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Effect of Aligned Magnetic Field on Unsteady Flow Between A Stretching Sheet and Oscillating Porous Plate with Constant Suction

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Abstract: The unsteady MHD flow of an incompressible viscous electrically conducting fluid between two horizontal parallel non conducting plates, where the lower one is stretching sheet and the upper one is oscillating porous plate is studied in the presence of an aligned magnetic field. Fluid motion is caused by the stretching of the lower sheet and a constant suction is applied at the upper plate which is oscillating in its own plane. The stretching velocity of the sheet is assumed to be a linear function of distance along the channel. The expressions relating to the velocity distribution are obtained and effects of different values of various physical parameters are calculated numerically and shown graphically.

Keywords: Aligned magnetic field, stretching sheet, Unsteady MHD flow.

1. Introduction

In recent years the study of flow over a stretching sheet has generated considerable interest because of its numerous industrial applications such as in the manufacture of sheeting material through an extrusion process, the cooling of bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets glass and polymer industries, fiber industry etc. cheng [10] seems to be the first to consider the problem of steady mixed convection in porous media along inclined surfaces. Both aiding and opposing flows were considered. Assuming a power law variation of the wall temperature similarity solutions were obtained for two cases. (i) a uniform flow along vertical isothermal flat plate and (ii) acceleration flow over a 45° inclined flat plate of constant heat flux. The heat transfer rate is found to approach asymptotically the forced and free convection limits as the value of the governing mixed convection parameter Rapa approaches zero and infinity respectively.

However, unsteady convective boundary layer flow problems have not, so far, received as much attention. Perhaps the first study on unsteady boundary layer flow on flat surfaces in porous media was made by Johnson and cheng [6], who found similarity solutions for certain variations of the wall temperature. The more common cases in general involve transient convection, which non similar and hence, more complicated mathematically. Nazer *et al.*, [7] studied the unsteady mixed convection flow near the stagnation point on a heated vertical flat plate embedded in a fluid saturated porous medium, in the presence of buoyancy forces. It is assumed that the unsteadyness is caused by the impulse start in motion of the free stream flow and by the sudden increase or decrease in the surface temperature.

Srivastava (2) has discussed the flow of second order fluids with heat transfer between plates, one moving and the other at rest. Crane (6) obtained the solution for the steady flow over a stretching surface in a quiescent fluid, and the same problem for the conducting fluid has been examined by Pavlov(7) in presence of transverse magnetic field. The heat and mass transfer over a stretching sheet with or without suction/blowing by considering various situations was studied by Gupta and Gupta (9). Chakrabarthi and Gupta (10) have investigated the motion of an electrically conducting fluid past a horizontal stretching sheet in the presence of a magnetic field.Chiam (11) consider the motion of micro polar fluids over a stretching sheeting.Borkakoti and Bharali(12) studied the flow and heat transfer problem in a conducting fluid between two horizontal parallel surfaces where the lower one is stretching and the upper one is a porous solid plate in the presence of a transverse magnetic field. Rajagopal *et al.* (13) discussed the flow of a visco elastic fluid over a stretching sheet. Agarwal *et al.*, (16) gave the solution of flow and heat transfer of a micro polar fluid over a stretching sheet using finite element technique. Anderson (17) considered the motion of power law fluids over a stretching sheet. Anderson (18) investigated the flow of electrically conducting visco elastic fluid past a flat and a impermeable elastic sheet. This work was extended by Ming–Ichar (21) to study heat as well as mass transfer. Chauhan (20) investigated the coupled stretching flow of a two dimensional viscous incompressible fluid through a channel bounded by naturally permeable bed. Takhar and Nath (24) have studued the unsteady flow over a stretching surface with amagnetic field in a rotating fluid. Kumari *et al.*, (25) gave the analytical solutionof the boundary layer equations over a stretching sheet with mass transfer using series method.

Nabil *et al.*, (26) obtained a solution of the flow problem in compact form for the motion due to linear velocity of stretching sheet. Sharma and Mishra (27) investigated steady MHD flow through horizontal channel. Lower being stretching sheet and upper being permeable plate bounded by porous medium. Subhas, A.M. *et al.* (28) studied the effect of magnetic field on the visco elastic fluid flow and heat transfer over a stretching surface with internal heat generation. Phukan (29) obtained the numerical solutions for power law velocity distribution of stretching plate. Bhardwaj (30) investigated the steady two dimensional of viscous , incompressible fluid through a channel bounded by a planarly stretched sheet and naturally permeable bed.an analysis of heat transfer taking dissipation function into account ,in boundary layer flow of a hydromagnetic fluid over a stretching sheet in the presence of uniform transverse magnetic field has been given by Lodha and Tak (31).

