Surface Cracks Modelling of Ultrasonic Non-Destructive Testing

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ABSTRACT

Non-destructive testing is becoming progressively important in an increasingly industrialized world. UTDefect is an example of programs that have been built based on the development of ultrasonic testing of thick walled material. Depending on the reciprocity argument, an ultrasonic signal can be sent through some probes to other recipient probes. Both interior cracks and surface breaking cracks are examined and compared in this experiment. Hypersingular integral method is applied to the cracks. Chebyshev functions are used to represent Crack opening displacement for each crack. The crack profile is different, with interior cracks having a COD that is a second root at the tip of the crack. On the other hand, surface cracks have a finite COD at the surface end. These details about the two sets of cracks are integrated into the analysis. Numerical comparisons demonstrate that significantly high frequencies the interior cracks exhibit behaviour similar to that of the surface cracks.

Keywords: Surface crack, Ultrasonic, Non -Destructive Testing,

1. INTRODUCTION

Testing of existence of structural flaws or cracks in a solid material presents a challenge to material scientists. One of the modern techniques that have been developed to test for cracks is the non-destructive testing. Alternative methods, which are sometimes less reliable, are destructive testing where forces are applied to a sample material to investigate the structural resilience of the material. One of the modern methods of testing for surface cracks is using ultrasonic on-destructive testing (Tse, 2015, p. 1562). The most important property of cracks in relation to ultrasonic non-destructive testing is the ability to scatter ultrasonic frequencies. Frequency approximation, application of integral equations and the use of finite element are some of the approaches that rely on this property of cracks (Schmerr, 1998, p. 490). For non-destructive testing, cracks considered are about two wavelengths in width and between 10 and 100 wavelengths in the distance of propagation relevant to the experiment. In FEM, 3D analysis results in indeterminate results. On the other hand, high frequency approaches have high efficiency, albeit with a significant probability of indeterminate accuracy.

2. LITERATURE REVIEW

Green, Ruud, & Kozaczek discuss the application of non-destructive testing in situations where destructive testing is not applicable. He gives an example of paintings and panels that often suffer cracks and delaminations that are difficult to assess and evaluate scientifically. The authors consider non-destructive testing a revolutionary approach to preserving items by avoiding catastrophic failure (Green, Ruud, & Kozaczek, 1994, p. 110). Further, the authors divide non-destructive testing into contact and non-contact methods. Yang song and Fan Wu elaborate further on the use of ultrasonicnon-destructive testing in the manual by Chang & Kopsaftopoulos. The authors cite railway as an important and common example of application of non-destructive testing in modern railway system in china (Chang & Kopsaftopoulos, 2015, p. 2061). In their article, the authors cite piezoelectric the method of choice in contact non-destructive testing in monitoring of railways.

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Pike, R., & Sabatier give an overview of the existing methods of non-destructive testing in their book "*Scattering: Scattering and inverse scattering in pure and applied science*". According to the authors, existing non-destructive testing approaches include piezoelectric, ultrasonic waves, magnetic testing, and optical systems (Pike & Sabatier, 2002, p. 594). The authors then go on to create a mathematical model that precedes experimental testing. In addition, the author's stress the role of scattering in ultrasonic non-destructive testing.

In perhaps the most important and relevant text to this paper, Langenberg, Marklein & Mayer discuss modeling of ultrasonic non destructive testing in respect to surface integral equation and the subdivision method of simplifying the field of investigation. Using electromagnetic modeling and the Kirchhoff approximation, the authors are able to show how ultrasonic methods can easily be applied to cracks with stress free boundary conditions(Langenberg, Marklein & Mayer, 2012, p. 19). This book lays the mathematical and geometrical foundation for ultrasonic testing for surface breaking cracks by presenting the relationship between the crack and the three dimensional Cartesian space.

Dominguez & Arizain investigate Hypersingular nature of fracture mechanics in the paper "*Hypersingular* and mixed boundary elements in fracture mechanics". The researchers use boundary integral equations to investigate three dimensional fracture mechanical characteristics. Their approach applies analysis to static and dynamic conditions (Dominguez & Ariza, 2003, p. 2). The Boundary element approach (BEM) can be applied with the exclusion of the singularity that occurs at the edge of the crack. Only simplistic theoretical cracks or their approximations are suitable candidates for application of BEM (Dominguez & Ariza, 2003, p. 115). Non-destructive testing is particularly important for evaluation of surface cracks or those that are near the surface, but not necessarily surface breaking.

