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Reswitching in a Model of Extensive Rent

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Abstract: This article presents an example of the reswitching of the order of fertility and of the order of rentability. Whether or not these orders differ from one another varies with distribution for certain parameter ranges in the example. This analysis emphasizes that more rent per acre is not necessarily associated with more fertile land and that the ranking of lands by fertility cannot, in general, be determined from only data on physical inputs and outputs for the available processes.

Keywords: Choice of technique, Rent, Cambridge Capital Controversy

JEL Code: B51, C67, D24, D33, Q13

INTRODUCTION

This article presents an example of the reswitching of the order of fertility and of the order of rentability, both of which arise in a model of extensive rent. The order of fertility is defined for specified techniques, in which specified qualities of land are farmed, with one quality only partially farmed. Being in excess supply, the partially farmed land pays no rent. At a given rate of profits, the qualities of land are ordered by wages, with the most fertile land paying the highest wage when only it is cultivated. The order of rentability specifies the sequence of different qualities of lands from high rent per acre to low rent per acre. Both orders may vary with the wage or the rate of profits. Perturbations of coefficients of production illustrate how these reswitchings can arise with technical change and highlight fluke cases.

This analysis builds on post-Sraffian price theory. The Cambridge capital controversy (CCC), surveyed in Harcourt (1972) and Lazzarini (2011), concentrated on models with labor as the only non-produced commodity. Quadrio-Curzio (1980) is a well-known reference on the implications of non-produced natural resources for post-Sraffian price theory. Schefold (1989) is another classic reference on the revival of classical political

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economy, including an analysis of rent. Baranzini et al. (2015) is a collection of articles building on this tradition. Kurz and Salvadori (1995) provide a comprehensive textbook treatment of the analysis of prices of production and of the choice of technique, including in models with circulating capital, fixed capital, and extensive and intensive rent. Parameter spaces for fluke switch points are explored by Vienneau (2021 and 2023). A knife edge case in which almost all perturbations of model parameters destroy its defining properties is a fluke case. Two wage curves tangent at a switch point is an example of a fluke switch point. Prices of production, in competitive markets, are such that operated processes can satisfy requirements for use after replacing commodity inputs required for production. A single rate of profits is made in all operated processes, and no extra profits are obtainable in non-operated processes.

The theory of extensive rent is an example of the analysis of joint production. More than one commodity, for example, wool and mutton, is produced in a process exhibiting joint production. Corn and an unchanged quantity and quality of land are produced commodities from processes considered here. As with models of pure fixed capital, some of the complications of full joint production do not appear in models of extensive rent. D'Agata (1983) highlights that, in the analysis of intensive rent, a cost-minimizing technique need not exist and, when one exists, it need not be unique even away from switch points. Bidard and Klimovsky (2004) note that the choice of technique cannot be analyzed in the general case of joint production by the construction of the outer frontier of the wage curves for all techniques.

The reswitching of the order of fertility and of the order of rentability are phenomena to append to the list of other Sraffa effects, which pose a challenge to marginal economics:

> Given sufficient refinement of analysis no doubt many other such 'assumptions', may have to be added. (One obvious candidate which has not been incorporated yet in neoclassical models is the absence of 'Sraffa effects' - though it may be difficult to formulate the necessary conditions explicitly.)' (Kaldor 1966)

Sraffa effects include the reswitching of techniques, capital-reversing, the truncation of the economic life of a long-lived machine at a lower rate of profits, reverse labor substitution, and the recurrence of a production process. A perturbation analysis of the parameters of models of prices of production suggests how these phenomena can emerge and that the temporal dynamics of market prices can dramatically alter suddenly.

Noting that the order of fertility and the order of rentability vary, in general, with distribution is not novel. Quadrio-Curzio (1980) has an example of the reswitching of the order of fertility, but he does not name it as such or give an explicit numerical example. As far as I know, the reswitching of the order of rentability has never previously been explicitly described in the literature. These orders are defined in an open model which does not include utility-maximizing consumers facing a budget constraint. Price theory can be explored without a focus on supply and demand curves. The analysis presented here abstracts from variations of quantities demanded by consumers as prices vary, for requirements for use are taken as given.

