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Testing Bienenfeld's Second-Order Approximation for the Wage-Profit Curve

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This paper constructs Bienenfeld's second-order approximation for the wage-profit curve and tests it using data from ten symmetric input-output tables of the Greek economy. The results suggest that there is room for using low-dimensional models as surrogates for actual single-product economies.

INTRODUCTION

Typical findings in many empirical studies of *single*-product systems are that, in the economically relevant interval of the profit rate, (i) the approximation of the production prices through Bienenfeld's (1988) linear and, *a fortiori*, quadratic formulae works pretty well; and (ii) the wage-profit curves (WPCs) are almost linear, i.e. the correlation coefficients between the distributive variables tend to be above 99%, and their second derivatives change sign no more than once or, very rarely, twice, irrespective of the numeraire chosen. As it has recently been argued, these findings could be connected to the skew distribution of the eigenvalues of the matrices of vertically integrated technical coefficients (Schefold, 2008, 2013; Mariolis and Tsoulfidis, 2009, 2011, 2014, 2015; Iliadi *et al*., 2014).

The present paper, drawing on Bienefeld's (1988) polynomial approximation of prices, constructs a 'proper rational' approximation for the WPC and tests its second-order form using data from the 19 x 19 Symmetric Input-Output Tables (SIOTs) of the Greek economy, spanning the period 1988-1997.[1] The selection of this dataset was based on its extensive use in a number of closely related studies (see Mariolis and Tsoulfidis, 2015a, chs 3 to 5, and the references therein).

The remainder of the paper is structured as follows. Section 2 presents the necessary preliminaries. Section 3 derives the approximation for the WPC. Section 4 brings in the empirical evidence. Finally, Section 5 concludes.

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PRELIMINARIES

Consider a closed, linear and viable system involving only single products, basic commodities (in the sense of Sraffa, 1960) and circulating capital. Under the usual assumptions, the wage-price-profit rate relations for the system may be written as

$$\mathbf{p} = w\mathbf{l} + (1+r)\mathbf{pA} \tag{1}$$

where $\mathbf{p}$ denotes a $1\times n$ vector of production prices, w the money wage rate, and $\mathbf{l}$ ($>\mathbf{0}$) the $1\times n$ vector of direct labour coefficients, r the uniform profit rate, and A the $n\times n$ matrix of direct technical coefficients.[2] After rearrangement, equation (1) becomes

$$\mathbf{p} = w\mathbf{v} + \rho\mathbf{pJ} \tag{2}$$

or, if $\rho < 1$,

$$\mathbf{p} = w\mathbf{v}[\mathbf{I} - \rho\mathbf{J}]^{-1} \tag{2a}$$

where $\mathbf{v} \equiv \mathbf{l}[\mathbf{I} - \mathbf{A}]^{-1}$ denotes the vector of vertically integrated labour coefficients or labour values, and $\mathbf{H} \equiv \mathbf{A}[\mathbf{I} - \mathbf{A}]^{-1}$ ($>\mathbf{0}$) the vertically integrated technical coefficients matrix. Moreover, $\rho \equiv rR^{-1}$, $0 \le \rho \le 1$, denotes the 'relative (or normalized) profit rate', which equals the share of profits in the Sraffian Standard system (SSS), and $R \equiv \lambda_{A1}^{-1} - 1 = \lambda_{H1}^{-1}$ the maximum profit rate, i.e. the profit rate corresponding to $[w = 0, \mathbf{p} > \mathbf{0}]$, which equals the ratio of the net product to the means of production in the SSS. Finally, $\mathbf{J} \equiv R\mathbf{H}$ denotes the normalized vertically integrated technical coefficients matrix, where $\lambda_{J1} = R\lambda_{H1} = 1$.

