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THE NUMBER OF REGIMES IN AGGREGATE AND INDIVIDUAL TIME SERIES IN MARKOV SWITCHING MODEL: A STATIC MODEL STUDY

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Abstract: *This paper aims to explore the issue of whether the number of regimes in aggregate time series is similar to those in individual time series. Two methods of aggregation, which areequal and value weighted, are considered. A Monte Carlo simulation is carried out with different settings to investigate possible source of changes that could affect the number of regimes in aggregate time series. The results show that the number of regimes in aggregate time series is a function of individual time series, regardless of the aggregation method. This result is consistent with Francq and Zakoian (2001, 2002).*

Keywords: *Aggregate time series; Individual time series; Markov switching model; Simulation; Static model*

JEL classification: C13, C15, C43

1. INTRODUCTION

This paper concerns determination of the number of regimes (hereafter called *r*) in aggregate time series in Markov switching model (hereafter called the MS model). Suppose that one individual time series ($y_{1,t}$) has two regimes and another series ($y_{2,t}$) has three regimes. When combining these two time series into one aggregate time series an interesting issue is whether aggregate time serieswould have a similar number of regimes to that of individual time series ($y_{1,t}$ or $y_{2,t}$). This question is not only interesting in theory but also important in practical applications. For instance, investment portfolios of most investors generally have more than one investment asset for the purpose of diversification. Assuming that there are two assets (common stocks and real estates, for example) in a portfolio and both of them have different regimes (e.g., one asset has two regimes while another asset has three regimes), would a combination of these two assets in a portfolio produce two or three regimes?

There are two general methods for aggregating time series: temporal and cross-sectional (Granger, 1988). The cross-sectional aggregation is a method by which several time series are summed to form an aggregate time series at a given point in

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time. On the other hand, when a time series is generated on a daily or weekly basis and summed in a form of a monthly series, it can be referred to as the temporal aggregation. Quarterly or annual time series, for instance, may be constructed from monthly series. In the literature, some studies, including Girardin and Liu (2007) and Chan and Chan (2008), have examined the latter case by using the MS model. Chan *et al.* (2009), for example, investigate the effects of temporal aggregation on the MS model. They suggest that if a model is closed under temporal aggregation, parameters of the lower frequency model (or aggregate time series) can be directly implied by those of the higher frequency model (or individual time series). Little study, however, has examined the cross-sectional aggregation on the MS model.

This paper considers particularly two possible cross-sectional methods of combining individual series into one aggregate time series. The first method (an equal-weighted method) assumes that all individual series have the same weight. The second method (a value-weighed method) assumes that individual series have a different weight. The primary focus of this paper is on the number of regimes in aggregate time series, which is formed by the cross-sectional aggregation.

There are four papers related to this issue. They are Rose (1977), Zhang and Stine (2001), and Francq and Zakoian (2001, 2002). On the issue of cross-sectional aggregation of time series, Rose (1977) shows that the (p, q) orders of aggregate time series are a function of the orders of individual ARMA processes. In addition, this aggregate time series could be formed directly from independent ARMA models. More specifically, the weighted sum of n independent ARMA processes of orders (p_i, q_i), $i = 1, 2, \ldots, n$ is an ARMA process of order (p, q) where k is the total number of root repetitions among the polynomials and

$$p \leq \sum_{i=1}^{n} p_i - k,\ q \leq \max_i (p + q_i).$$

For example, if there are three independent time series without root repetitions and p_i for each of them is one, then when combining these three models into one aggregate time series, the order $p$of the aggregate time series would be less than or equal to three. Zhang and Stine (2001) first show that the auto-covariance function of a second-order stationary MS model without intercept term can be represented as that of VARMA model and the orders of VARMA model are directly linked with the number of regimes. Francq and Zakoian (2001, 2002) extend this result to *any* second-order stationary MS model (with or without intercept term). The results can be summarised as follows: (i) in the case of MS static model where both p and q are zero, there exists an ARMA(r–1, r–1) representation; (ii) in the case of MS model that allows autoregressive parameters to be varying, both p and q orders will be a function of the p, r, and the dimension of the processes (hereafter called h), which may be calculated as $r + r\,(hp)^2 – 1$ and $r + r\,(hp)^2 – 2$, respectively.