In this chapter the effect of aligned magnetic field on unsteady MHD flow of a viscous in compressible electrically conducting fluid between two horizontal parallel non conducting plane surfaces is considered. The lower surface is a stretching sheet and the upper one is an oscillating porous plate. The fluid motion is caused by stretching of the sheet and suction at the upper porous plate which is oscillating in its own plane. The stretching velocity of the sheet is assumed to be linear function of distance along the channel and the induced magnetic filed is considered negligible in comparisons to the applied magnetic filed.

2. Formulation of the problem

The unsteady flow of viscous incompressible electrically conducting fluid between two horizontal parallel non conducting plane surfaces is considered. Let X-axis be taken along the lower stretching sheet in the flow direction and y-axis is taken normal to the sheet. The lower plate is stretched by introducing two equal and opposite forces along the x-axis, so that the position of the origin remains unchanged. The fluid is sucked through the upper oscillating porous plate, with constant velocity V_0. A constant magnetic field B_0 is applied at angle θ with the x-axis. In the analysis of the flow the following assumption are made (i). The stretching velocity of the sheet is a linear function of distance along the channel. And (ii) Electric field and the induced magnetic field are neglected.

The governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial \bar{t}}+\upsilon\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}+\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma_e B_0^2\sin^2\theta}{\rho}u+\frac{\sigma_e B_0^2\sin\theta\cos\theta}{\rho}v \tag{2}$$

$$\frac{\partial v}{\partial \bar{t}}+\upsilon\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{-1}{\rho}+\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma_e B_0^2\cos^2\theta}{\rho}v+\frac{\sigma_e B_0^2\sin\theta\cos\theta}{\rho}u \tag{3}$$

The boundary conditions are

$$u = cx, \quad v = 0 \quad \text{at} \quad y = 0 \tag{4}$$

$$u = U_0(1 + \varepsilon e\overline{i\omega t}), \quad v = V_0 \quad \text{at} \quad y = h \tag{5}$$

Non dimensionalization

Introducing the following dimensionless quantities

$$\eta = \frac{y}{h},\ t = \frac{t}{c},\ \bar{\omega} = \omega c,\ \text{Re} = \frac{ch^2}{\upsilon},\ M^2 = \frac{B_0^2 h^2 \sigma_e}{\rho\upsilon} \tag{6}$$

Such that the equation of continuity admits the self similar solution

$$u = cxf^1(\eta, t) \quad \text{and} \quad v = -chf(\eta, t) \tag{7}$$

Using (6) and (7) into the equations (2) and (3), we get

$$\frac{\partial f^1}{\partial t} + f1^2 - ff^{11} = \frac{-1}{\rho c^2 x}\frac{\partial p}{\partial x} + \frac{1}{\text{Re}} f^{111} - \frac{M^2}{\text{Re}}\sin^2\theta f^1 - \frac{M^2 h}{\text{Re}\, x}\sin\theta\, os\,\theta f \tag{8}$$

$$-\frac{\partial f}{\partial t} + ff^1 = \frac{-1}{\rho c^2 h^2}\frac{\partial p}{\partial x} - \frac{1}{\text{Re}} f^{11} + \frac{M^2 x}{\text{Re}\, h}\cos\theta\sin\theta f^1 + \frac{M^2}{\text{Re}}\cos^2\theta f \tag{9}$$

Where prime denotes the differentiation with respect to η.

Eliminating the pressure from equations (8) and (9), we have

$$\frac{\partial^2 f}{\partial \eta^4} - \text{Re}\frac{\partial^3 f}{\partial \eta^2 \partial t} + \text{Re}\frac{\partial^3 f}{\partial \eta^3} - \text{Re}\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta^2} - M_1^2\frac{\partial^2 f}{\partial \eta^2} = 0 \tag{10}$$

Where $M_1 = M\sin\theta$.

