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FUZZY METRIC ON ORDER INTERVALS OF FUZZY NUMBER

Abstract: *In this paper we study the norm induced by the supremum metric on the space of order intervals of fuzzy numbers And then we propose a method for constructing a norm on the quotient space of order intervals of fuzzy numbers. Finally we investigate some properties of this metric.*

Keywords: *Fuzzy number, Order Interval, norm*

1. INTRODUCTION

Chang and Zadeh [5] introduced the concept of fuzzy number with the consideration of the properties of probability functions.Since then a lot of mathematicians have been studying on fuzzy number, and have obtained many results [3,6,8,10,13,14].

In classical mathematics, if X is a normed space with norm $\|\cdot\|$ defines a metric on *X*. Thus a normed space is naturally a metric space and all metric space concepts are meaningful.

The upper and lower bounds of such an order intervalof fuzzy number corresponds to the maximum and minimum imaginable decisions, respectively. Thus the order intervals signifies the extent of tolerance that quantity could possibly take.However,the ordering of fuzzy numbers is not always easy to specify, and hence it is a basic problem in fuzzy optimization and fuzzy decision making systems. Since the set of fuzzy numbers is not totally ordered , there are many different methods proposed in the literature to compare fuzzy numbers. Some of these ordering methods can be found in [2, 4, 11, 12] and [15].

In this paper to propose a method for constructing a norm of an order interval of fuzzy numbers. Further this norm is very natural and works well with the induced on the quotient space.

2. PRELIMINARIES

Definition 2.1

A fuzzy number is a function $X:\mathbb{R}\to[0,1]$ satisfying the following properties:

(i) X is normal,that is there exists a $t_0\in\mathbb{R}$ such that $X(t_0)=1$

(ii) X is fuzzy convex, that is $t,u\in\mathbb{R}$ and $\lambda\in[0,1]$ we have $X(\lambda t+(1-\lambda)u)\geq\min\{X(t),X(u)\}$;

(iii) X is upper semi-continuous;

(iv) The closure of the set $\{t\in\mathbb{R}:X(t)>0\}$, denoted by X^0, is compact.

These properties imply that for each $\alpha\in(0,1]$, the α-level set $X^\alpha=\{t\in\mathbb{R}:X(t)\geq\alpha\}=\left[\underline{X}^\alpha,\overline{X}^\alpha\right]$ is a non-empty compact convex subset of $\mathbb{R}$, as the support X^0.

A real number r can be considered as a fuzzy number $\tilde{r}$ defined by

$$\tilde{r}(t)=\begin{cases}1, & if\ \ t=r\\ 0, & t\neq r\end{cases}$$

The set of all fuzzy numbers is usually denoted by $L(\mathbb{R})$.

Definition 2.2

Let $L(\mathbb{R})$ be a set of all fuzzy number. A function $D:L(\mathbb{R})\times L(\mathbb{R})\to\mathbb{R}_0^+=\{x\in\mathbb{R}:x\geq 0\}$ defined by $D(X,Y)=\sup\limits_{\alpha\in[0,1]}\max\left\{\left|\underline{X}^\alpha-\underline{Y}^\alpha\right|,\left|\overline{X}^\alpha-\overline{Y}^\alpha\right|\right\}$ is called the supremum metric.

Definition2.3

An order interval $[X,Y]$ of fuzzy numbers can be defined as $[X,Y]=\{\mathrm{z}\in\mathrm{L}(\mathbb{R}):X\leq Z\leq Y\}$ where $X,Y\in L(\mathbb{R})$. This interval is an ordinary (non-fuzzy) set whose elements are fuzzy numbers.

