

# An Adaptive Online Deterministic Algorithm for Multiprocessor Scheduling with Rejection

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## ABSTRACT

In this paper we have proposed an algorithm called as Rejection Penalty with Dynamic Processing Time Barrier (RP-DPTB) which has been minimizing the ratio of online performance to optimal offline performance by using an adaptive deterministic scheduling for multi-processor with rejection techniques. Here we have taken random jobs of different numbers and all jobs are associated with random Values of Penalty (VP) and the random values of Processing Time (PT). On the basis of rejection condition, part of the total jobs are rejected and accepted jobs are allowed to section into two, one with PT less then barrier value ' $\beta$ ' and another with PT more or equal to barrier value ' $\beta$ '. The value of barrier ' $\beta$ ' is not fixed, it is changing each time with a new job arrival which is providing a dynamic barrier state, that allow scheduling in the machine dynamically in according to the input processing time of the jobs. We have implemented our approached model in MATLAB and have found a reduced competitive ratio as compared to other existing deterministic algorithm.

**Keywords:** RP-DPTB, VP, PT, Offline scheduling, Online scheduling, Competitive ratio.

## 1. INTRODUCTION

The technique used to organize data and store data within the computer storage so that it can be used efficiently can be termed as scheduling. That decides how to commit resources among different possible tasks. Real time scheduling are meant for deciding which of the processes or jobs in the ready queue that to be allocated to the processor or machine. Real time scheduling requires robust and efficient algorithm design so that it can run dynamically with no delay as input arrives. Scheduling is basically of two types, one is online scheduling and another is offline scheduling. In offline scheduling the complete knowledge of all tasks are given from the beginning whereas in online scheduling algorithm, it does not have entire input at the beginning.

In Multi Processor with Rejection (MSR) techniques each job is accepted or rejected, the accepted jobs are allowed to process by one of the machines that we have taken from the beginning of the system design and the rejected jobs are use to paid rejection penalty. The basic objective of such scheduling is to lower the makespan of jobs that are accepted and to lower all rejected jobs' total rejection penalty [6]. This type of machines scheduling with rejection are some time called as order acceptance.

The scheduling researcher are giving more attention towards such techniques of scheduling along with them the production managers are doing it in their field since been last few decades. In production industries this practice of rejecting order which will advent their farm is often, due to having small span of deliver dates and having production capacity constraints. To deal with such situation the manufacture need to reject some of the orders, so they usually reject the orders having high processing time with relatively low profits. Above is one of the most relevant practical applications of scheduling with rejection techniques. Another application area is when outsourcing is an accessory path, scheduling with rejection combines both the outsourcing and industrial production scheduling idea together [6].

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Basically these scheduling are classified into two category based on the way they works, online scheduling need real time processing where processor needs to formulate such algorithm which can work on any scale of incoming data, where data are not predicated before the job arrival, so it has been a challenging job for all researcher working on this field to determine and optimize the laps time of scheduling so that the overall competitive ratio results a lower value. Where as in offline scheduling we have complete idea regarding the behavior and characteristic of jobs that to be scheduled. For such scheduling the basic constrains are like deadlines, precedence and computational time [2]. And here the scheduling is based on the pre decided parameters, and the tasks are scheduled much before of their arrival. Performance of online algorithm also can be judged from the central processing unit of the system's utilization, the throughput, time of waiting and another time constrain is Turn-Around-Time(TAT).

Competitive ratio is a key criteria to measure the performance and efficiency of online scheduling algorithm which is nothing but like a approximation ratio that compare the objective online performance value to the optimal objective performance value. Mathematically [2]

$$\text{Competitive ratio} = \frac{\text{On line performance}}{\text{optimal off line performance}} \quad (1)$$

## 2. LITERATURE SURVEY

Literature of this paper includes the survey of the paper that deals with scheduling, multi-processor scheduling, scheduling with rejection and regarding competitive ration minimization and the papers that has been picking the challenges and applications of the said techniques.