The boundary conditions (4) and (5) in non dimensional form are

$$f = 0;\quad f^1 = 1 \quad \text{at} \quad \eta = 0 \tag{11}$$

$$f = -\beta;\quad f^1 = \alpha(1 + \varepsilon e^{i\omega t}) \quad \text{at} \quad \eta = 1 \tag{12}$$

Where $\alpha = \frac{U_0}{cx},\quad \beta = \frac{V_0}{ch}.$

3. Method of Solution

In view of the boundary conditions and assuming

$$f(\eta, t) = f_0(\eta) + \varepsilon e^{i\omega t} f_1(\eta) \tag{13}$$

Substituting (13) into the equations (10) to (12) and separating the steady and unsteady parts, we get

$$\text{Re} f_0^1 f_0^{11} - \text{Re} f_0 f_0^{111} = f_0^{iv} - M^{12} f_0^{11} \tag{14}$$

$$\text{Re}\, i\omega f_1^{11} + \text{Re}(f_0^1 f_1^{11} + f_0^{11} f_1^1) - \text{Re}(f_0 f_1^{111} + f_1 f_0^{111}) = f_1^{iv} - M^{12} f_1^{11} \tag{15}$$

The corresponding boundary conditions are

$$f_0 = 0,\quad f_1 = 0,\quad f_0^1 = 1,\quad f_1^1 = 0 \quad \text{at} \quad \eta = 0 \tag{16}$$

$$f_0 = \beta, \quad f_1 = 0, \quad f_0^1 = \alpha, \quad f_1^1 = \alpha \quad \text{at} \quad \eta = 1 \tag{17}$$

Taking Reynolds number as small, a regular perturbation scheme can be developed as

$$f_0(\eta) = f_{00}(\eta) + \mathrm{Re}\, f_{01}(\eta) + O(\mathrm{Re}^2)$$

and

$$f_1(\eta) = f_{10}(\eta) + \mathrm{Re}\, f_{11}(\eta) + O(\mathrm{Re}^2) \tag{18}$$

Substituting (18) into equations (14) to (17) and equating like powers of Re, We have the following sets of equations with the corresponding boundary conditions.

Zeroth Order Equations

$$f_{00}^{iv} - M_1^2 f_{00}^{11} = 0 \tag{19}$$

$$f_{10}^{iv} - M_1^2 f_{10}^{11} = 0 \tag{20}$$

The corresponding boundary conditions are

$$f_{00} = 0, \quad f_{10} = 0, \quad f_{00}^1 = 1, \quad f_{10}^1 = 0 \quad \text{at} \quad \eta = 0 \tag{21}$$

$$f_{00} = \beta, \quad f_{10} = 0, \quad f_{00}^1 = \alpha, \quad f_{10}^1 = \alpha \quad \text{at} \quad \eta = 1 \tag{22}$$

First Order Equations

$$f_{01}^{iv} - M_1^2 f_{01}^{11} = f_{00}^1 f_{00}^{11} - f_{00} f_{00}^{111} \tag{23}$$

$$f_{11}^{iv} - M_1^2 f_{11}^{11} = i\omega f_{10}^1 + f_{00}^1 f_{10}^{11} + f_{00}^{11} f_{10}^1 - f_{00} f_{10}^{111} - f_{10} f_{00}^{111} \tag{24}$$

The corresponding boundary conditions are

$$f_{01} = 0, \quad f_{11} = 0, \quad f_{01}^1 = 0, \quad f_{11}^1 = 0 \quad \text{at} \quad \eta = 0 \tag{25}$$

$$f_{01} = 0, \quad f_{11} = 0, \quad f_{01}^1 = \alpha, \quad f_{11}^1 = 0 \quad \text{at} \quad \eta = 1 \tag{26}$$