Definition 2.4

Let $I[L(\mathbb{R})]$ be the set of all order intervals of fuzzy numbers. A metric $D_I : I[L(\mathbb{R})]\times I[L(\mathbb{R})] \to \mathbb{R}_0^+$ is defined on $D_I\left([X_1,Y_1],[X_2,Y_2]\right) = \max\left\{D(X_1,X_2), D(Y_1,Y_2)\right\}$

3. MAIN RESULTS

Definition 3.1

The norm structure on $I[L(\mathbb{R})]$ is $\|.\|: I[L(\mathbb{R})] \to \mathbb{R}$ defined by

$$\|[X_1,Y_1]\| = D_I([X_1,Y_1],0)$$

$$= \max\{D(X_1,0), D(Y_1,0)\}$$

$$= \max\left\{\sup\max\{|\underline{X_1}^\alpha|,|\overline{X}_1^\alpha|\}, \sup\max\{|\underline{Y_1}^\alpha|,|\overline{Y}_1^\alpha|\}\right\}$$

Theorem 3.2

The function $\|.\|$ is a norm on $I[L(\mathbb{R})]$

Proof

(i) It is obvious that $\|[X_1,Y_1]\| \geq 0$ for all $[X_1,Y_1] \in I[L(\mathbb{R})]$,

$$\max\left\{\sup\max\{|\underline{X_1}^\alpha|,|\overline{X}_1^\alpha|\}, \sup\max\{|\underline{Y_1}^\alpha|,|\overline{Y}_1^\alpha|\}\right\} = 0$$

Therefore $\|[X_1,Y_1]\| = 0$ then there exits $k \in [0,1]$

such that $\left[[\underline{X_1}(k),\underline{Y_1}(k)],[\overline{X_1}(k),\overline{Y_1}(k)]\right] \neq \{0\}$

So $\max\left\{\sup\max\{|\underline{X_1}^\alpha|,|\overline{X}_1^\alpha|\}, \sup\max\{|\underline{Y_1}^\alpha|,|\overline{Y}_1^\alpha|\}\right\} \neq 0$

Therefore $\|[X_1,Y_1]\| > 0$

(ii) For all $[X_1,Y_1] \in I[L(\mathbb{R})]$ and $b \in \mathbb{R}$

then $\|b[X_1,Y_1]\| = D_I\left(b[X_1,Y_1],0\right)$

$$= \max\left\{\sup\max\{|b|\left|\underline{X_1}^{\alpha}\right|, \left|\overline{X}_1^{\alpha}\right|\}, \sup\max\{|b|\left|\underline{Y_1}^{\alpha}\right|, \left|\overline{Y}_1^{\alpha}\right|\}\right\}$$

$$= |b|\max\left\{\sup\max\{\left|\underline{X_1}^{\alpha}\right|, \left|\overline{X}_1^{\alpha}\right|\}, \sup\max\{\left|\underline{Y_1}^{\alpha}\right|, \left|\overline{Y}_1^{\alpha}\right|\}\right\}$$

$$= |b|\,\|[X_1, Y_1]\|$$

(iii) For all $[X_1, Y_1], [X_2, Y_2] \in I[L(\mathbb{R})]$

then $\|[X_1, Y_1] + [X_2, Y_2]\| = \|[X_1 + X_2, Y_1 + Y_2]\|$

$$= D_I\left([X_1 + X_2, Y_1 + Y_2], 0\right)$$

$$= \max\left\{\sup\max\{\left|\underline{(X_1 + X_2)}^{\alpha}\right|, \left|\overline{(X_1 + X_2)}^{\alpha}\right|\}, \sup\max\{\left|\underline{(Y_1 + Y_2)}^{\alpha}\right|, \left|\overline{(Y_1 + Y_2)}^{\alpha}\right|\}\right\}$$

$$\leq \max\left\{\sup\max\{\left|\underline{X_1}^{\alpha}\right| + \left|\underline{X_2}^{\alpha}\right|, \left|\overline{X}_1^{\alpha}\right| + \left|\overline{X}_2^{\alpha}\right|\}, \sup\max\{\left|\underline{Y_1}^{\alpha}\right| + \left|\underline{Y_2}^{\alpha}\right|, \left|\overline{Y}_1^{\alpha}\right| + \left|\overline{Y}_2^{\alpha}\right|\}\right\}$$