The paper written by Prativa Satpathy et al. [2] is saying that the competitive ratio of multi-processor with rejection techniques can be minimized by using hybrid algorithm that form by fusion of choose and rejection penalty algorithm. They have named it as improved rejection penalty algorithm and their competitive ratio is 1.819. Scott F. Smith and James F. Frenzel [3], both have designed a multiprocessor which is sharing memory within a single chip which is best utilize by bioinformatics jobs. Where they have analyzed the architecture where Smith –Waterman alignment algorithm is running that reveals the penalty due to enhanced bus latency which is compared to a synchronous architecture design will found to be less. Ji Tian et al. [4], have presented an optimal possible dense algorithm where they have taken m number of parallel batch machines for minimizing the makespan with unbounded batch capacity and the process time of their jobs get known on the job's arrival. Enrique Gerstl and Gur Mosheiov [5] have solevd the scheduling problems that join the processing time which is position dependent and the rejection of jobs. In this paper they have proposed a technique of scheduling on identical parallel machine with rejection of jobs techniques that also possess processing time which is position dependent. Jinwen Ou et al. [6], have studied regarding various classical parallel machine scheduling model with m different machines and with n different jobs, where each job either accepted and then the accepted job processed to one of the machine or it is rejected and the rejected penalty is paid. They have studied basically the parallel machine with rejection techniques. Marc P. Renault et al. [7], in this paper they have compliment their outcome by providing a lower bound which reveals for all online mode of algorithm with advice to be minimal for the different scheduling problem mention in their paper. They have basically worked on the scheduling problems and bin packing advice based on online algorithm. Ji Tian et al. [8], they have considered online algorithm by taking m unbounded machine with time of delivery, here they have consider a drop line batch machine which can process many tasks in a batch such that the longest processing time of the jobs present in batch will be the processing time of the batch, which have same starting point. They have eyed to minimize the jobs deliver time. The research done by Peihai Liu and Xiqen Lu [9] proposed for flow shop scheduling where machine m is two, they have designed a online algorithm for above said problem by minimizing makespan. They have proposed online algorithm having competitive ratio equal to 1.6180. Danyu Bai et al. [10], have minimized the makespan considering the problem of open shop scheduling for

which they have proposed an algorithm named as general dense scheduling (GDS) which is providing accelerate convergence for large scale problems and for moderate scale problems they have employed differential evolution algorithm for better results. R. Al-Omari et al. [11] have conducted different method of study like simulation based and analytical taking different fault and deadline scenarios, and they have found that their adaptive scheme for fault tolerant scheduling in multiprocessor taking soft real time tasks providing better SR index than that of the non-adaptive schemes. Huangxin Wang et al. [12], have designed increasing weighted throughput for both online and offline algorithm, and they have also analyzed performance guarantees of the algorithm with respect to respective worst case scenarios throughput.

### 3. RESEARCH METHOD

The main objective of our proposed algorithm is to lower the makespan which is defined as the total time required by any system having ‘m’ number of machine to complete accepted job scheduling. Another objective is to minimize the total rejected jobs’ penalties. Rejection Penalty ( $RP(\alpha)$ ) is the existing algorithm where value of  $\alpha$  is fixed and is equal to 0.618, which is an online algorithm for multiprocessor scheduling with rejection problems, which can be applied for both fixed number and arbitrary number of machine. The scheduling in conventional algorithm has not justified the state of dynamicity in terms of selecting the barrier for lower level and higher level accepted job selection. So it has been an open area of research to make the existing algorithm to match and to make it adaptive to the present incoming accepted jobs processing time.

In our proposed algorithm we have tried to make our barrier dynamic so as to minimize the makespan and competitive ratio to make the online scheduling more efficient and reliable. The accepted jobs total load store separately, in highly loaded machine the value with higher processing time then the barrier ‘ $\beta$ ’ will store sequentially. In our proposed model we have total machine ‘m’ is equal to 3. where one machine is completely dedicated to deal with high spike values that usually halt the total scheduling process, which hold the processor for a longtime to schedule, that makes wait for other smaller jobs that to be scheduled. In low loaded machines holds the accepted job’s processing time whose value is less or equal to ‘ $\beta$ ’ value on the basis of balanced load concept. The lower jobs will schedule on the machine having small total load. In our proposed model there are two machines which are dedicated to schedule lower jobs.

#### 3.1. Notation and Preliminaries

$Z$	=	Total set of Jobs
$Z_i$	=	individual jobs where $i = 1, 2, \dots, l$ (length of the total job) Each $Z_i$ is associated with one processing time $PT_j$ And value of penalty $VP_j$
$PT_j$	=	Processing time of each jobs, $j = i$ for individual job
$VP_j$	=	value of penalty of each jobs, $j = i$ for individual job
$m$	=	the number of machines.
CR	=	condition of rejection If $(\alpha \times PT_j \geq VP_j)$ rejected Else job is accepted.
$\Phi$	=	Golden ratio = $(\sqrt{5} + 1)/2$ $= 1.618$