Solving equations (19), (20),(23) and (24) with respect to the boundary conditions (21),(22), (25) and (26) we get

$$f_{00} = c_1 + c_2\eta + c_3 e^{M_1\eta} + c_4 e^{-M\eta} \tag{27}$$

$$f_{01} = c_5 + c_6\eta + (c_7 - L_1) e^{M_1\eta} + (c_8 - L_2) e^{-M\eta} + L_3 \eta e^{M_1\eta}$$

$$- L_4 \eta e^{-M_1\eta} - L_5 \eta^2 e^{M_1\eta} - L_6 \eta^2 e^{-M_1\eta} \tag{28}$$

$$f_{10} = c_9 + c_{10}\eta + c_{11} e^{M_1\eta} + c_{12} e^{-M_1\eta} \tag{29}$$

$$f_{11} = f_{11R} + i f_{11I} \tag{30}$$

Where

$$f_{11R} = C_{13} + C_{14}\eta + C_{15} e^{M_1\eta} + C_{16} e^{-M_1\eta} + C_{17}\eta^2 e^{M_1\eta}$$

$$+ C_{18}e^{-M_1\eta} + C_{19}e^{-M_1\eta} + C_{20}\eta^2 e^{-M_1\eta} \quad (31)$$

$$f_{11I} = D_{13} + D_{14}\eta + D_{15}\eta^2 + D_{16}e^{M_{11}\eta} + D_{17}e^{-M_1\eta} + D_{18}\eta e^{M_1\eta} + D_{19}\eta e^{-M_1\eta} \quad (32)$$

Hence the required solution is

$$f(\eta, t) = f_{00}(\eta) + \varepsilon\,[M_r \cos \omega t] - Mi \sin \omega t \quad (33)$$

Where $$Mr = f_{10}(\eta) + \mathrm{Re}\, f_{11R}(\eta) \quad (34)$$

And $$Mi = \mathrm{Re}\, f_{11i}(\eta) \quad (35)$$

Where C_1 to C_{20}, L_1 to L_6, and D_{13} to D_{19} are constants.

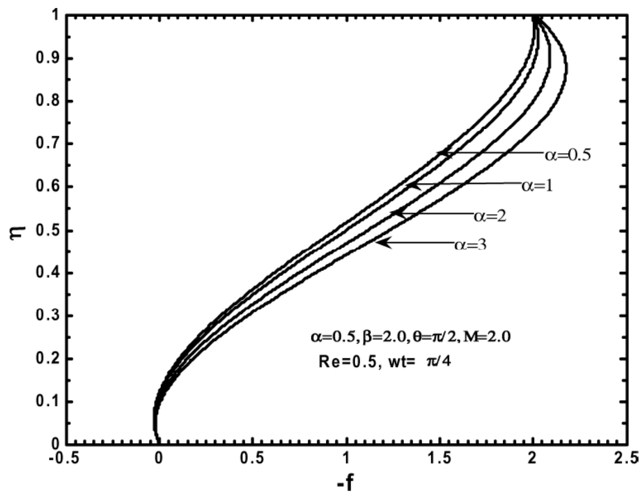

Figure 1: Variations of $-f(\eta, t)$ for different values of α

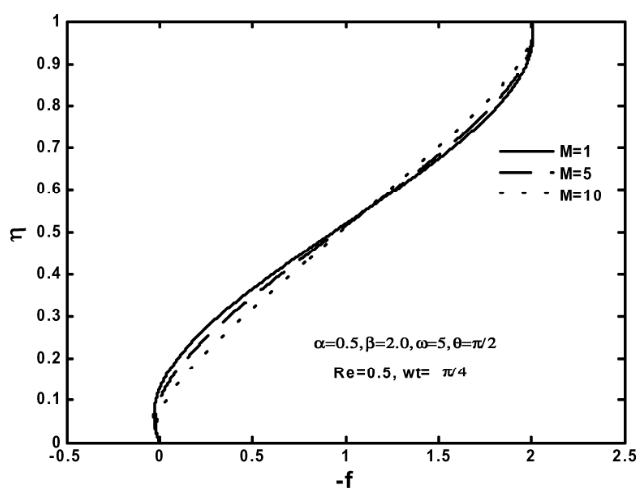

Figure 2: Variations of $-f(\eta, t)$ for different values of M

4. Results and Discussion

The fluid flow along the y-direction at the stretching sheet, characterized by $-f(\eta, t)$ is shown from figures (1) to (5) for fixed values Re = 0.5, $\omega = 5$ and $wt = \pi/4$, with the variations of α, β, M, θ. In figure (1), the fluid flow along the y-direction is showed with the variation of α for fixed values of $M = 2.0$, $\theta = \pi/2$, $\beta = 2.0$. It is noticed that the fluid flow is affected by the back flow whose magnitude is reduced with the increase of α. In figure (2) variations of $-f(\eta, t)$ is shown for various values of Magnetic parameter M, for fixed values of $\theta = \pi/4$, $\alpha = 0.5$, $\beta = 2.0$. It is noticed that the fluid flow is affected by back flow whose magnitude decreases with the increase of Magnetic parameter M. and the reverse effect is observed near the moving plate.