3. BACKGROUND

To formulate a problem that can be investigated. We can consider a crack that is perpendicular to the surface. The crack has a height *a* an imaginary line perpendicular to the crack can be assumed to be the X-axis, while the *y* axis goes along the crack. The Z axis should be along the depth of the crack. Linear



Figure 1: A Surface breaking crack in three-dimensional space

elasticity and uniform density are assumed for the material in which the crack occurs (Langenberg, Marklein, & Mayer, 2012, p. 17). Constants μ and γ are applied to the material and density is assigned the symbol ρ . For the signal, k_p represents the longitudinal waves and k_s represents the transverse waves. e^{-iwt} is the factor used to allow presumption of a time harmonic condition in the system. The diagrammatic representation of the crack is illustrated below.

The crack will be analyzed in half space parameters, therefore avoids any additional scattering of frequencies. The thickness of the plate on which the crack is inserted for investigation should be at least twice the wavelengths of the ultrasonic frequency used. An integral equation can now be formulated for the crack considering the field of displacement *u*. Assuming that the boundaries of the crack do not experience stress, the integral is formed as follows:

$$\lim_{x \to 0} \frac{k_s}{u} \int_{-\infty}^{\infty} dy' \int_0^a dz' \Delta u(r') \cdot \sum (r', r) = -t^i(r)$$
(1)

The ultrasonic incident field has traction $-t^i$ and the crack opening displacement (COD) is represented by Δu . Integration is executed first and then the limit is inspected within the integral set up. The above equation, the Green stress tensor, is expanded in the double Fourier transform while the crack opening displacement is expanded in Chebyshev functions. Green stress tensor is as follows when expanded:

$$\sum_{n'n} (r', r) = 2i\mu^{2} \sum_{j} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dqdp}{k_{j}h_{j}} F_{jn'}F_{jn} e^{i(h_{j}|x'-x|+p(y'-y)+q(z'-z))} + 2i\mu^{2} \sum_{jj'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dqdp}{k_{j}h_{j}} G_{jn'}R_{jj'}G_{j'n}^{\dagger} e^{i(q(x'-x)+p(y'-y)+h_{j}z'-h_{j}z)}$$
(2)

As observed above, (2), Fourier double transform has been used to represent the Green stress tensor in the first term of the equation as essentially the coordinates of the tangents to the crack. Consequently, when the co-ordinates are perfectly situated on the crack the term |x' - x| tends to zero. The succeeding term is now the remaining plausible solution in the half space. Now the crack opening displacement is as follows:

$$\Delta u_{n'}(y',z') = \sum_{m'} \int_{-\infty}^{\infty} \frac{dp'}{k_s} \beta_{n'm'}(p') \mathcal{O}_{m'}(z') e^{-ip'y'}$$
(3)

In the above equation, m' is asum over all real integer numbers. The following equation is used for expansion:

$$\emptyset_m(z) = \cos\left((2m-1)\arcsin\left(\frac{z}{a}\right)\right), \text{ where mis 1, 2, 3}$$
 (4)

When the functions (3) and the expansion of Green stress tensor are substituted in the equation (1), it appears as follows:

$$I_m(\gamma) = \int_0^1 \cos((2m-1)\arcsin t)e^{i\gamma t}dt$$
(5)

For a significantly large value of γ the analytical form of the equation can be computed. However, for small value of γ computation is not possible. This equation is for surface cracks while a similar equation for interior cracks is a Bessel function. Computing the Fourier transform of the function (1) with (2) and (3) substituted into it results in the following integral function:

$$\sum_{n'm'} Q_{nnm'm'}(p) \beta_{n'm'}(p) = \frac{1}{2\pi\mu a} \int_{-\infty}^{\infty} dy \int_{0}^{a} dz t_{n}^{i}(y,z) Q_{m}(z) e^{ipy}$$
(6)