- $\alpha$  =  $(\sqrt{5}+1)/2 - 1 = 0.618$   
 $\beta$  = Barrier value  
 $PT_{max}$  = maximum processing time of all the jobs  
 $CR$  = Competative ratio =  $\frac{M^{ON}}{M^{OPT}}$   
 $M^{ON}$  = Online makespan performance  
 $M^{OPT}$  = Offline makespan performance

### 3.2. RP Online Scheduling Algorithm

Rejection Penalty is a well know algorithm in the field of online scheduling where all jobs appeared with a random value of processing time (PT) and value of penalty (VP) associated with each job.

$$Z \in (PT_j, VP_j), \text{ where } j = \text{job number} = 1, 2.. 1 \text{ (length of total job)}$$

The parameter to get accepted jobs or rejected jobs RP use alpha value equal to 0.618, mathematically

$$\alpha = (\sqrt{5} + 1) / 2 - 1 = 0.618$$

In RP algorithm the jobs are rejected by the following formula

If  $\alpha \times PT_j \geq VP$ , rejected

Else job is accepted and the jobs are scheduled in the least loaded machine (m).

After all the jobs get scheduled, the makespan can be calculated by evaluating online makespane  $M^{ON}$  and optimal offline makespan  $M^{OPT}$ , where the  $M^{ON}$  is the highest loaded machine sum of processing time plus the summation of rejected jobs. Whereas the  $M^{OPT}$  is the largest processing time between all jobs. And the competitive ratio is given by

$$CR = \text{Competitive ration} = \frac{\text{On line performance}}{\text{optimal off line performance}}$$

### 3.3. Motivation

The existing algorithms for multiprocessor scheduling available including rejection penalty algorithm are giving higher competitive ratio i.e. higher value of makespan . the higher value of makes pan indicating that the scheduling of multiprocessor using online algorithm are not much effective, it needs to further analyze and the algorithm need to make dynamic, as the jobs are having random values so the program also need to work dynamically as according to the arrival processing time. We have proposed and algorithm that will work dynamically as according to the arrival processing time of the jobs. The barrier 'β' value for higher and lower job selection is not fixed for entire scheduling process, each time a higher value arrives the barrier value changes accordingly, that is based on present and recent past processing time value. This dynamicity gives rise to our algorithm, named as Rejection Penalty with Dynamic Processing Time Barrier (RP-DPTB) algorithm.

## 4. PROPOSED ALGORITHM

Our proposed algorithm is the advanced form of rejection penalty algorithm, named as rejection penalty with dynamic processing time barrier (RP-DPTB), we have taken the same golden ratio that has been taken

by RP algorithm is that 1.618 and same value of  $\alpha$ , 0.618. Our algorithm is for online scheduling with multiprocessor with rejection techniques, the high processing time which is not in advent with the process get rejected by a rejection condition and the rejected jobs paid to the total penalty. Accepted jobs then taken and divided into two section based on the value of their processing time, if the processing time is more than barrier ' $\beta$ ' value then it is treated as SPIKE jobs or high processing time jobs and the processing time less the barrier ' $\beta$ ' value is treated as NORMAL or low processing value.

Here all the jobs are taken randomly to satisfy the condition of unpredictable processing time and penalty which is associated to each jobs. Here we have taken three machines where one machine is dedicated to SPIKE jobs and NORMAL jobs are scheduled to low loaded machine with rest two machines. We found after simulation the competitive ratio is less than all existing algorithm. We have taken different number of jobs randomly from 10 to 100000 for validating our proposed algorithm.

#### 4.1. RP-DPTB Algorithm

- Random set of jobs  $Z$  where  $Z_i$  is associated with  $PT_j$  and  $VP_j$ ,  $i = j = 1, 2, \dots, N$   
 $N =$  length of the total job,  $k =$  upper limit of  $PT$
- Apply CR to all ' $N$ ' jobs  
 If  $(\alpha \times PT_j \geq VP_j)$   
     Job is rejected  
 Else  
     Job is accepted.
- Initialize  $\beta = k \times 3/4$ ,  $m = 3$
- Accepted jobs classification  
 If *accepted\_jobs*  $PT \geq \beta$   
     SPIKE jobs or high jobs  
      $\beta = (\beta + PT_j)/2 - 1$   
 Else  
     NORMAL jobs or low jobs
- Load balancing Jobs Scheduling:  
      $m(1) =$  SPIKE jobs,  
      $m(2)$  and  $m(3) =$  NORMAL jobs
- Calculate  $M^{ON} = \max(m) + \sum \text{rejected\_job\_VP}$
- Calculate  $M^{OPT} = \frac{\sum_i^N PT_j}{m}$
- Calculate  $CR = \frac{M^{ON}}{M^{OPT}}$