In figure (3) variation of $-f(\eta, t)$ is shown with the variation of θ for fixed values of $M = 5$, $\beta = 2.0$. It is noticed that the magnitude of the back flow decreases

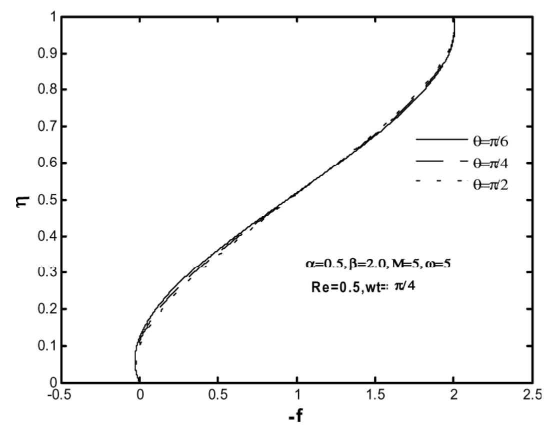

Figure 3: Variations of $-f(\eta, t)$ for different values of θ

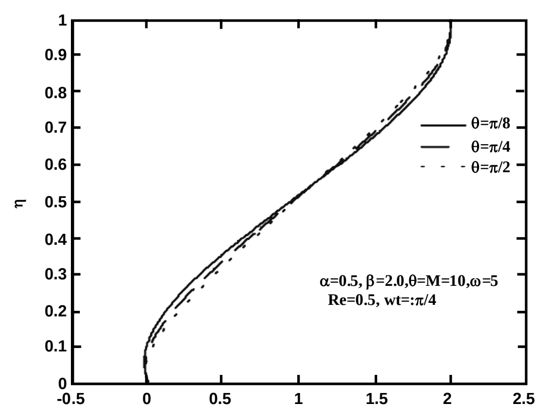

Figure 4: Variations of $-f(\eta, t)$ for different values of θ

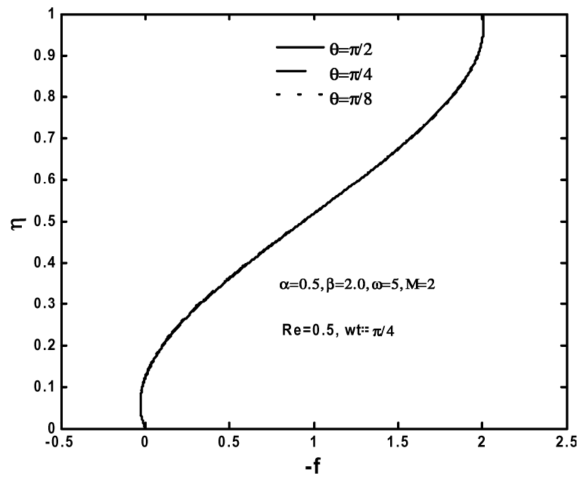

Figure 5: Variations of $-f(\eta, t)$ for different values of θ

as angle changed from $\theta = \pi/6$ to $\theta = \pi/2$, but reverse action is seen near the moving plate. The effect of the aligned angle is studied through figure (4), for fixed values of $M = 10$, $\alpha = 0.5$, $\beta = 2$. It is noticed that the back flow of the fluid decreases with the increase of θ for $0 < \eta < 0.5$ and the reverse action is noticed near the moving plate from $0.5 < \eta < 1$. In figure (5) variation of $-f(\eta, t)$ with θ is showed for small values of Hartmann number M, i.e. $M = 2$. It is noticed that the back flow of the fluid is not much influenced for the change in θ.