If commodity $\mathbf{z}^{\mathrm{T}}$, with $\mathbf{vz}^{\mathrm{T}} = 1$, is chosen as the numeraire, i.e. $\mathbf{pz}^{\mathrm{T}} = 1$, then equation (2a) implies

$$w^{\mathrm{Z}} = (\mathbf{v}[\mathbf{I} - \rho\mathbf{J}]^{-1}\mathbf{z}^{\mathrm{T}})^{-1} \tag{3}$$

and

$$\mathbf{p}^{\mathrm{Z}} = (\mathbf{v}[\mathbf{I} - \rho\mathbf{J}]^{-1}\mathbf{z}^{\mathrm{T}})^{-1}\mathbf{v}[\mathbf{I} - \rho\mathbf{J}]^{-1} \tag{4}$$

Post-multiplying equation (2) by SSC, i.e. $\mathbf{s}^{\mathrm{T}} \equiv [\mathbf{I} - \mathbf{A}]\mathbf{x}_{\mathbf{A}1}^{\mathrm{T}}$, with $\mathbf{l}\mathbf{x}_{A1}^{\mathrm{T}} = 1$, and rearranging terms, gives

$$w^{\mathrm{Z}} = (1-\rho)\mathbf{p}^{\mathrm{Z}}\mathbf{s}^{\mathrm{T}} \tag{3a}$$

where $\mathbf{p}^z\mathbf{s}^T$ represents the price of net output, measured in terms of commodity $\mathbf{z}^T$, of the SSS. Equations (3) and (4) gives the WPC and the production prices, measured in terms of commodity $\mathbf{z}^T$, as functions of ρ, respectively. It then follows that:

(i) At $\rho = 0$ we obtain $w^z(0) = 1$ and $\mathbf{p}^z(0) = \mathbf{v}$, while in the other extreme case, i.e. at $\rho = 1$, we obtain $w^z(1) = 1$ and $\mathbf{p}^Z(1) = (\mathbf{y}_{J1}\mathbf{z}^T)^{-1}\mathbf{y}_{J1}$.

(ii) The WPC is an improper rational function of degree n, and strictly decreasing in $0 \leq \rho \leq 1$ (see equations (3) and (3a)):

$$\dot{w}^Z \equiv dw^Z / d\rho = -w^Z\mathbf{p}^Z\mathbf{J}[\mathbf{I} - \rho\mathbf{J}]^{-1}\mathbf{z}^T$$

$$\dot{w}^Z(0) = -\mathbf{p}^Z(0)\mathbf{J}\mathbf{z}^T = -\mathbf{v}\mathbf{J}\mathbf{z}^T \tag{3b}$$

$$\dot{w}^Z(1) = -\mathbf{p}^Z(1)\mathbf{s}^T = -(\mathbf{y}_{J1}\mathbf{z}^T)^{-1}\mathbf{y}_{J1}\mathbf{s}^T \tag{3c}$$

where $\mathbf{vJz}^T$ equals the ratio of the capital-net output ratio (measured in terms of labour values) in the vertically integrated industry producing the numeraire commodity to the capital-net output ratio, R^{-1}, in the SSS. Finally, the WPC may admit up to $3n - 6$ inflection points (though it is not certain that they will all occur in $0 \leq \rho \leq 1$; Garegnani, 1970, p. 419).

(iii) If 1 ($\mathbf{v}$) is the P-F eigenvector of $\mathbf{A}$ ($\mathbf{J}$) or if SSC is chosen as the numeraire, then

$$w^S = 1 - \rho \tag{5}$$

i.e. the WPC is linear, as in a one-commodity world. The former case corresponds to the Ricardo-Marx-Samuelson 'equal value compositions of capital' case, and implies that $\mathbf{p} = \mathbf{v}$, i.e. the 'pure labour theory of value' holds true. In the latter case,

$$\mathbf{p}^S = (1-\rho)\mathbf{v}[\mathbf{I} - \rho\mathbf{J}]^{-1} = \mathbf{v}\sum_{h=0}^{+\infty}(1-\rho)(\rho\mathbf{J})^h \tag{6}$$

which is the reduction of prices to 'dated quantities of embodied labour' (Kurz and Salvadori, 1995, p. 175) in terms of SSC. Differentiation of equation (6) with respect to ρ finally gives

$$\dot{\mathbf{p}}^S(0) = -\mathbf{v} + \mathbf{vJ} \tag{6a}$$

APPROXIMATION FOR THE WAGE-PROFIT CURVE

Assume that prices are measured in terms of SSC. As is well known, for any semi-positive n–vector $\mathbf{y}$, $\mathbf{yJ}^h$ tends to the left P-F eigenvector of $\mathbf{J}$ as h tends to infinity, i.e.