$$\leq \max\left\{\sup\max\{\left|\underline{X_1}^{\alpha}\right|, \left|\overline{X}_1^{\alpha}\right|\}, \sup\max\{\left|\underline{Y_1}^{\alpha}\right|, \left|\overline{Y}_1^{\alpha}\right|\}\right\} +$$

$$\max\left\{\sup\max\{\left|\underline{X_2}^{\alpha}\right|, \left|\overline{X}_2^{\alpha}\right|\}, \sup\max\{\left|\underline{Y_2}^{\alpha}\right|, \left|\overline{Y}_2^{\alpha}\right|\}\right\}$$

$$= \|[X_1, Y_1]\| + \|[X_2, Y_2]\|.$$

Hence $\|.\|$ is a norm on $I[L(\mathbb{R})]$

Theorem 3.3

The function D_I satisfies following properties

(i) $D_I([X_1, Y_1], [X_2, Y_2]) \geq 0$ for any $[X_1, Y_1], [X_2, Y_2] \in I[L(\mathbb{R})]$

(ii) $D_I([X_1, Y_1], [X_2, Y_2]) = D_I([X_2, Y_2], [X_1, Y_1])$ for any $[X_1, Y_1], [X_2, Y_2] \in I[L(\mathbb{R})]$

(iii) $D_I([X_1,Y_1],[X_2,Y_2]) \le D_I([X_1,Y_1],[X_3,Y_3]) + D_I([X_3,Y_3],[X_2,Y_2])$ for any $[X_1,Y_1],[X_2,Y_2],[X_3,Y_3] \in I[L(\mathbb{R})]$

(iv) $D_I([X_1,Y_1],[X_1,Y_1]) \ne 0$ for any fuzzy number $[X_1,Y_1] \notin \mathbb{R}$

Proof

(i) By the definition of D_I it is obvious $D_I([X_1,Y_1],[X_2,Y_2]) \ge 0$ for any $[X_1,Y_1],[X_2,Y_2] \in I[L(\mathbb{R})]$

(ii) For any $[X_1,Y_1],[X_2,Y_2] \in I[L(\mathbb{R})]$

then $D_I\left([X_1,Y_1],[X_2,Y_2]\right) = \max\left\{D(X_1,X_2), D(Y_1,Y_2)\right\}$

$$= \max\left\{\sup\max\{\left|\underline{(X_1-X_2)}^{\alpha}\right|, \left|\overline{(X_1-X_2)}^{\alpha}\right|\}, \sup\max\{\left|\underline{(Y_1-Y_2)}^{\alpha}\right|, \left|\overline{(Y_1-Y_2)}^{\alpha}\right|\}\right\}$$

$$= \max\left\{\sup\max\{\left|\underline{(X_2-X_1)}^{\alpha}\right|, \left|\overline{(X_2-X_1)}^{\alpha}\right|\}, \sup\max\{\left|\underline{(Y_2-Y_1)}^{\alpha}\right|, \left|\overline{(Y_2-Y_1)}^{\alpha}\right|\}\right\}$$

$$= \max\left\{D(X_2,X_1), D(Y_2,Y_1)\right\}$$

$$= D_I\left([X_2,Y_2],[X_1,Y_1]\right)$$

(iii) To prove triangle inequality for any fixed $\alpha \in [0,1]$ and for any $[X_1,Y_1],[X_2,Y_2],[X_3,Y_3] \in I[L(\mathbb{R})]$

$$D_I\left([X_1,Y_1],[X_2,Y_2]\right) = \left\|[X_1,Y_1]-[X_2,Y_2]\right\|$$

$$= \left\|[X_1-X_2, Y_1-Y_2]\right\|$$

$$= D_I([X_1-X_2, Y_1-Y_2], 0)$$

$$= \max\{D(X_1-X_2,0), D(Y_1-Y_2,0)\}$$

$$\le \max\left\{D(X_1,X_3)+D(X_3,X_2), D(Y_1,Y_3)+D(Y_3,Y_2)\right\}$$

$$\le \max\left\{D(X_1,X_3), D(Y_1,Y_3)\right\} + \max\{D(X_3,X_2), D(Y_3,Y_2\right\}$$