The above ideas when combined together imply that any second-order stationary MS models can be represented as the VARMA model, where both p and q can be calculated from r. In particular, the (p, q) orders of the VARMA model are a function of that of individual ARMA models and the number of regimesin the MS models is also a function of the orders of individual ARMA models. For example, if the true model of individual time series is a two-regime MS autoregressive model of order one ($r_i = 2$; $p_i = 1$), Francq and Zakoian (2001, 2002) suggest that individual time series admits an ARMA(1, 1) representation. According to Rose (1977), the aggregate time series also has an ARMA representation with $p \leq 3$ and $q \leq 4$ which translates into a MS autoregressive model with $r \leq 0.04$ and $p \leq 3$.

The motivation for this paper is stated as follows: First, the above analytical results only provide an upper bound with which the number of regimes in aggregate time series can be identified. It is interesting to see whether this bound holds true or not when confronted with actual data. Second, they may not apply to a MS static model where there are exogenous variables. Third, they may not apply to cases where some individual time series has number of regimes different from the others. A Monte Carlo simulation with different settings is proposed to investigate possible sources of changes that could affect the number of regimes in aggregate time series. These settings include a difference in number of individual series in aggregate time series, sample size, parameters, and noise level. The results suggest that the number of regimes in aggregate time series is a function of that of individual time series, regardless of whether the aggregation method is equal or value weighted. For example, if aggregate time series has two individual series (e.g., one series has two regimes and one exogenous variable, while another time series has three regimes and one exogenous variable), the numbers of regimes and variables in aggregate time series would be two and one, respectively. This result is consistent with Francq and Zakoian (2001, 2002).

The reminder of the paper is organised as follows. Section 2 presents the class of the MS models under consideration. Sections 3 and 4 present the set-up of Monte Carlo studies and their simulation results. Section 5 concludes the paper.

2. MODELS AND ASSUMPTIONS

This paper considers the MS static model of the following form:

$$y_t = v^{(s_t)} + A_l^{(s_t)} x_t + \sigma^{(s_t)} e_t, \tag{1}$$

where s_t are unobservable random variables that take values in the finite set $\{1, 2, \ldots, r\}$ and are independent of e_t and

e_t is independent and identically distributed random variables such that $E(e_t) = 0$.

The random variables, s_t, (hereafter referred to as regime variables) are assumed to be a temporally homogeneous first-order Markov chain on {1, 2, …, r} with transition matrix P = p_{ij}, i and $j \in$ {1, 2, …, r}, where p_{ij} = Prob($s_{t+1} = j \mid s_t = i$). It is also assumed that s_t are periodic and irreducible. Notice that s_t may or may not be stationary. If the stationarity assumption is imposed, the above conditions guarantee a unique row-stochastic vector $\pi = (\pi_1, \pi_2, \ldots, \pi_r)'$ such that $\pi P = \pi$ and π_i = Prob($s_t = i$) > 0 for all $i \in$ {1, 2, …, r} and all t. Under these assumptions, the model could be referred as a r-regime MS static model (or MS(r) static model).

It is assumed that r of the MS static model are unknown, so the interesting issue is to estimate r on the basis of a finite segment $y_n = (y_1, y_2, \ldots, y_T)$ of length T from Equation (1). This paper identifies r by using Schwarz (1978) information criterion (hereafter called BIC) since Awirothananon and Cheung (2009) show that BIC outperforms other information criteria (including Akaike (1974) information criterion, HQC (Hannan and Quinn, 1979), and MSC (Smith et al., 2006)) in joint determination of the numbers of states and variables. BIC can be calculated as $BIC = -2L + k \log(T)$, where L, k, and T are the maximised log likelihood value, the number of estimated parameters, and the total observations, respectively.

The MS model could be estimated by using a maximum likelihood (hereafter called ML) procedure. The ML algorithm of this model is based on the expectation maximisation algorithm discussed in Krolzig (1997). This algorithm is originally described by Dempster et al. (1977) as a general approach to iteratively compute the ML estimation technique. This technique is designed for general models where observed variables are dependent on some unobserved variables, s_t.

3. SIMULATION DESIGN

This paper considers two possible cases of forming aggregate time series. The first case assumes that all individual time series have the same regime (in this case is two regimes). Another case is that all individual time series have a different regime. All variables are independent and identically distributed random variables with zero mean and unit variance. Possible factors that could affect regime in aggregate time series are also included in the simulation settings as follows:

(i) MS static model: the true model consists of two regimes (or $r = 2$) and one exogenous variable with an intercept term in each regime. The true regression coefficients are $v^{(st)} = (0, 1)$ and $A^{(st)} = (0.3, 0.9)$. The r-regime variable, s_t, is a Markov chain with transition probabilities $p_{11} = 0.6$ and $p_{22} = 0.4$ and the initial probabilities are set to 0.1 and 0.9. The total number of observations, T, is initially set to 200 and $e_t \sim N(0, 1)$ and $\sigma^{(st)} = (0.5, 0.5)$.