$$\lim_{h \to +\infty} \mathbf{yJ}^h = [(\mathbf{yx}_{J1}^{T})(\mathbf{y}_{J1}\mathbf{x}_{J1}^{T})^{-1}]\mathbf{y}_{J1}$$

while an upper bound on the rate of convergence is given by $\|\mathbf{K}\|_\infty < bc^h$, where

$$\mathbf{K} \equiv \mathbf{J}^h - (\mathbf{y}_{J1}\mathbf{x}_{J1}^{T})^{-1}\mathbf{x}_{J1}^{T}\mathbf{y}_{J1}$$

the norm $\|\mathbf{K}\|_\infty$ denotes the maximum of the absolute values of the elements of **K**, and b represents a positive constant, which depends on **J** and c, for any c such that $|\lambda_{J2}| < c < 1$ (see, e.g. Horn and Johnson, 1990, p. 501). Therefore, for a sufficiently large value of m such that

$$\mathbf{vJ}^m \approx \mathbf{vJ}^{m+1} \approx \ldots \approx \mathbf{p}^S(1) \tag{7}$$

it follows from equation (6) that

$$\mathbf{p}^S \approx \mathbf{v}\sum_{h=0}^{m-1}(1-\rho)(\rho\mathbf{J})^h + (1-\rho)\rho^m(1+\rho+\rho^2+\ldots)\mathbf{p}^S(1)$$

or

$$\mathbf{p}^S \approx \mathbf{p}^A \equiv \mathbf{v} + \sum_{h=1}^{m-1}\rho^h(\mathbf{vJ}^h - \mathbf{vJ}^{h-1}) + \rho^m(\mathbf{p}^S(1) - \mathbf{vJ}^{m-1}) \tag{8}$$

The vector $\mathbf{p}^A$ is Bienenfeld's (1988) polynomial approximation for the price vector measured in terms of SSC:

(i) It is exact at the extreme, economically significant, values of ρ.

(ii) For $m \geq 2$, it gives, the correct slope of the actual $p_j - \rho$ curves at $\rho = 0$ (consider equation (6a)).

(iii) Its accuracy is directly related to the rate of convergence in (7), which in its turn is directly related to the magnitudes of $|\lambda_{Jk}|^{-1}$. In the extreme case where the non-labour conditions of production in all industries are linearly dependent on each other, it holds that *rank*[**J**] = 1, which implies $\mathbf{J} = (\mathbf{y}_{J1}\mathbf{x}_{J1}^{T})^{-1}\mathbf{x}_{J1}^{T}\mathbf{y}_{J1}$ and $|\lambda_{J2}| = 0$. Thus, $\mathbf{vJ} = \mathbf{p}^S(1)$ and Bienenfeld's approximation becomes linear and exact for all ρ, i.e.

$$\mathbf{p}^S = \mathbf{p}^A = \mathbf{v} + \rho(\mathbf{p}^S(1) - \mathbf{v})$$

Equations (3) to (6) imply

$$(w^{Z})^{-1}(\mathbf{p}^{Z}\mathbf{z}^{T}) = (w^{S})^{-1}(\mathbf{p}^{S}\mathbf{z}^{T})$$

or

$$w^{Z} = w^{S}(\mathbf{p}^{S}\mathbf{z}^{T})^{-1}$$

Thus, invoking approximation (8), we obtain

$$w^{Z} \approx w^{S}(\mathbf{p}^{A}\mathbf{z}^{T})^{-1}$$

or

$$w^{Z} \approx w^{A} \equiv w^{S}\{1+\sum_{h=1}^{m-1}\rho^{h}(\mathbf{vJ}^{h}-\mathbf{vJ}^{h-1})\mathbf{z}^{T}+\rho^{m}(\mathbf{p}^{S}(1)-\mathbf{vJ}^{m-1})\mathbf{z}^{T}\}^{-1} \quad (9)$$

The approximate w^{A} curve has the following attributes:

(i) It is a *proper* rational function of degree m, and exact at the extreme values of ρ, while its accuracy is directly related to the magnitudes of $\left|\lambda_{Jk}\right|^{-1}$.