$$= D_I\left([X_1,Y_1],[X_3,Y_3]\right) + D_I\left([X_3,Y_3]\right),[X_2,Y_2]$$

(iv) For all $[X_1, Y_1] \in I[L(\mathbb{R})]$

then $D_I\left([X_1,Y_1],[X_1,Y_1]\right) = \left\|[X_1,Y_1]-[X_1,Y_1]\right\|$

$= D_I([X_1 - X_1, Y_1 - Y_1], 0)$

$= \max\left\{\sup\max\{\left|\underline{(X_1+X_1)}^\alpha\right|, \left|\overline{(X_1+X_1)}^\alpha\right|\}, \sup\max\{\left|\underline{(Y_1+Y_1)}^\alpha\right|, \left|\overline{(Y_1+Y_1)}^\alpha\right|\}\right\}$

Since $\left\|[X_1,Y_1]\right\| \neq 0$ there exists $k \in [0,1]$ such that $\left[[\underline{X_1}(k), \underline{Y_1}(k)], [\overline{X_1}(k), \overline{Y_1}(k)]\right] \neq \{0\}$

$= \max\left\{\sup\max\{\left|\underline{(X_1+X_1)}^\alpha\right|, \left|\overline{(X_1+X_1)}^\alpha\right|\}, \sup\max\{\left|\underline{(Y_1+Y_1)}^\alpha\right|, \left|\overline{(Y_1+Y_1)}^\alpha\right|\}\right\} \neq \{0\}$

therefore $D_I\left([X_1,Y_1],[X_1,Y_1]\right) = \neq \{0\}$.

Definition 3.4

Define a function $D_I : I[L(\mathbb{R})] \times I[L(\mathbb{R})] \to \mathbb{R}_0^+$ by

$$D_I\left([X_1,Y_1],[X_2,Y_2]\right) = \left\|[X_1,Y_1]-[X_2,Y_2]\right\|$$

$$= \inf_{\substack{X \in [X_1,Y_1] \\ Y \in [X_2,Y_2]}} D(X_1, Y_1)$$

$$= \inf_{\substack{X \in [X_1,Y_1] \\ Y \in [X_2,Y_2]}} \sup\max\left\{\left|\underline{X}^\alpha - \underline{Y}^\alpha\right|, \left|\overline{X}^\alpha - \overline{Y}^\alpha\right|\right\}$$

Definition 3.5

Define a function $\|.\| : \mathcal{F} \to \mathbb{R}$ as $\left\|[X_1,Y_1]\right\| = D_I([X_1,Y_1],0)$

$$= \inf_{X \in [X_1,Y_1]} \sup\max\left\{\left|\underline{X}^\alpha\right|, \left|\overline{X}^\alpha\right|\right\}$$

Theorem 3.6

The function $\| . \|$ norm in (3.5) is norm on $L(\mathbb{R})$

Proof

(i) It is obvious that $\|[X_1, Y_1]\| \geq 0$ for all $[[X_1, Y_1]] \in I[L(\mathbb{R})]$ and

$$= \inf_{X \in [X_1, Y_1]} \sup \max \left\{ \left| \underline{X}^{\alpha} \right|, \left| \overline{X}^{\alpha} \right| \right\} = \{0\}$$

Therefore $\|[X_1, Y_1]\| = 0$

Converslyif $\|[X_1, Y_1]\| \neq 0$ then there exits $k_0 \in [0,1]$ suchthat $\{ \left| \underline{X}^{\alpha} \right|, \left| \overline{X}^{\alpha} \right| \} \neq \{0\}$

So $\inf_{X \in [X_1, Y_1]} \sup \max \left\{ \left| \underline{X}^{\alpha} \right|, \left| \overline{X}^{\alpha} \right| \right\} \neq \{0\}$