## 5. IMPLEMENTATION

We have implemented our proposed model in MATLAB and have checked whether our proposed algorithm is giving better result in context to competitive ratio is giving better result or not. Our main objective of the implementation to get low value makespan, low value of rejected penalty and low competitive ratio. Apart

from that it has been also a key part in any multiprocessor scheduling is balancing the load in the machines. Here we have taken 3 machines, where one is especially dedicated to deal with the spike jobs and rest two is taken to schedule for low processing time jobs on the basis of low load machine first. We have taken many sets of nodes to validate our outcomes; we have taken random value of 10 jobs with random value of PT and VP with a highest range  $k$ .

Basic demonstration OF RP-DPTB taking 10 random jobs with first row defines as the job number, second row is the associated PT and third row is the VP. Each column is a job subset, shown from the simulation of the matlab in Table 1.

There are ten jobs, according to the condition of rejection job number 3, 7 and 8 are rejected and are marked in red in the Tabel 1. and rest all jobs are accepted jobs, that will allow to schedule in all three machine. The highest range  $k$  here is 20.

**Table 1**  
**Random PT and VP for 10 jobs with reded value indicating the job rejection (3, 7, 8).**

Z	1	2	3	4	5	6	7	8	9	10
PT	17	3	13	6	20	4	20	17	9	16
VP	19	19	2	11	20	20	10	3	19	20

Accepted jobs = [1, 2, 4, 5, 6, 9, 10]

Spike jobs = [1, 5]

Normal jobs = [2, 4, 6, 9, 10]

$\beta = k \times 3/4 = k \times 3/4 = 15$  (initial value)

$\beta_1$  = first updated  $\beta$  after arrival of processing time 17 of job 1  
 $= (15 + 17)/2 - 1 = 15$

$\beta_2$  = first updated  $\beta$  after arrival of processing time 20 of job 5  
 $= (15 + 20)/2 - 1 = 16.5$

$CR = 1.2480$

$m_1 = 17 + 20 = 37$

$m_2 = 3 + 4 + 16 = 23$

$m_3 = 6 + 9 = 15$

From the above, it is clearly elucidated that the barrier for scheduling is dynamic nature which certainly enhancing the performance by considerably reducing the CR and to make the machine nearly balanced. In

**Table 2**  
**RP-DPTB algorithm based multiprocessor scheduling performance analysis**

No. of jobs	$M^{ON}$	$M^{OPT}$	CR
10	47	43	1.0930
100	354	331	1.0695
1000	3817	3501	1.0901
10000	36524	34825	1.0488
100000	367577	350180	1.0497
Average competitive ratio of RP-DPTB is =			1.0702

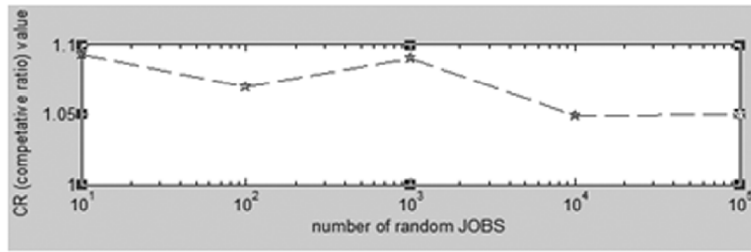


Figure 1: The top one is the region filling, middle one is image filling and the last one is the bounding box outcome graph

Table 3  
RP-DPTB algorithm based multiprocessor scheduling machine load balance analysis

No. of jobs	<i>m1</i>	<i>m3</i>	<i>m3</i>
10	18	11	20
100	235	202	196
1000	1642	2440	2433
10000	17513	22771	22769
100000	171329	229630	229624
Avg. machine Load	38147	51011	51008

the above simulation if we were not at dynamic barrier then the processing time of job number 10 is 16 which is more than the initial  $\beta$  value, that will imbalance the machine load and certainly to the CR value.

In Table 2, the simulated value of online makespan, optimal offline makespan and the competitive ratio from Matlab is tabulated for taking different number of random jobs with random value of processing time and penalty associated to each job. The average competitive ratio for our proposed algorithm comes out is 1.0702. In figure 1. We have drawn the log value of no of jobs taken for the simulation in x-axis and the linear competitive ratio in the y- axis, the value of competitive ratio is lying between 1.093 to 1.048. The average of which we have found is 1.0702.