The stretching effects on the fluid velocity in x-direction characterized by $f^1(\eta, t)$ is shown from figures (6) to (10) for fixed values of Re = 0.5, $\omega = 5$ and $wt = \pi/4$. In figure (6) variation of $f^1(\eta, t)$ is shown with the variation of α, for fixed values of M

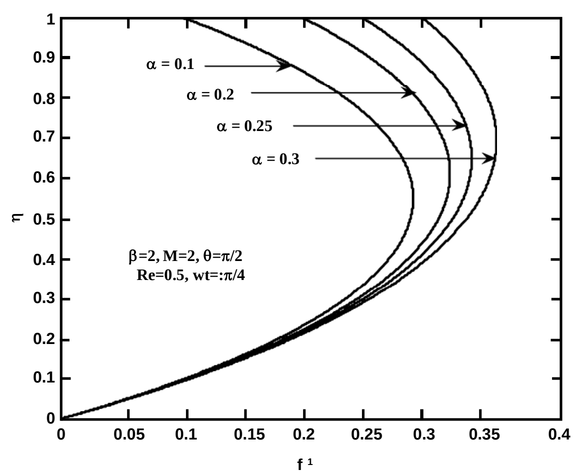

Figure 6: Variations of $f^1(\eta, t)$ for different values of α

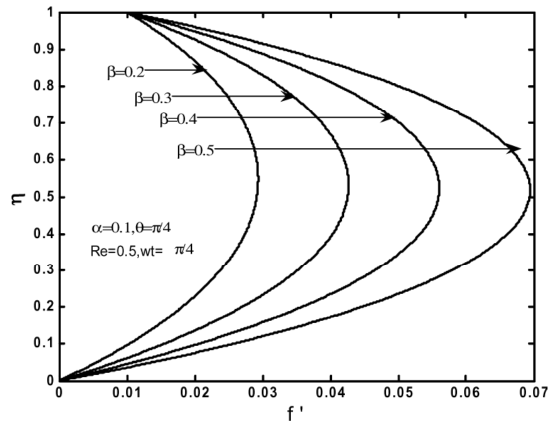

Figure 7: Variations of $f^1(\eta, t)$ for different values of β.