(ii) It gives the correct slope of the actual WPC at $\rho = 1$ (consider equations (9) and (3c)):

$$\dot{w}^{A}(1) = -(\mathbf{p}^{A}(1)\mathbf{z}^{T})^{-1} = -(\mathbf{p}^{S}(1)\mathbf{z}^{T})^{-1}$$

or

$$\dot{w}^{A}(1) = -(\mathbf{y}_{J1}\mathbf{z}^{T})^{-1}\mathbf{y}_{J1}\mathbf{s}^{T} = \dot{w}^{Z}(1)$$

For $m \geq 2$, it also gives the correct slope at $\rho = 0$ (consider equations (9) and (3b)):

$$\dot{w}^{A}(0) = -\mathbf{p}^{S}(1)\mathbf{z}^{T}, \text{ for } m = 1$$

$$\dot{w}^{A}(0) = -\mathbf{vJz}^{T} = \dot{w}^{Z}(0), \text{ for } m \geq 2$$

(iii) For $m \geq 2$, it may admit up to $2m - 1$ inflection points.

Setting $m = 1$, w^{A} reduces to the homographic function

$$w^{A} = w^{S}[1+\rho(\mathbf{p}^{S}(1)\mathbf{z}^{T}-1)]^{-1} \quad (9a)$$

which has exactly the same algebraic form as the WPC for the Samuelson-Hicks-Spaventa or 'corn-tractor' model (see Spaventa, 1970). Moreover, $\ddot{w}^{A} < 0$ for $0 \leq \rho \leq 1$ iff $\mathbf{p}^{S}(1)\mathbf{z}^{T} < 1$ or, since $\mathbf{p}^{S}(1)\mathbf{J} = \mathbf{p}^{S}(1)$, $\mathbf{p}^{S}(1)\mathbf{Hz}^{T} < R^{-1}$, i.e. at $\rho = 1$ the vertically integrated industry producing $\mathbf{z}^{\mathrm{T}}$ is labour intensive relative to the SSS. When $rank[\mathbf{J}]=1$, this approximation becomes exact for all ρ.[3]

Finally, setting $m = 2$, w^{A} reduces to

$$w^{\mathrm{A}} = w^{\mathrm{S}}[1+\rho(\mathbf{vJz}^{\mathrm{T}}-1)+\rho^{2}(\mathbf{p}^{\mathrm{S}}(1)\mathbf{z}^{\mathrm{T}}-\mathbf{vJz}^{\mathrm{T}})]^{-1} \tag{9b}$$

Let $\rho_{\kappa}, \kappa = 1,2,3$ be the roots of equation $\ddot{w}^{\mathrm{A}} = 0$, let Δ be the discriminant of the numerator of $\ddot{w}^{\mathrm{A}}$, and set $\alpha \equiv \mathbf{vJz}^{\mathrm{T}}$ and $\beta \equiv \mathbf{p}^{\mathrm{S}}(1)\mathbf{z}^{\mathrm{T}}$, $\alpha \neq \beta$. It is easily checked that

$$\rho_1\rho_2\rho_3 = (\alpha^2-\beta)(\alpha-\beta)^{-2}$$

$$\rho_1\rho_2+\rho_1\rho_3+\rho_2\rho_3 = 3\alpha(\alpha-\beta)^{-1}$$

$$\rho_1+\rho_2+\rho_3 = 3$$

$$\Delta = -432\beta^2[(1+\alpha)^2-4\beta](\alpha-\beta)^4$$

It then follows that:

(i) All the roots are real and distinct iff $\beta > 4^{-1}(1+\alpha)^2$, where $\alpha^2 < \alpha < 4^{-1}(1+\alpha)^2$ for $0<\alpha<1$, while $\alpha < 4^{-1}(1+\alpha)^2 < \alpha^2$ for $\alpha > 1$.

(ii) The second-order approximate curve (9b) has at *most* one inflection point in the interval $0 \le \rho \le 1$. More specifically, that inflection point occurs when $\alpha <(>)\ 1$ and $\beta \ge\ (\le)\ 2^{-1}(1+\alpha)$ or $\beta \le\ (\ge)\ \alpha^2$.

EMPIRICAL EVIDENCE

The test of the second-order approximate WPC, measured in terms of the 'actual' real wage rate, with data from the flow SIOTs of the Greek economy, spanning the period 1988-1997, gave the results summarized in Table 1. This table reports:

(i) The actual, ρ^{*}, and the estimated, ρ^{e}, values of ρ, $0 \le \rho \le 1$, at which the inflection points occur.