Therefore $\|[X_1, Y_1]\| \geq 0$

(ii) For all $[X_1, Y_1] \in I[L(\mathbb{R})]$ and $b \in \mathbb{R}$,then $\|b[X_1, Y_1]\| = D_I \left(b[X_1, Y_1], 0 \right)$

$$= \inf_{X \in [X_1, Y_1]} \sup \max \left\{ |b| \left| \underline{X}^{\alpha} \right|, |b| \left| \overline{X}^{\alpha} \right| \right\}$$

$$= |b| \inf_{X \in [X_1, Y_1]} \sup \max \left\{ \left| \underline{X}^{\alpha} \right|, \left| \overline{X}^{\alpha} \right| \right\}$$

$$= |b| \|[X_1, Y_1]\|$$

(iii) For all $[X_1, Y_1], [X_2, Y_2] \in I[L(\mathbb{R})]$ then

$$\|[X_1, Y_1] + [X_2, Y_2]\| = \|[X_1 + X_2, Y_1 + Y_2]\|$$

$$= D_I \left([X_1 + X_2, Y_1 + Y_2], 0 \right)$$

$$= \inf_{\substack{X \in [X_1, Y_1] \\ Y \in [X_2, Y_2]}} \sup \max \left\{ \left| \underline{X}^{\alpha} + \underline{Y}^{\alpha} \right|, \left| \overline{X}^{\alpha} + \overline{Y}^{\alpha} \right| \right\}$$

$$\leq \inf_{\substack{X\in[X_1,Y_1]\\ Y\in[X_2,Y_2]}} \sup \max\left\{\left|\underline{X}^{\alpha}\right| + \left|\underline{Y}^{\alpha}\right|, \left|\overline{X}^{\alpha}\right| + \left|\overline{Y}^{\alpha}\right|\right\}$$

$$= \inf_{\substack{X\in[X_1,Y_1]\\ Y\in[X_2,Y_2]}} \sup \max\left\{\left|\underline{X}^{\alpha}\right|, \left|\overline{X}^{\alpha}\right|\right\} + \inf_{\substack{X\in[X_1,Y_1]\\ Y\in[X_2,Y_2]}} \sup \max\left\{\left|\underline{Y}^{\alpha}\right|, \left|\overline{Y}^{\alpha}\right|\right\}$$

$$= \left\|[X_1, Y_1]\right\| + \left\|[X_2, Y_2]\right\|$$

Definition 3.7

The diameter of an order interval $[X_1, Y_1] \in I[L(\mathbb{R})]$ is defined by $diam[X_1, Y_1] = D(X_1, Y_1) \ = D_I([X_1, Y_1], 0)$

$$= \left\|[X_1, Y_1]\right\|$$

$$diam[X_1, Y_1] = \left\|[X_1, Y_1]\right\|$$

Also $diam[X_1, Y_1] = \sup D(X_1, Y_1)$

$$= \sup \left\|[X_1, Y_1]\right\|$$

Theorem 3.8

Let $[X_1, Y_1]$ be the element of $I[\mathcal{F}(\mathbb{R})]$ and $\in > 0$ then $\left\|[X_1, Y_1]\right\| < \in$ if and only if for every $X \in [X_1, Y_1]$ $\left\|X\right\| < \in$

Proof

Assume $\left\|[X_1, Y_1]\right\| < \in$

$$\Rightarrow \inf_{X\in[X_1,Y_1]} \left\|X\right\| < \in$$

$$\Rightarrow \left\|X\right\| < \in$$

Theorem 3.9

Let $[X_1,Y_1]\,[X_2,Y_2]$, be the element of $I[\mathcal{F}(\mathbb{R})]$ and $\in\, > 0, \|[X_1 - X_2, Y_1 - Y_2]\| < \in$

there exist $X \in [X_1,Y_1]\; Y \in [X_2,Y_2]$ suchthat $\left|\underline{X}^{\alpha} - \underline{Y}^{\alpha}\right| < \in \& \left|\overline{X}^{\alpha} - \overline{Y}^{\alpha}\right| < \in$