In the CCC, economists demonstrated that agents do not earn more because of the greater productivity of the factors of production that they supply. The rate of profits, also known as the interest rate, is generally unequal to the marginal product of capital. The ranking of lands by fertility cannot, in general, be determined from only data on physical inputs and outputs for the available processes, including processes that require inputs of land. A higher rent per acre need not reflect the greater fertility of a plot of land. The reswitching of the order of fertility and of the order of rentability further emphasize this disconnection. Since economic rent is a more general concept than in the model of extensive rent in this article, this finding may have wider applicability. For example, the salary of a corporate executive or of a professional basketball player, such as Wilt Chamberlain, presumably contains a large component of economic rent. A higher income which contains a large component of economic rent need not, in general, reflect higher productivity.

The remainder of this article consists of four sections and two appendices. The next section specifies the parameters for a numeric example of a model of extensive rent. The section after that illustrates the reswitching of the order of fertility and how it arises and vanishes with perturbations of selected coefficients of production. The penultimate section is an illustration of the reswitching of the order of rentability, with a similar perturbation analysis. The final section concludes.

TECHNOLOGY, ENDOWMENTS OF LAND, AND REQUIREMENTS FOR USE

Table 1 presents the coefficients of production for numerical examples of a model of extensive rent. Appendix A outlines the more general model illustrated here. Iron and corn are produced commodities in this simple economy. One process for producing iron and three for producing corn are available. Each corn-producing process is operated on a different type of land. Each column specifies the coefficients of production for that process, that is, the person-years of labor, the acres of each type of land, the tons of iron, and the bushels of corn required as inputs to produce a unit output, either a ton iron or a bushel corn, for that process. Local perturbations of two parameters, $a_{0,2}$ and $a_{1,2}$, are explored below.

Multiple types of land, each of a given quality, are available. Each process takes a year to complete. This is an example of joint production in that, for each process producing corn, the same quantity of land used as input is also an output of that process. Since this is a model of extensive rent, exactly one process for producing corn is available for each type of land. Only one process is available for producing the non-agricultural commodity.

Table 1: An Example with Extensive Rent

Input	Iron Industry	Corn Industry		
	I	**II**	**III**	**IV**
Labor	$a_{0,1} = 1$	$a_{0,2}$	$a_{0,3} = \frac{91}{250}$	$a_{0,4} = \frac{67}{100}$
Type 1 Land	0	$c_{1,2} = \frac{49}{100}$	0	0
Type 2 Land	0	0	$c_{2,3} = \frac{59}{100}$	0
Type 3 Land	0	0	0	$c_{3,4} = \frac{9}{20}$
Iron	$a_{1,1} = \frac{9}{20}$	$a_{1,2}$	$a_{1,3} = \frac{9}{10000}$	$a_{1,4} = \frac{67}{1000}$
Corn	$a_{2,1} = 2$	$a_{2,2} = \frac{6}{125}$	$a_{2,3} = \frac{27}{100}$	$a_{2,4} = \frac{3}{20}$

The amount of corn that can be produced is constrained by the available quantities of each type of land. Endowments of land and requirements for use must be among the givens to analyze the choice of technique in this example. Suppose the quantities of the different types of land and net output are such that all three types of land must be farmed. One type will be only partially farmed. The iron-producing process must be operated in each of the three economically viable techniques. Table 2 describes which type of lands are fully cultivated and which type of land is left partially fallow in each of the Alpha, Beta, and Gamma techniques.

Table 2: Selected Techniques

Technique	Land		
	Type 1	Type 2	Type 3
Alpha	Fully farmed	Fully farmed	Partially farmed
Beta	Partially farmed	Fully farmed	Fully farmed
Gamma	Fully farmed	Partially farmed	Fully farmed