(ii) The signs of the discriminant, Δ, and of $1 - \alpha$.

(iii) The mean of the absolute error in this approximation, i.e.

$$MAE \equiv \int_0^1 \left| w^{\mathrm{Z}} - w^{\mathrm{A}} \right| d\rho$$

(iv) The Euclidean angles (measured in degrees) between $\mathbf{p}^{\mathrm{S}}(1)$ and $\mathbf{vJ}^{m}$, $m = 0, 1, 2$, which are denoted by θ_m.

(v) The modulus of the subdominant eigenvalue, $|\lambda_{J2}|$, and the arithmetic (AM) and geometric (GM) means of the moduli of the non-dominant eigenvalues.

Table 1
Indicators and determinants of the accuracy of the second-order approximate wage-profit curve

	1988	*1989*	*1990*	*1991*	*1992*	*1993*	*1994*	*1995*	*1996*	*1997*
ρ^*	0.269	0.300	0.360	0.302	0.534	0.385	0.336	0.110	0.200	– ($\ddot{w}^z < 0$)
ρ^e	0.275	0.298	0.311	0.299	0.410	0.325	0.302	0.216	0.247	0.136
Δ	<0	<0	<0	<0	<0	<0	<0	<0	<0	<0
$1-\alpha$	>0	>0	>0	>0	>0	>0	>0	>0	>0	>0
MAE	0.007	0.009	0.008	0.007	0.004	0.007	0.008	0.006	0.006	0.005
θ_0	29.3°	31.1°	30.4°	30.0°	25.3°	26.5°	27.8°	28.1°	27.3°	26.4°
θ_1	15.0°	16.6°	15.7°	14.8°	10.3°	13.3°	13.7°	13.1°	12.5°	10.8°
θ_2	6.4°	8.3°	7.7°	7.2°	4.3°	6.7°	6.9°	5.9°	5.7°	4.7°
$\lvert\lambda_{J2}\rvert$	0.643	0.683	0.675	0.657	0.624	0.667	0.678	0.655	0.664	0.641
AM	0.168	0.175	0.176	0.176	0.185	0.174	0.171	0.161	0.167	0.157
GM	0.086	0.086	0.088	0.087	0.089	0.081	0.088	0.074	0.074	0.073

From this table, the associated numerical results and the hitherto analysis we arrive at the following conclusions:

(i) Although the systems deviate considerably from the equal value compositions of capital case (see the values of θ_0), the approximation works pretty well. With the exception of the year 1997, both curves switch from convex to concave at 'adjacent' values of the relative profit rate (the error $|\rho^e - \rho^*|$ is in the range of 0.002 to 0.124), and the MAE is no greater than 0.010. The graphs in Figure 1 are sufficiently representative and display the actual (depicted by a solid line) and the approximate (depicted by a dotted line) WPCs for the years 1989 (where MAE, θ_m and $|\lambda_{J2}|$ exhibit their highest values) and 1997 (where $\ddot{w}^Z < 0$).

(ii) In all systems, $\mathbf{vJ}^m$ tend quickly to $\mathbf{p}^S(1)$, and the moduli of the first non-dominant eigenvalues fall quite rapidly, whereas the rest constellate in much lower values forming a 'long tail'. In fact, as it has already been pointed out (Mariolis and Tsoulfidis, 2011, pp. 104-105), the moduli of the eigenvalues of each system matrix follow an exponential pattern of the form

$$y = \gamma_0 + \gamma_1 \exp(x^{-0.2})$$

where $-1.738 \le \gamma_0 \le -1.827$, $0.986 \le \gamma_1 \le 1.040$ and the R – squared is in the range of 0.975 to 0.991. These findings, which are in absolute accordance with those detected in other studies of quite diverse actual economies (see Mariolis and Tsoulfidis, 2015a, chs 5-6, and the references therein), indicate that the accuracy of this second-order approximation would *not* be so sensitive to the numeraire choice.