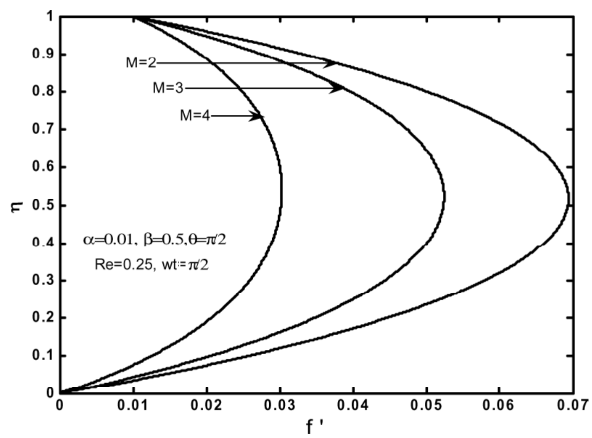

Figure 8: Variations of $f^1(\eta, t)$ for different values of M

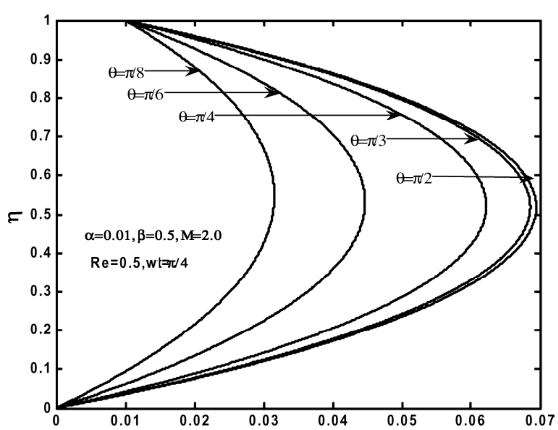

Figure 9: Variations of $f^1(\eta, t)$ for different values of θ

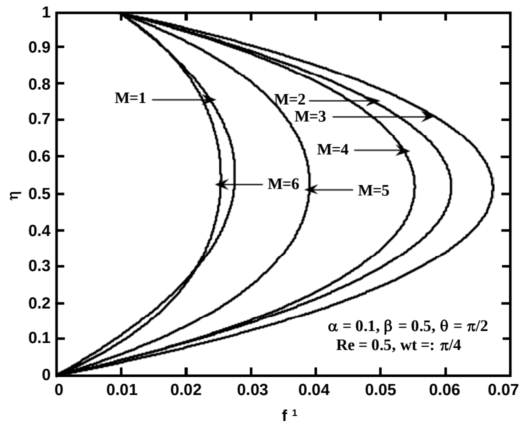

Figure 10: Variations of $f^{1}(\eta, t)$ for different values of M

$= 2$, $\theta = \pi/2$, $\beta = 0.5$. It is observed that along the main stream the velocity is more prominent in the back flow. The magnitude of the back flow increases with the increase of α throughout the width of the channel from the stretching sheet and the reserve action is observed near the moving plate. In figure (7) variation of $f^{1}(\eta, t)$ is shown with the variation of suction parameter β, for fixed values of $M = 2$, $\theta = \pi/2$ and $\alpha = 0.5$. It is noticed that the back flow increases with the increase of β.

In figure (8) variation of $f^{1}(\eta, t)$ is shown with the variation of M for fixed values of $\theta = \pi/2$, $\alpha = 0.01$, $\beta = 0.5$. It is noticed that the velocity decreases with the increase of magnetic parameter M. In figure (9) variation of $f^{1}(\eta, t)$ is shown with the variation of θ for fixed values of $M = 2$, $\alpha = 0.01$, $\beta = 0.5$. It is observed that the velocity increases with the increase of θ. In figure (10) variation of $f^{1}(\eta, t)$ is shown with the variation of M for fixed $\theta = \pi/2$, $\alpha = 0.1$, $\beta = 0.5$. It is noticed that velocity increases up to $M = 3$ later as M increases velocity decreases.

5. Appendix

$$C_1 = -(C_3 + C_4); \qquad C_2 = 1 - M_1(C_3 - C_4); \qquad C_3 = \frac{A_4B_5 - A_5B_4}{A_3B_4 - A_4B_3};$$

$$C_4 = \frac{A_5B_3 - A_3B_5}{A_3B_4 - A_4B_3}; \qquad C_5 = (C_7 + C_8 + K_1); \qquad C_6 = M_1(C_8 - C_7) - K_2;$$

$$C_7 = \frac{A_4K_6 - B_4K_5}{A_3B_4 - A_4B_3}; \qquad C_8 = \frac{B_3K_5 - A_3K_6}{A_3B_4 - A_4B_3}; \qquad C_9 = \frac{\alpha(B_3 - B_4)}{A_1B_2 - A_2B_1};$$

$$C_{10} = \frac{-\alpha M_1(B_3 + B_4)}{A_1B_2 - A_2B_1}; \qquad C_{11} = \frac{\alpha B_4}{A_1B_2 - A_2B_1}; \qquad C_{12} = \frac{\alpha B_3}{A_1B_2 - A_2B_1};$$

$C_{13} = p_{13}; \quad C_{14} = p_{14}; \quad C_{15} = p_{15} - \frac{r_1}{M_1^4} + \frac{17r_3}{8M_1^5}; \quad C_{16} = \frac{r_1}{2M_1^3} - \frac{5r_3}{4M_1^4};$

$C_{17} = \frac{r_3}{4M_1^3}; \quad C_{18} = p_{16} - \frac{r_2}{M_1^4} + \frac{17r_4}{8M_1^5}; \quad C_{19} = \frac{-r_2}{2M_1^3} + \frac{5r_4}{4M_1^4};$

$C_{20} = \frac{r_4}{4M_1^3}; \quad A_3 = M_1(e^{M_1} - 1); \quad A_4 = -M_1(e^{-M_1} - 1); \quad A_5 = 1 - \alpha;$

$B_3 = e^{M_1} - 1 - M_1; \quad B_4 = e^{-M_1} - 1 + M_1; \quad B_5 = \beta + 1; \quad M_1 = M \sin\theta;$

$L_1 = \frac{17C_2C_3}{8M_1^2}; \quad L_2 = \frac{17C_2C_4}{8M_1^2}; \quad L_3 = \frac{7C_2C_3 - 2M_1C_1C_3}{4M_1};$

$L_4 = \frac{7C_2C_4 - 2M_1C_1C_4}{4M_1}; \quad L_5 = \frac{C_2C_3}{4}; \quad L_6 = \frac{C_2C_4}{4}; \quad K_1 = -(L_1 + L_2);$