(ii) MS static model with small/large coefficient: this paper considers variation from the setting (i). First, this paper changes the coefficient for both regimes to the same value: $v^{(st)}$ = (0, 1) and $A^{(st)}$ = (0.3, 0.3). Second, setting the intercept term for both regimes to the same value: $v^{(st)}$ = (0, 0) and $A^{(st)}$ = (0.3, 0.9).

(iii) MS static model with high/low transition probability: this paper also considers variations from the settings (i) and (ii) by changing the transition probability p_{11} from 0.6 to 0.9 and p_{22} from 0.4 to 0.1 and the initial probabilities from (0.1, 0.9) to (0.4, 0.6). To examine the impact of persistence in $\{s_t\}$, the transition probabilities p_{11} and p_{22} are also set to 0.9 so that their sum is greater than one.

(iv) MS static model with small/large sample and high/low noise level: the following variations from the settings (i), (ii), and (iii) are considered. First, to examine the impact of sample size on performance, this paper conducts the above simulations using T = 100, 400, and 500, respectively. Second, $s^{(st)}$= (1.0, 1.0) and (0.5, 1.0) for all T to understand the effect of a change in noise level.

The second case where a different regime is set to each individual time series is also investigated. In particular, this paper considers only two time series; one series has two regimes while another time series has three regimes. The simulation designs for this case are discussed below:

(v) For two-regime time series, the initial probabilities are set to 0.1 and 0.9 with the transition probabilities as $p11$ = 0.6; $p22$ = 0.4. The true regression coefficients are $v^{(st)}$ = (0, 1) and $A^{(st)}$= (0.3, 0.9). For the regime-specific case, the $s^{(st)}$is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2. For the three-regime time series, initial probabilities are set to 0.1, 0.3, and 0.6 with the transition probabilities as p_{11}= 0.6; p_{22}= 0.3; p_{33}= 0.1. The true coefficients are $v^{(st)}$ = (0, 0.4, 0.6) and $A^{(st)}$= (0.3, 0.4, 0.5). For the regime-specific case, $\sigma^{(st)}$is set to 1/3, 1/3, and 1/3 for s_t = regime 1, regime 2, and regime 3, respectively.

(vi) The models with small/large coefficient: the coefficient in the setting (v) will be set as follows: First, setting the coefficient for all regimes to the same value: $v^{(st)}$ = (0, 1) and $A^{(st)}$= (0.3, 0.3) for two-regime time series, while $v^{(st)}$ = (0, 0.4, 0.6) and $A^{(st)}$= (0.3, 0.3, 0.3) are assigned to three-regime time series. Second, setting the intercept coefficient for all regimes to the same value: $v^{(st)}$ =(0, 0) and $A^{(st)}$= (0.3, 0.9) are set to the two-regime time series, while the three-regime series has $v^{(st)}$ = (0, 0, 0) and $A^{(st)}$= (0.3, 0.4, 0.5).

(vii) The models with small/large sample and high/low noise level: the following variation from the settings (v) and (vi) is considered to change. First, to examine the impact of sample size on performance, the above simulations is conducted using T = 100, 400, and 500, respectively. Second, setting $\sigma^{(st)}$ to (1, 1) and (0.5, 0.1) for the two-regime time series, while the three-regime time series has $\sigma^{(st)}$= (1, 1, 1) and (1/3, 0.5, 1.0).

(viii) The models with high/low transition probabilities: the transition probabilities in the settings (v) to (vii) will be changed from (0.6, 0.4) and (0.6, 0.3, 0.1) to (0.9, 0.1) and (0.9, 0.08, 0.02), respectively. This paper also changes the initial probabilities from (0.1, 0.9) and (0.1, 0.3, 0.6) to (0.4, 0.6) and (0.4, 0.1, 0.5) for the two-regime and three-regime time series, respectively.

The number of individual time series, n, in this paper will be set to 3, 10, 30, 100, and 500, respectively. The simulations proceed by first generating an artificial time series, y_t, and exogenous variables, x_t, of length 500 + T according to the settings (i) and (viii) and setting initial values to zero. The first 500 pseudo-data points then are discarded in order to eliminate start-up effects, while the remaining T points are used to determine r minimising BIC over r regimes (r = 2 and 3). Since computations are very intensive, 1000 Monte Carlo replications are carried out for each setting to assess how often BIC selects the model with r regimes.