The matrix for the whole model becomes:

$$Q_{nmn'm'}(p) = -4\pi i a \int_{-\infty}^{\infty} dq \left\{ \sum_{j} \frac{1}{k_{j}h_{j}} F_{jn'}F_{jn}^{*}I_{m}(qa)I_{m'}(-qa) + \sum_{jj'} \frac{1}{k_{j}h_{j}} F_{jn'}R_{jj'}G_{j'n}^{\dagger}I_{m}(h_{j'a})I_{m'}(-h_{j}a) \right\}$$
(7)

It can be observed that the limit in the function (1) has been eliminated in the equation (6) because of the convergence of function q. Using a reciprocity argument, the difference in the transmitted signal at both ends can be obtained. Taking P as the signal power that reached the recipient probe, the equation can be presented as follows:

$$\delta T = \frac{i\omega}{4P} \int_{-\infty}^{\infty} \int_{0}^{a} \Delta u_{j}(y, z) \sigma_{1j}(y, z) dy dz$$
⁽⁷⁾

In practical application the δT is the value evaluated. It represents that reflected signal at the recipient probe owing to the crack. The same value can be better presented in the re-factored equation below:

$$\delta T = \int_{-\infty}^{\infty} T_{nm}(p) Q_{nmn'm'}^{-1}(p) T_{n'm'}(p) dp$$
(8)

The transmitter signal is represented by the following matrix:

$$T_{nm}(p) = \int_{-\infty}^{\infty} \sum_{j=1}^{3} L_{nmj}(p,q) e^{i(-qd_x + h_jd_z)} dq$$
(9)

 L_{nmj} is used for a crack whose mouth is closed. The above analysis provides a comprehensive mathematical model of the experiment.

4. EXPERIMENTAL PROCEDURE

- 1. The crack to be investigated was placed on a plate 30mm deep.
- 2. A 2 megahertz transmitter was used. The probe was circular with a 20mm diameter.

Results were recorded after calibration against a drilled hole of 2mm and 30mm in diameter and depth respectively.

- 3. Both the results of the crack and those of the hole used in the control procedure were plotted on graphs.
- 4. For comparison, the rectangular crack data was analysed against simulation data of strip like cracks.

5. DISCUSSION AND CONCLUSION

The results as seen in the figure 2 are that of a crack 2mm deep. The crack used to approximate a real crack is strip-like. A rectangular shaped crack is assumed for calculations. While this may not be the case for a natural strip like crack where the crack used for the experiment is smaller than the wavelength of the ultrasonic wave employed, the approximation of the dimensions of the crack should be approximated with precision (Meyendorf, N., Nagy, P. B., & Rokhlin, S. I. (2004). This is because a significant error may affect the repeatability of the experiment. For this experiment, the middle frequency of the response spectrum

is used for calculations. The centre frequency of the probe spectrum is the choice for calculation. For simplicity, A fixed frequency is assumed for to avoid the attenuation at the extremes of the frequency range (Thompson & Chimenti, 1997, p. 41). The comparison between a rectangular crack data and strip like simulation data reveals the effect of shape on the investigation. The type of data as viewed after collection reveals to the technician the possible shape of the crack being investigated.

The following data is that of a simulation versus experimental results.



Figure 2: This is the plot of the data for the 2mm deep crack having broken the surface. The continuous line is the characteristic of a closed mouth crack while the dotted line is the characteristic of a open mouth crack.



Figure 3: This is a plot of data from a 0.5 millimetre crack. The dotted line represents the characteristics of an open crack while the continuous line represents the data from a closed line.

Height (mm)	Experimental	Simulation (numerical integration)	Simulation (stationary-phase method)
2	8.0	7.9	6.9
5	13.3	12.7	13.4
10	13.7	13.7	20.4
20	13.5	13.0	25.9

Figure 4: Comparison of simulation versus experimental data.

Surface breaking strip like cracks were successfully investigated by application of ultrasonic frequencies. This proves that this is a method that can be applied on cracks where visual examinations are not applicable. It is also evident that cracks are not easily seen by naked eye can be detected using this method. In addition, it is possible to obtain more data in non-destructive testing.

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