Proof

Assume that $\|[X_1,Y_1]-[X_2,Y_2]\| < \in$ for $\in > 0$

$\|[X_1 - X_2, Y_1 - Y_2]\| < \in$

$$\Rightarrow \inf_{\substack{X\in[X_1,Y_1]\\ Y\in[X_2,Y_2]}} \sup \max\left\{\left|\underline{X}^{\alpha} - \underline{Y}^{\alpha}\right|, \left|\overline{X}^{\alpha} - \overline{Y}^{\alpha}\right|\right\} < \in$$

$$\Rightarrow \max\left\{\left|\underline{X}^{\alpha} - \underline{Y}^{\alpha}\right|, \left|\overline{X}^{\alpha} - \overline{Y}^{\alpha}\right|\right\} < \in$$

$$\Rightarrow \left|\underline{X}^{\alpha} - \underline{Y}^{\alpha}\right| < \in \& \left|\overline{X}^{\alpha} - \overline{Y}^{\alpha}\right| < \in.$$

Theorem 3.10

The pair $\left(I(\mathcal{F}(\mathbb{R}), \|\,.\,\|\right)$ is a complete normed linear space.

Proof

Let $\{[X_n, Y_n]\}_{n\in N}$ be a Cauchy sequence in $\left(I(\mathcal{F}(\mathbb{R}), \|\,.\,\|\right)$

Then for given $\in > 0$ there exists N such that $\|[X_n, Y_n] - [X_m, Y_m]\| < \in$ for all $n, m \geq N$

Since $\left(I(\mathcal{F}(\mathbb{R}), \|.\|\right)$ is complete there exists $X, Y \in \mathcal{F}(\mathbb{R})$ such that

$\left|\underline{X_n}^{\alpha} - \underline{X}^{\alpha}\right| < \in \& \left|\overline{X_n}^{\alpha} - \overline{X}^{\alpha}\right| < \in$

as $n \to \infty$ Hence $[X_n, Y_n] \to [X, Y] \in I(\mathcal{F}(\mathbb{R})$ is normed linear space.

Therefore $\left(I(\mathcal{F}(\mathbb{R}), \|\,.\,\|\right)$ is a Banach space.

Definition 3.11

Diameter of a set $[X_1, Y_1]$ in $\left(I(\mathcal{F}(\mathbb{R}), \|.\|\right)$ is defined as $diam[X_1, Y_1] = \sup_{X,Y \in [X_1, Y_1]} \|X - Y\|$

Theorem 3.12

The metric diameter of $[X_1, Y_1] \in I\ (\mathcal{F}(\mathbb{R})$ is equal to $\|X_1 - Y_1\|$

Proof

By definition $diam[X_1, Y_1] = \sup_{X,Y \in [X_1, Y_1]} \|X - Y\|$

$\|X_1 - Y_1\| = \inf \sup\max_{\alpha \in [0,1]} \left\{ \left|\underline{X}^\alpha - \underline{Y}^\alpha\right|, \left|\overline{X}^\alpha - \overline{Y}^\alpha\right| \right\}$ so for $X, Y \in [X_1, Y_1]$.

$= \inf \sup\max_{\alpha \in [0,1]} \left\{ \left|\underline{X}^\alpha - \underline{Y}^\alpha\right|, \left|\overline{X}^\alpha - \overline{Y}^\alpha\right| \right\} \le \sup_{\alpha \in [0,1]} \left\{ \left|\underline{X}^\alpha - \underline{Y}^\alpha\right|, \left|\overline{X}^\alpha - \overline{Y}^\alpha\right| \right\}$

(ie) $\|X - Y\| \le \|X_1 - Y_1\|$ for every $X, Y \in [X_1, Y_1]$

Hence $diam[X_1, Y_1] \le \|X - Y\|$

By definition $\|X - Y\| \le diam[X_1, Y_1]$ and hence result follows.

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