4. SIMULATION RESULTS

All results in this section are generated by using Ox Metrics version 3.40 (Doornik, 2002) and the MSVAR package version 1.32a (Krolzig, 1998).The data generating process is a two-regime MS static model with one exogenous variable for different sample size, parameters, and noise level. To ease interpretation, this paper hereinafter refers to any MS static model that observes the relationship as the expected model. The results will show how many times that aggregate time series has r regimes (r = 2, 3), if the true model of individual time series is a two-regime MS static model of one exogenous variable (r_i = 2) for different settings.These settings include a difference in number of individual series in aggregate time series, sample size, parameters, and noise level. Notice that only value-weighted method is firstly consideredto form aggregate time series. In particular, Table 1 shows the benchmark case where the initial probabilities of state 1 and state 2 are set to 0.4 and 0.6, respectively, and the transition probability of moving from state 1 (2) to state 2 (1) is set to 0.4 (0.6). Table 1first allows for change in regression coefficient only. The case allowing for change in intercept term is presented in Table 2. This paper further allows for both intercept term and regression coefficient to be regime-specific, which is reported in Table 3. In addition, Table 4 considers the case where there is a change in transition probabilities from (0.6, 0.4) to (0.9, 0.1), while Table 5 deals with a change in the initial probabilities. Table 6 considers the case where $\{s_t\}$ is allowed to be persistent.

Before proceeding into the details, two main noticeable results can be seen. First, an increase in noise level could reduce the frequency of two regimes and one exogenous variable selection. Second, increasing sample size does help to identify the true model. Allowing for change in regression coefficient only, Table 1shows that aggregate time series has more than 90% chance of having two regimes and one exogenous variable. Remarkably, the frequency of three-regime selection is zero when the sample size becomes larger. With 100 observations and the noise level of 0.5, for instance, the number of times that aggregate time series of three individual series has two regimes and one exogenous variable is 928 out of 1000. This frequency reduces to 911, when the noise level is high.

Table 2 exhibits a similar result; an increasing in sample size leads to a better frequency of the true model identification. As an example, if the sample size is 100 and the noise level is high, Table 2 indicates that the frequency of two regimes and one exogenous variable selection for 30 individual series in aggregate time series is 907. The frequency increases significantly to 970 when the sample size becomes 500 observations. Regardless of the number of individual time series being considered, increasing sample size or reducing the noise level could improve the chance of the true model selection, while allowing for change in variance could not.

Table 3, where the model allows for regime change in both intercept term and regression coefficient, basically reinforce observations from Tables 1 and 2. Compared to Table 1, however, Table 3 shows that the frequency of two regimes and one exogenous variable selection is generally higher. In the case of 200 observations with low noise level, for instance, the frequency of the true model identification for 30 individual series in aggregate time series is 970 out of 1000 in Table 3. This number is slightly higher than Table 1 because the corresponding number is 965.

Change in transition probabilities from (0.6, 0.4) to (0.9, 0.1), as shown in Table 4, generally leads to a slight increase in frequency of two regimes and one exogenous variable selection. The results reveal that change in initial probabilities from (0.1, 0.9) to (0.4, 0.6) leads to a better identification of the model with two regimes and one exogenous variable. With 200 observations and high noise level, for instance, the frequency of true model selection for 30 individual series in aggregate time series is 952. This number is higher than Table 1 since the corresponding number is 943 out of 1000. In addition, persistence in $\{s_t\}$ generally increases the frequency of selecting the model with two regimes and one exogenous variable, as depicted in Table 6.

This paper also considers another case, where all individual series having the same weight when forming aggregate time series. Similar assumptions of value-weighted model (see Tables 1 to 6) are employed. The results suggest that the results of equal-weighted method are generally higher than those of value-weighted method, regardless of changes in regression coefficient or an intercept term or in both. With 400 observations and low noise level, for example, the frequency of the model identified with two regimes and one exogenous variable for 500 individual series in aggregate time series is 989. This number is marginally higher than that allows for changing in intercept term only, which is 986.