$K_2 = M_1(L_2 - L_1) + (L_3 - L_4); \quad K_3 = -(L_1 - L_3 + L_5)e^{M_1} - (L_2 + L_4 + L_6)e^{-M_1};$

$K_4 = [-M_1L_1 + (M_1+1)L_3 - (M_1+2)L_5)]e^{M_1} + [M_1L_2 + (M_1-1)L_4 - (M_1-2)L_6]e^{-M_1};$

$K_5 = K_4 - K_2; \quad K_6 = K_3 - K_2 + K_1; \quad D_{13} = q_{13} - \frac{s_0}{M_1^4}; \quad D_{14} = q_{14}; \quad D_{15} = \frac{s_0}{2M_1^2};$

$D_{16} = q_{15} - \frac{s_1}{M_1^4}; \quad D_{17} = q_{16} - \frac{s_2}{M_1^4}; \quad D_{18} = \frac{s_1}{2M_1^3}; \quad D_{19} = \frac{-s_2}{2M_1^3}; \quad S_0 = \omega C_{10};$

$S_1 = \omega M_1 C_{11}; \quad S_2 = -\omega M_1 C_{12}; \quad r_1 = M_1^2(C_2C_{11} + C_3C_{10} - M_1(C_1C_{11} + C_3C_9));$

$r_2 = M_1^2(C_2C_{12} + C_4C_{10} + M_1(C_1C_{12} + C_4C_9)); \quad r_3 = M_1^3(C_2C_{11} + C_3C_{10});$

$r_4 = M_1^3(C_1C_{12} + C_4C_9); \quad p_1 = \frac{17(r_3 - r_4) - 8M_1(r_1 + r_2)}{8M_1^5};$

$p_2 = \frac{4M_1(r_2 - r_1) + 7(r_3 + r_4)}{8M_1^4}; \quad p_3 = l_1r_1 + l_2r_2 + l_3r_3 + l_4r_4;$

$p_4 = l_1^1r_1 + l_2^1r_2 + l_3^1r_3 + l_4^1r_4; \quad p_5 = A_3; \quad p_6 = A_4;$

$q_1 = \frac{-(S_0 + S_1 + S_2)}{M_1^4}; \quad q_2 = \frac{(S_2 + S_1)}{2M_1^3}; \quad q_3 = l_0S_0 + l_1S_1 + l_2S_2;$

$q_4 = \frac{-S_0}{M_1^2} + l_1^1S_1 + l_2^1S_2; \quad q_5 = B_3; \quad q_6 = B_4;$

$l_0 = \frac{-(2 + M_1^2)}{2M_1^4}; \quad l_1 = \frac{(M_1 - 2)e^{M_1}}{2M_1^4}; \quad l_2 = \frac{-(2 + M_1)e^{M_1}}{2M_1^4};$

$$l_3 = \frac{(17-10M_1-2M_1^2)e^{M_1}}{8M_1^5}; \qquad l_4 = \frac{-(17+10M_1+2M_1^2)e^{-M_1}}{8M_1^5};$$

$$l_1^1 = \frac{(2M_1^4 l_1 + e^{M_1})}{2M_1^3}; \qquad l_2^1 = \frac{-(2M_1^4 l_2 + e^{-M_1})}{2M_1^3};$$

$$l_3^1 = M_1 l_3 + \frac{(2M_1-5)e^{M_1}}{4M_1^4}; \qquad l_4^1 = M_1 l_e + \frac{(2M_1+5)e^{-M_1}}{4M_1^4};$$

$$D_1 = p_5 q_6 - p_6 q_5; \qquad p_{13} = -(p_{15} + p_{16} + p_1);$$

$$p_{14} = M_1(p_{16} - p_{15}) - p_2; \qquad p_{15} = [(p_2 - p_4)q_6 - p_6(p_3 - p_2 - p_1)]/D_1;$$

$$p_{16} = [(p_4 - p_2)q_5 - p_5(p_3 - p_2 - p_1)]/D_1; \qquad q_{13} = -(q_{15} + q_{16} + q_1);$$

$$q_{14} = M_1(q_{16} - q_{15}) - q_2; \qquad q_{15} = [(q_2 - q_4)q_6 - p_5(q_3 - q_2 - q_1)]/D_1;$$

$$q_{16} = [(q_4 - q_2)q_5 - p_5(q_3 - q_2 - q_1)]/D_1.$$

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