A PARAMETRIZATION WITH THE ORDER OF RENTABILITY INVARIANT

Prices of production are specified by a system of equations with one degree of freedom. Suppose a bushel corn is the numeraire. Let p be the price of a ton of iron; r the rate of profits; w the wage; and ρ_1, ρ_2, and ρ_3 the rent per acre on Type 1, Type 2, and Type 3 lands. Prices of production satisfy the system of equations in Displays [1] through [4]:

$$\left(a_{1,1} \cdot p + a_{2,1}\right) \cdot (1 + r) + a_{0,1} \cdot w = p \quad [1]$$

$$\left(a_{1,2} \cdot p + a_{2,2}\right) \cdot (1 + r) + c_{1,2} \cdot \rho_1 + a_{0,2} \cdot w = 1 \quad [2]$$

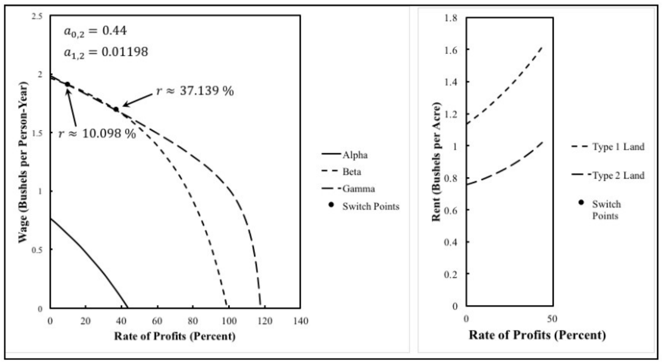
$$\left(a_{1,3} \cdot p + a_{2,3}\right) \cdot (1 + r) + c_{2,3} \cdot \rho_2 + a_{0,3} \cdot w = 1 \quad [3]$$

$$\left(a_{1,4} \cdot p + a_{2,4}\right) \cdot (1 + r) + c_{3,4} \cdot \rho_3 + a_{0,4} \cdot w = 1 \quad [4]$$

The parameters in these equations are coefficients of production taken from Table 1. The person-years of labor needed to produce a unit output in each process is denoted as $a_{0,j}$; $j = 1,2,3,4$. The coefficients of production $a_{i,j}$; $i = 1,2$; $j = 1,2,3,4$; denote the tons iron or bushels corn need to produce a unit output in each process. The coefficients $c_{1,2}$, $c_{2,3}$, $c_{3,4}$ are the acres of Type 1, Type 2, and Type 3 land, respectively, needed to produce a bushel corn in each of the last three processes. Each process is assumed to exhibit constant returns to scale. Display [5] constrains rents such that at least one type of land is free:

$$\rho_1 \cdot \rho_2 \cdot \rho_3 = 0 \quad [5]$$

Figure 1: Wage Curves and Rent for an Example of Extensive Rent



One can solve for prices of production for each technique (Appendix B). The left pane in Figure 1 depicts wage curves for each technique, as functions of the rate of profits. The direct labor and iron inputs, $a_{0,2}$ and $a_{1,2}$, required to produce a bushel corn on Type 1 land are as indicated. Suppose the Alpha technique were operated. No rent would be paid for Type 3 land, and, given the rate of profits, Displays [1] and [4] would be a system of two equations in two variables, p and w. A solution, with a positive price of iron and a positive wage, exists for a nonnegative rate of profits up to a maximum associated with the technique. The wage curve on the left pane in Figure 1, labeled 'Alpha', shows the solution to this system for wages. The wage curves for the Beta and Gamma techniques are similarly derived.

Rent must be such that processes operated on lands that are fully farmed make the same rate of profits as the type of land that is not scarce. For the Alpha technique, the equation in Display [2] can be solved for rent per acre on Type 1 land. The equation in Display [3] yields rent per acre for Type 2 land. The resulting rent curves are graphed on the right pane in Figure 1. Rent curves need not be monotone in the general case.

The Choice of Technique

Techniques are chosen on the basis of cost, where the prices must be such that no land pays a negative rent. At a given rate of profits, order the wage curves on the left pane in Figure 1 from high wages to low wages. This is the order of fertility, that is, the order in which different types of land would be introduced as requirements for use expand. Suppose the rate of profits were a specific value, for example zero, and that requirements for use were such that they could be satisfied by farming only one type of land. The wage curve for a postulated technique in which only Type 2 land is farmed is equal to the wage curve for the Gamma technique, in which all three types of land are farmed and Type 2 land pays no rent. If only one type of land were farmed at a rate of profits of zero, only Type 2 land would be farmed, and no rent would be paid. The wage curve for Gamma has the highest wage at a rate of profits of zero.