Table 1

Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Regression Coefficient

This table summarises frequencies of selection of a value weighted MS(r) static model based on Schwarz (1978) information criterion (BIC) as:

$$\mathrm{BIC} = -2L + k\log(T),$$

where L, T, and k are the maximised log likelihood value, the total number of observations, and the number of estimated parameters, respectively.
The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_t = v^{(st)} + A^{(st)}x_t + s^{(st)}e_t.$$

The initial probabilities are set to 0.1 and 0.9 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(st)} = (v^{(1)}, v^{(2)})$ and $A^{(st)} = (A^{(1)}, A^{(2)})$, where $v^{(st)} = (0, 0)$ and $A^{(st)} = (0.3, 0.9)$. Notice that the total replication is 1000 and for the regime-specific case, $s^{(st)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2.

No. of individual	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
3	0.5	2	928	69	973	27	986	14	989	11
		3	2	1	0	0	0	0	0	0
	1.0	2	911	89	921	79	945	58	970	30
		3	0	0	0	0	0	0	0	0
	Mix	2	916	83	964	36	976	24	986	14
		3	0	1	0	0	0	0	0	0
10	0.5	2	953	47	975	25	983	17	993	7
		3	0	0	0	0	0	0	0	0
	1.0	2	937	63	972	28	973	27	985	15
		3	0	0	0	0	0	0	0	0
	Mix	2	950	49	973	27	981	19	992	78
		3	0	1	0	0	0	0	0	0
30	0.5	2	942	58	965	35	975	25	988	12
		3	0	0	0	0	0	0	0	0
	1.0	2	906	94	943	57	947	53	967	31
		3	0	0	0	0	0	0	2	0
	Mix	2	926	73	948	52	955	45	978	22
		3	0	1	0	0	0	0	0	0
100	0.5	2	956	44	971	29	972	28	976	24
		3	0	0	0	0	0	0	0	0

Table 1
Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Regression Coefficient (cont'd)

No. of individual	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
100	1.0	2	902	98	922	88	932	66	956	44
		3	0	0	0	0	0	2	0	0
	Mix	2	927	73	933	67	960	40	964	36
		3	0	0	0	0	0	0	0	0
500	0.5	2	953	47	975	25	980	20	982	18
		3	0	0	0	0	0	0	0	0
	1.0	2	905	95	928	72	938	62	974	26
		3	0	0	0	0	0	0	0	0
	Mix	2	925	74	953	47	958	42	977	23
		3	0	1	0	0	0	0	0	0

Table 2
Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Intercept Term

This table summarises frequencies of selection of a value weighted MS(r) static model based on Schwarz (1978) information criterion (BIC) as:

$$BIC = -2L + k\log(T),$$

where L, T, and k are the maximised log likelihood value, the total number of observations, and the number of estimated parameters, respectively. The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_t = v^{(st)} + A^{(st)}x_t + s^{(st)}e_t.$$

The initial probabilities are set to 0.1 and 0.9 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(st)} = (v^{(1)}, v^{(2)})$ and $A^{(st)} = (A^{(1)}, A^{(2)})$, where $v^{(st)} = (0,1)$ and $A^{(st)} = (0.3, 0.3)$. Notice that the total replication is 1000 and for the regime-specific case, $s^{(st)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2.

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
3	0.5	2	961	39	972	28	982	18	996	4
		3	0	0	0	0	0	0	0	0
	1.0	2	861	139	893	107	979	21	984	16
		3	0	0	0	0	0	0	0	0
	Mix	2	920	80	960	40	981	19	985	15
		3	0	0	0	0	0	0	0	0

Table 2
Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Intercept Term (cont'd)

No. of individual	*Sigma*	*No. of regimes imposed*	*100 observations* 1 *variable*	*100 observations* 2 *variables*	*200 observations* 1 *variable*	*200 observations* 2 *variables*	*400 observations* 1 *variable*	*400 observations* 2 *variables*	*500 observations* 1 *variable*	*500 observations* 2 *variables*
10	0.5	2	954	46	976	24	983	17	994	6
		3	0	0	0	0	0	0	0	0
	1.0	2	929	71	958	42	976	21	983	17
		3	0	0	0	0	3	0	0	0
	Mix	2	946	54	972	28	979	21	992	8
		3	0	0	0	0	0	0	0	0
30	0.5	2	950	50	966	34	994	6	998	2
		3	0	0	0	0	0	0	0	0
	1.0	2	907	81	964	36	969	31	970	30
		3	0	12	0	0	0	0	0	0
	Mix	2	925	75	965	35	983	17	995	5
		3	0	0	0	0	0	0	0	0
100	0.5	2	954	46	989	11	992	8	995	5
		3	0	0	0	0	0	0	0	0
	1.0	2	946	54	956	44	964	36	990	10
		3	0	0	0	0	0	0	0	0
	Mix	2	947	53	962	38	990	10	992	8
		3	0	0	0	0	0	0	0	0
500	0.5	2	939	61	979	21	986	14	993	7
		3	0	0	0	0	0	0	0	0
	1.0	2	886	113	901	99	946	54	974	26
		3	0	1	0	0	0	0	0	0
	Mix	2	931	70	967	33	985	15	990	10
		3	0	0	0	0	0	0	0	0