The load of three machines has been tabulated in Table III. For different random in put value of jobs having random processing time and random value of penalty, the corresponding graph has been drawn and shown in fig. 2.

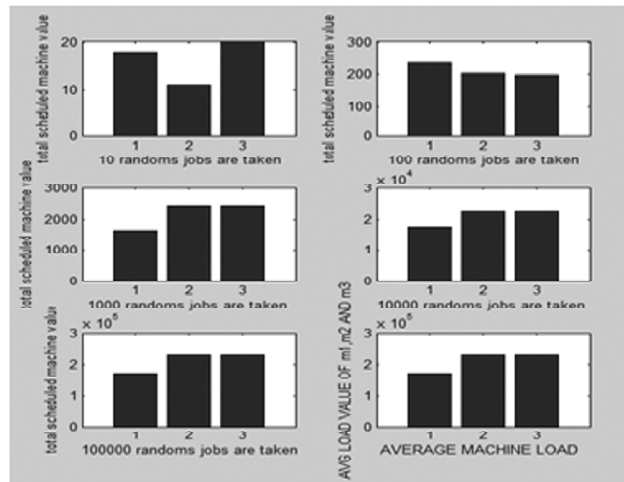
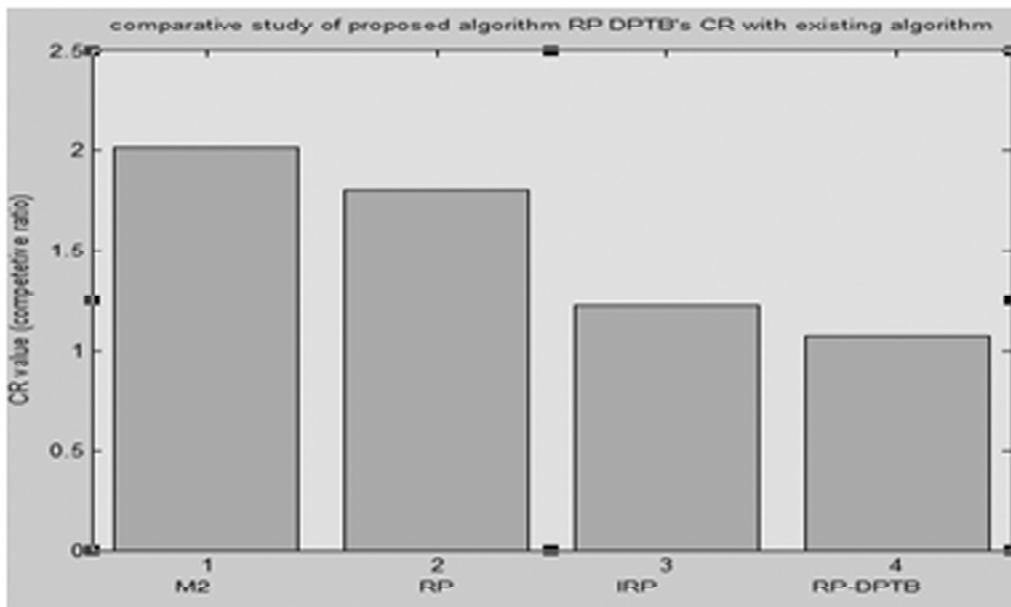


Figure 2: Load of all three machine with different number of random input jobs, the last subplot is showing the avg. machine load *m1*,*m2* and *m3*

We have done the comparative study between the existing online algorithms for multiprocessor scheduling with rejection to our proposed algorithm in Table IV. And also done a bar graph to show the difference in CR value between these algorithms that shown in Fig. 3. Here M2 and RP are pre existing graph and IRP algorithm [2] is made Prativa Sathpathy and team, after comparing all we found that our proposed model is giving the best competitive ratio.

**Table 4**  
**RP-DPTB algorithm based multiprocessor scheduling machine load balance analysis**

Sl. No	Algorithm / Model name	Comptative ratio (CR)
01.	M2 algorithm	2.012
02.	RP algorithm	1.8017
03.	IRP algorithm [ref-2]	1.224
04.	Proposed algorithm RP-DPTB	1.0702



**Figure 3:** Load of all three machine with different number of random input jobs, the last subplot is showing the avg. machine load m1,m2 and m3

## 6. CONCLUSION

We have simulated our proposed algorithm RP-DPTB in Matlab and we have found a better competitive ratio value as compared to all other existing algorithm of its kind. We have also able to minimize the makespan of accepted jobs and the penalty of the rejected jobs. Our algorithm is giving less CR value for all scale of input random jobs with random processing time and penalty.

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