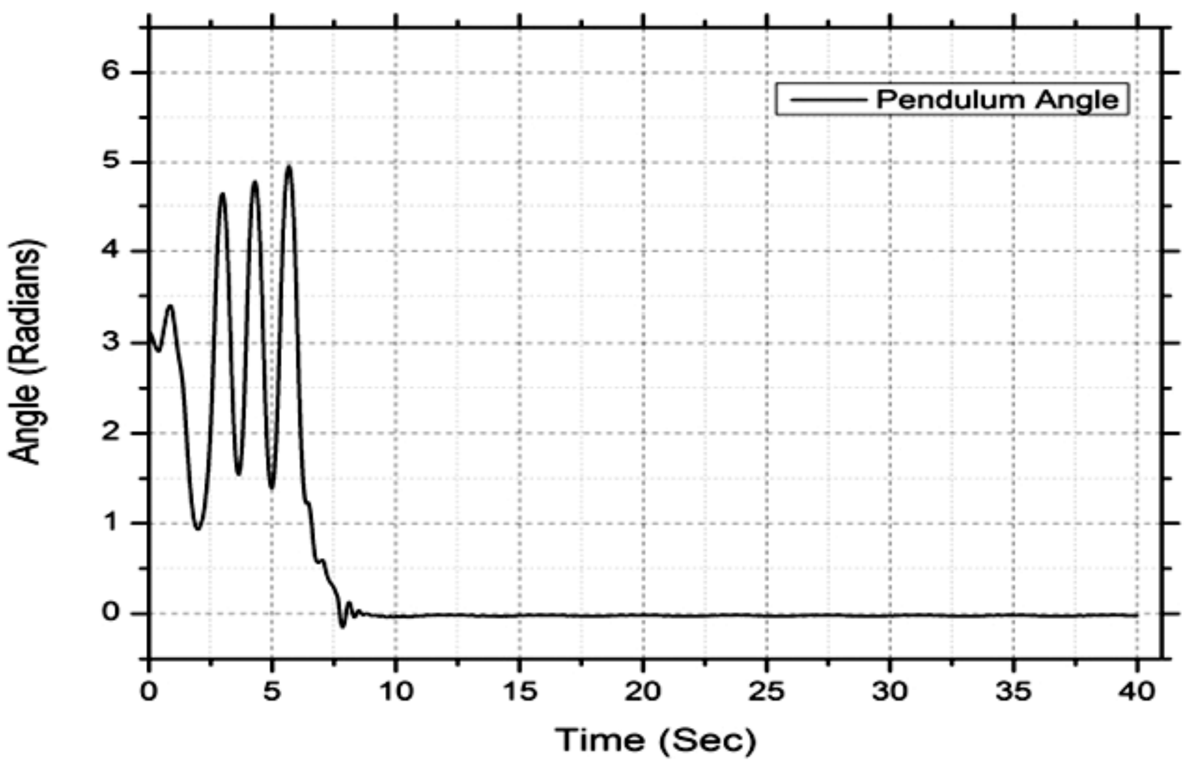

Figure 10: Pendulum angle for FLC

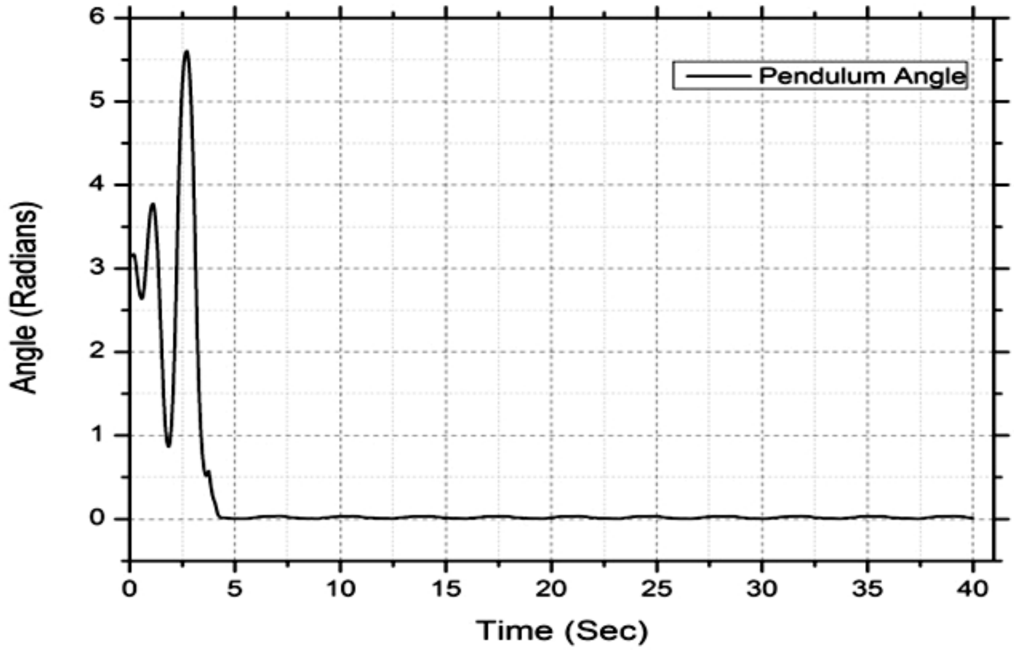

Figure 11: Pendulum angle for Optimized FLC

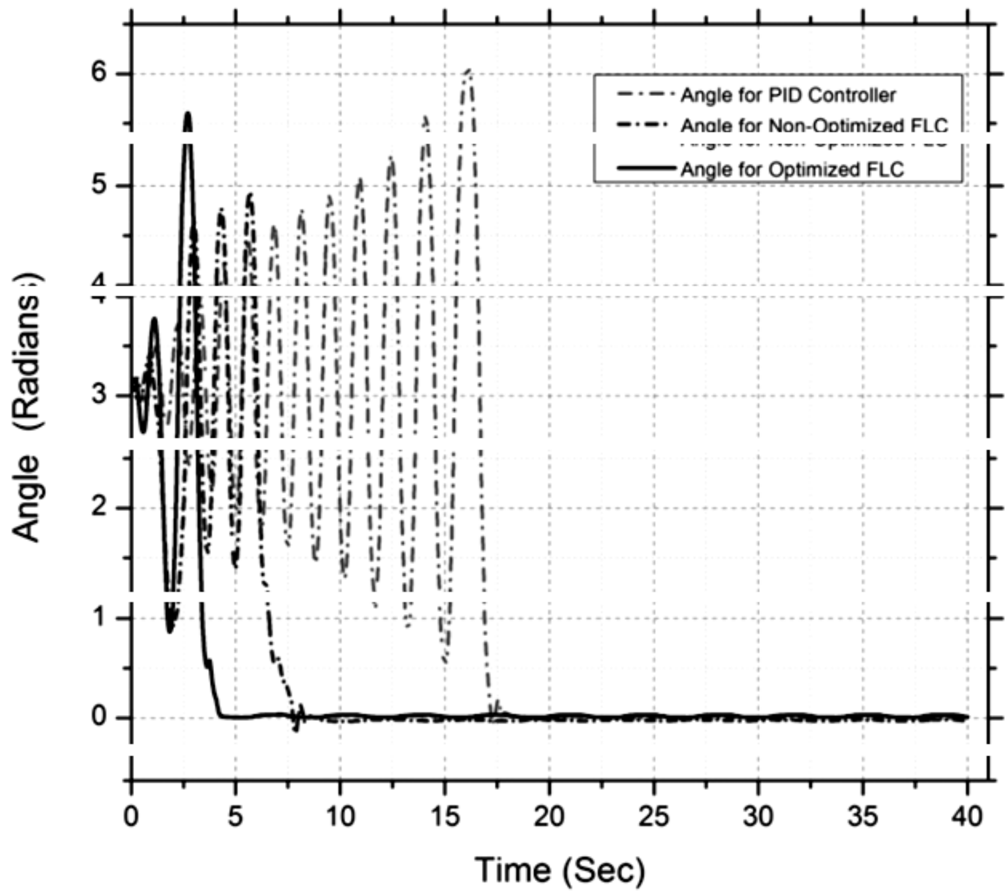

Figure 12: Comparison of performance for PID, FLC, and optimized FLC

Pendulum angle stabilisation comparison as depicted in Figure 13 clearly indicates that steady state error for all the controllers is "0". It can however, be concluded that, pendulum angle is stabilized within a shorter duration for optimized FLC as compared to PID control or FLC. Table 4 summarizes "Settling time (t_s)" and "Peak Value (M_p)" for the three controllers.

Table 4
Settling time and Peak Value comparison for pendulum angle stabilization

Controller	*t_s(Sec)*	*M_p(Radians)*
PID	17.3	6.01
FLC	7.4	4.9
Optimized – FLC	4.6	5.52

Table 5 summarizes error indices (error in pendulum angle) for the three controllers; these indices include [29]:

Table 5
Performance indices

Error indices	*ISE*	*ITSE*	*IAE*	*ITAE*
PID	204.3	1902	54.75	477
FLC	60.99	215.2	19.72	83.95
Optimized FLC	39	76.17	11.57	34.99

As the steady-state error for the three (PID, FLC, and Optimized FLC) controllers is: "0", the comparisons have been carried out with respect to: Settling time, Peak value and error indices. Optimized FLC exhibits minimum settling time (fast convergence) amongst the three controllers. The peak value is however only marginally different for the three controllers, which is quite an unavoidable phenomenon in swing up angle stabilization of an inverted pendulum. Also minimum performance indices are demonstrated by optimized FLC, hence it is innocuous to say that the optimized FLC is an efficient and effective controller, and shows an improvement over benchmark PID control and simple FLC.

6. CONCLUSION

In this paper, a new method has been proposed to optimize FS based on Fuzzy Entropy function. The proposed method uses predefined FS and optimizing the support of the set by maximizing objective function. Once the FS are presumed the new FS* are formed by displacing the predefined setsby standard deviation. The new formed FS are then optimized using GA to obtain sets having maximum entropy function value.The proposed control scheme is applied for real-time control ofpendulum angle in a classic inverted pendulum problem. The results indicate an improvement in angle stabilisation and various parameters (like t_s, M_p etc.) for optimized-FLCas compared to FLC or PID.

The proposed methodology is used to optimize FS having triangular MF, the same technique can be applied to fuzzy logic based systems having non-triangular MFstoo (Gaussian, trapezoidal, s-function etc.)The applicability of this technique is limited to systems having availability of reference data as a constrained requirement.For majority of practical systems datais available as reference set or can be determined with the help of simulation/experimental analysis, therefore availability of data willnot be a major constraint for applicability of the proposed technique.

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