Table 3

Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Both Regression Coefficient and Intercept Term

This table summarises frequencies of selection of a value weighted MS(r) static model based on Schwarz (1978) information criterion (BIC) as:

$$\text{BIC} = -2L + k\log(T),$$

where L, T, and k are the maximised log likelihood value, the total number of observations, and the number of estimated parameters, respectively.

The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_t = v^{(st)} + A^{(st)}x_t + s^{(st)}e_t.$$

The initial probabilities are set to 0.1 and 0.9 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(st)} = (v^{(1)}, v^{(2)})$ and $A^{(st)} = (A^{(1)}, A^{(2)})$, where $v^{(st)} = (0, 1)$ and $A^{(st)} = (0.3, 0.9)$. Notice that the total replication is 1000 and for the regime-specific case, $s^{(st)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2.

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
3	0.5	2	952	48	967	33	986	14	989	11
		3	0	0	0	0	0	0	0	0
	1.0	2	910	89	945	55	946	54	967	34
		3	0	1	0	0	0	0	0	0
	Mix	2	950	50	958	42	969	31	985	15
		3	0	0	0	0	0	0	0	0
10	0.5	2	930	70	989	11	991	9	994	6
		3	0	0	0	0	0	0	0	0
	1.0	2	925	75	938	62	980	20	987	13
		3	0	0	0	0	0	0	0	0
	Mix	2	926	73	980	20	983	17	993	7
		3	0	1	0	0	0	0	0	0
30	0.5	2	958	42	970	30	986	14	993	7
		3	0	0	0	0	0	0	0	0
	1.0	2	916	84	959	41	973	27	987	13
		3	0	0	0	0	0	0	0	0
	Mix	2	926	71	968	32	982	18	990	10
		3	0	0	0	0	0	0	0	0
100	0.5	2	944	56	974	26	983	17	993	7
		3	0	0	0	0	0	0	0	0

Table 3
Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Both Regression Coefficient and Intercept Term (cont'd)

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
100	1.0	2	904	96	916	84	980	20	984	16
		3	0	0	0	0	0	0	0	0
	Mix	2	933	67	949	51	981	19	993	7
		3	0	0	0	0	0	0	0	0
500	0.5	2	951	49	983	17	989	11	995	5
		3	0	0	0	0	0	0	0	0
	1.0	2	909	91	941	39	953	47	991	1
		3	0	0	0	0	0	0	0	0
	Mix	2	943	57	956	44	977	23	993	7
		3	0	0	0	0	0	0	0	0

Table 4
Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Transition Probabilities and Regression Coefficient

This table summarises frequencies of selection of a value weighted MS(r) static model based on Schwarz (1978) information criterion (BIC) as:

$$\text{BIC} = -2L + k\log(T),$$

where L, T, and k are the maximised log likelihood value, the total number of observations, and the number of estimated parameters, respectively. The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_t = v^{(st)} + A^{(st)}x_t + s^{(st)}e_t.$$

The initial probabilities are set to 0.1 and 0.9 with the transition probabilities as $p_{11} = 0.9$; $p_{22} = 0.1$. The true regression coefficients are $v^{(st)} = (v^{(1)}, v^{(2)})$ and $A^{(st)} = (A^{(1)}, A^{(2)})$, where $v^{(st)} = (0, 0)$ and $A^{(st)} = (0.3, 0.9)$. Notice that the total replication is 1000 and for the regime-specific case, $s^{(st)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2.