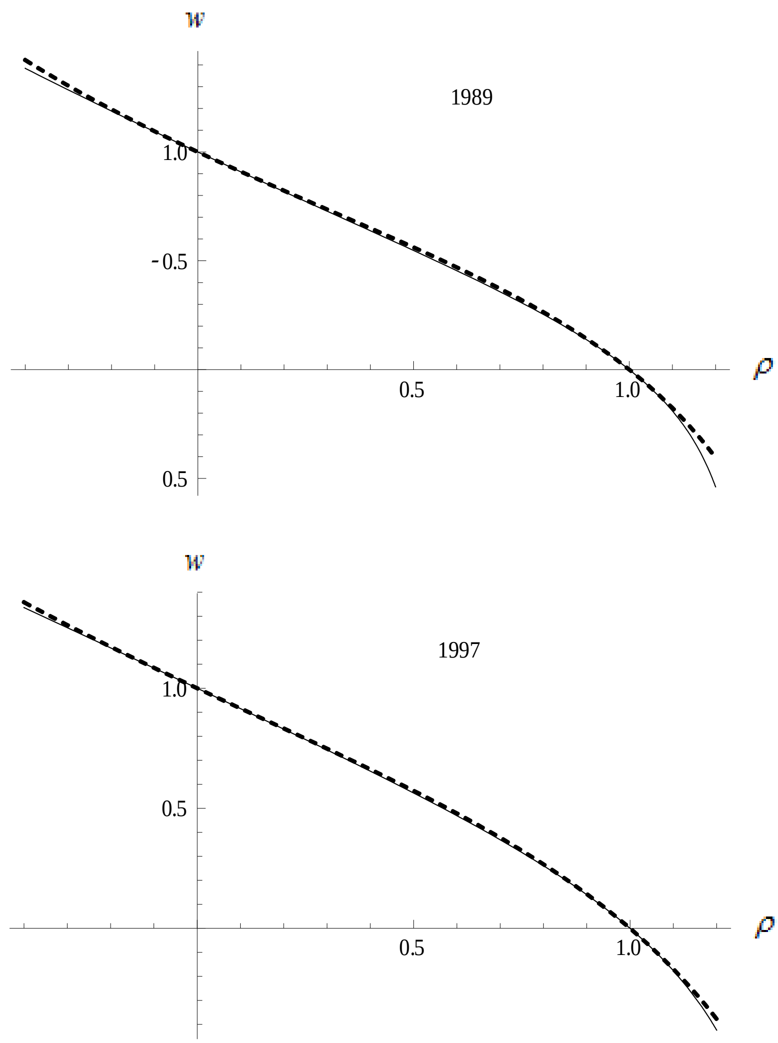

Figure 1: Actual and approximate wage-relative profit rate curves; years 1989 and 1997

CONCLUDING REMARKS

Using data from ten symmetric input-output tables of the Greek economy, it has been argued that Bienenfeld's second-order approximate wage-profit curve, which has at most one inflection point in the economically relevant interval of the profit rate, is accurate enough. This argument reduces to the skew distribution of the eigenvalues of the matrices of vertically integrated technical coefficients, and suggests that low-dimensional models, with no more than three industries, can be profitably used as surrogates for actual single-product economies.

Future research efforts should be directed to the 'inverse problem', that is, to (i) determine the conditions that notional production systems should fulfil in order to generate Bienenfeld's-like low-order wage-price-profit rate curves; and (ii) explore the structural interfaces of those systems with the actual ones.

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Notes

1. For the available data and the construction of the relevant variables, see Tsoulfidis and Mariolis (2007, pp. 435-437).
2. The transpose of a $\mathbf{1}\times n$ vector $\mathbf{y} \equiv [y_j]$ is denoted by $\mathbf{y}^{\mathrm{T}}$. Furthermore, $\lambda_{\mathbf{A}1}$ denotes the Perron-Frobenius (P-F) eigenvalue of a semi-positive $n\times n$ matrix $\mathbf{A}$, and $(\mathbf{x}_{\mathbf{A}1}^{\mathrm{T}}, \mathbf{y}_{\mathbf{A}1})$ denote the corresponding eigenvectors, while $\lambda_{\mathbf{A}k}$, $k = 2, ..., n$ and $|\lambda_{\mathbf{A}2}| \geq |\lambda_{\mathbf{A}3}| \geq ... \geq |\lambda_{\mathbf{A}n}|$, denote the non-dominant eigenvalues.
3. For the derivation of an alternative homographic approximation, i.e. $w^{\mathrm{S}}[1+\rho(\mathbf{vJz}^{\mathrm{T}}-1)]^{-1}$, see Mariolis (2015).

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