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
3	0.5	2	962	38	979	21	993	7	994	6
		3	0	0	0	0	0	0	0	0
	1.0	2	937	62	952	48	977	23	980	20
		3	0	1	0	0	0	0	0	0
	Mix	2	940	60	965	35	980	20	984	16
		3	0	0	0	0	0	0	0	0

Table 4
Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Transition Probabilities and Regression Coefficient (cont'd)

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
10	0.5	2	950	50	967	33	981	19	997	3
		3	0	0	0	0	0	0	0	0
	1.0	2	904	88	918	82	963	37	986	14
		3	0	8	0	0	0	0	0	0
	Mix	2	914	86	921	79	970	30	988	12
		3	0	0	0	0	0	0	0	0
30	0.5	2	938	62	975	25	978	22	992	8
		3	0	0	0	0	0	0	0	0
	1.0	2	908	92	939	61	906	94	980	20
		3	0	0	0	0	0	0	0	0
	Mix	2	933	65	966	34	970	30	985	15
		3	0	2	0	0	0	0	0	0
100	0.5	2	957	43	974	26	975	25	983	17
		3	0	0	0	0	0	0	0	0
	1.0	2	886	113	899	89	907	93	979	21
		3	0	1	6	6	0	0	0	0
	Mix	2	905	95	943	57	973	27	982	18
		3	0	0	0	0	0	0	0	0
500	0.5	2	925	75	968	32	991	9	994	6
		3	0	0	0	0	0	0	0	0
	1.0	2	890	109	952	48	979	21	962	38
		3	0	1	0	0	0	0	0	0
	Mix	2	901	98	956	44	988	12	992	8
		3	1	0	0	0	0	0	0	0

Table 5

Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Initial Probabilities and Regression Coefficient

This table summarises frequencies of selection of a value weighted MS(r) static model based on Schwarz (1978) information criterion (BIC) as:

$$\text{BIC} = -2L + k\log(T),$$

where L, T, and k are the maximised log likelihood value, the total number of observations, and the number of estimated parameters, respectively. The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_t = v^{(st)} + A^{(st)}x_t + s^{(st)}e_t.$$

The initial probabilities are set to 0.4 and 0.6 with the transition probabilities as $p_{11} = 0.6$; $p_{22} = 0.4$. The true regression coefficients are $v^{(st)} = (v^{(1)}, v^{(2)})$ and $A^{(st)} = (A^{(1)}, A^{(2)})$, where $v^{(st)} = (0, 0)$ and $A^{(st)} = (0.3, 0.9)$. Notice that the total replication is 1000 and for the regime-specific case, $s^{(st)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2.

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
3	0.5	2	948	52	968	32	983	17	985	15
		3	0	0	0	0	0	0	0	0
	1.0	2	921	76	950	50	947	53	981	19
		3	0	3	0	0	0	0	0	0
	Mix	2	925	75	950	50	969	31	984	16
		3	0	0	0	0	0	0	0	0
10	0.5	2	950	50	961	39	964	36	982	18
		3	0	0	0	0	0	0	0	0
	1.0	2	943	56	946	54	954	46	965	35
		3	0	1	0	0	0	0	0	0
	Mix	2	946	54	961	39	963	37	970	30
		3	0	0	0	0	0	0	0	0
30	0.5	2	944	56	966	34	979	21	984	16
		3	0	0	0	0	0	0	0	0
	1.0	2	906	94	952	48	957	43	982	18
		3	0	0	0	0	0	0	0	0
	Mix	2	934	66	961	39	970	30	983	10
		3	0	0	0	0	0	0	7	0
100	0.5	2	947	53	971	29	990	10	998	2
		3	0	0	0	0	0	0	0	0

Table 5
Frequencies of Selection of a Value Weighted MS(2) Static Model – Change in Initial Probabilities and Regression Coefficient (cont'd)

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
100	1.0	2	926	73	928	72	977	23	977	23
		3	1	0	0	0	0	0	0	0
	Mix	2	940	60	946	54	989	11	980	20
		3	0	0	0	0	0	0	0	0
500	0.5	2	963	37	974	26	981	19	988	12
		3	0	0	0	0	0	0	0	0
	1.0	2	940	60	964	36	959	41	982	18
		3	0	0	0	0	0	0	0	0
	Mix	2	939	61	970	30	975	25	983	17
		3	0	0	0	0	0	0	0	0

Table 6
Frequencies of Selection of a Value Weighted MS(2) Static Model – Persistence in $\{s_t\}$

This table summarises frequencies of selection of a value weighted MS(r) static model based on Schwarz (1978) information criterion (BIC) as:

$$\text{BIC} = -2L + k\log(T),$$

where L, T, and k are the maximised log likelihood value, the total number of observations, and the number of estimated parameters, respectively. The true data generating process is the MS static model with two regimes and one exogenous variable, as follows:

$$y_t = v^{(st)} + A^{(st)}x_t + s^{(st)}e_t.$$

The initial probabilities are set to 0.1 and 0.9 with the transition probabilities as $p_{11} = 0.9$; $p_{22} = 0.9$. The true regression coefficients are $v^{(st)} = (v^{(1)}, v^{(2)})$ and $A^{(st)} = (A^{(1)}, A^{(2)})$, where $v^{(st)} = (0, 0)$ and $A^{(st)} = (0.3, 0.9)$. Notice that the total replication is 1000 and for the regime-specific case, $s^{(st)}$ is set to 0.5 if s_t = regime 1 and 1 if s_t = regime 2.

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
3	0.5	2	962	38	972	28	980	20	987	13
		3	0	0	0	0	0	0	0	0
	1.0	2	906	94	921	77	955	45	955	45
		3	0	0	2	0	0	0	0	0
	Mix	2	913	86	925	75	963	37	964	36
		3	0	1	0	0	0	0	0	0

Table 6
Frequencies of Selection of a Value Weighted MS(2) Static Model – Persistence in $\{s_t\}$ (cont'd)

No. of individual time series	*Sigma*	*No. of regimes imposed*	*100 observations*		*200 observations*		*400 observations*		*500 observations*	
			1 variable	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*	*1 variable*	*2 variables*
10	0.5	2	935	64	966	34	987	13	992	8
		3	0	1	0	0	0	0	0	0
	1.0	2	909	90	935	65	964	36	976	24
		3	0	1	0	0	0	0	0	0
	Mix	2	917	82	938	62	970	30	978	22
		3	1	0	0	0	0	0	0	0
30	0.5	2	947	53	976	24	987	13	995	5
		3	0	0	0	0	0	0	0	0
	1.0	2	943	57	949	51	965	35	980	20
		3	0	0	0	0	0	0	0	0
	Mix	2	943	57	976	24	979	21	985	15
		3	0	0	0	0	0	0	0	0
100	0.5	2	970	30	975	25	991	9	992	8
		3	0	0	0	0	0	0	0	0
	1.0	2	923	77	951	41	966	34	970	30
		3	0	0	0	0	0	0	0	0
	Mix	2	942	58	959	41	968	32	973	27
		3	0	0	0	0	0	0	0	0
500	0.5	2	951	49	974	26	983	17	998	2
		3	0	0	0	0	0	0	0	0
	1.0	2	906	94	908	92	958	42	979	21
		3	0	0	0	0	0	0	0	0
	Mix	2	912	88	967	33	975	25	980	20
		3	0	0	0	0	0	0	0	0

Other three possible factors that could influence the number of regimes in aggregate time series are changes in transition and initial probabilities and allowing for persistence in $\{s_t\}$. The results suggest that changes in transition and initial probabilities lead to an increase in frequency of the true model identification. In addition, allowing for persistence in $\{s_t\}$ does increase the frequency. As an example, if the sample size is 400 and low noise level, the results indicate that the frequency of two regimes and one exogenous variable selection for 10 individual series in aggregate time series is 992 out of 1000. This number is slightly higher than that allows for changing in regression coefficient only, which is 983.

This paper further investigates an additional factor where individual time series have a different *r*. All previous settings are used with this case. Only two time series are considered for this case; one series has two regimes, while another series has three regimes. The results suggest that aggregate time series has more than 90% chance of having two regimes and one exogenous variable. In addition, aggregate time series tends to have two regimes rather than three regimes. With 100 observations and the noise level of 0.5, for instance, the chance that aggregate time series with individual series equally weighted, has two regimes and one exogenous variable is 95.4%. This frequency slightly increases to 99.5% when the sample size is 500 observations, while the chance of having three regimes for aggregate time series is zero.This paper also explores changes in transition and initial probabilities that could affect *r* in aggregate time series. The same settings as the above are employed to investigate the effect from these two changes. The results suggest that these changes could not alter the main conclusion.

5. CONCLUSION

This paper investigates the issue of whether the number of regimes in aggregate time series is similar to those in individual time series. A Monte Carlo simulation is carried out with different settings to investigate possible sources of changes that could affect the number of regimes in aggregate time series. For aggregation purpose, this paper considers both equal and value weighted methods. The results suggest that the number of regimes in aggregate time series are a function of individual series, regardless of whether the aggregation method is equal or value weighted. For example, if aggregate time series has two individual series (e.g., one series has two regimes and one exogenous variable, while another time series has three regimes and one exogenous), the numbers of regimes and variables in aggregate time series would be two and one, respectively. This result is consistent with Francq and Zokoian(2001, 2002).

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