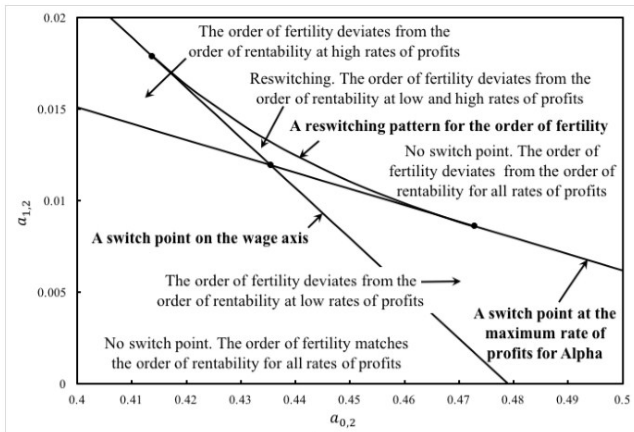
If requirements for use could be satisfied only by farming two types of land at a rate of profits of zero, Type 2 land would be fully cultivated, and Type 1 would enter into the production of corn. The wage curve for Beta is the next lower wage curve, and it is constructed under the assumption that Type 1 land pays no rent. The first, second, and third processes defined in Table 1 would be operated. A single rate of profits can be obtained by operating these processes in parallel, with one wage and one price of iron ruling throughout the economy, only if Type 2 land pays a positive rent.

The order of fertility is then the order in which different types of land enter into cultivation when ever larger produced quantities of the agricultural product are needed to satisfy requirements for use. In the example, in which all three types of land are assumed to be cultivated, at least somewhat, the Alpha technique is always cost-minimizing. The wage frontier is the inner envelope of the three wage curves. Rents on fully farmed land are positive for the Alpha technique, but not for the Beta and Gamma techniques. In the analysis of the choice of technique with circulating capital and no scarce natural resources, the outer envelope of the wage curves is the wage frontier. The possibility of wage curves on the outer envelope intersecting more than once leads to the reswitching of techniques in the case of simple (non-joint) production and the reswitching of the order of fertility in the case of extensive rent.

The order of fertility is also known as the order of efficiency. The order of rentability is the order downwards of rent curves on the right pane in Figure 1. For low and high rates of profits, the order of fertility is Type 2, Type 1, and Type 3 lands. For intermediate rates of profits, the order of fertility is Type 1, Type 2, and Type 3 lands. The order of fertility matches the order of rentability at intermediate rates of profits but not outside the range defined by the switch points. One could call this an example of the reswitching of the order of fertility.

Notice that no quantity flows vary, in this example, with the variation in the order of fertility. For any feasible distribution of income, Type 1 and Type 2 lands are fully farmed. The number of acres of Type 3 land that are left fallow also does not vary. Prices are all that varies. Since quantity flows do not vary, it is an abuse of language to call these intersections of wage curves on the outer frontier 'switch points', albeit I do this throughout this article.

Figure 2: The Parameter Space for an Example of Extensive Rent



Perturbations of Selected Coefficients of Production

Since graphs on paper are limited to two dimensions, perturbations of only two coefficients of production are considered in this article. Accordingly, Figure 2 shows a partition of part of the parameter space around the example illustrated in Figure 1. Perturbations of these parameters modify the location of the wage curve for Beta and the rent per acre obtained for Type 1 land under the Alpha technique. Intersections of rent curves do not exist for parameters in any regions in this small two-dimensional slice of the parameter space. Type 1 land is always higher here than Type 2 land in the order of rentability. Type 3 land pays no rent. On the other hand, qualitative variations in the intersections of the wage curves for the Beta and Gamma techniques exist in this region of the parameter space.

The parameters with which the reswitching of the order of fertility is illustrated in Figure 1 are taken from the central region in Figure 2. Each of the one-dimensional loci in Figure 2 represent a fluke case. At a reswitching pattern, two wage curves, in this case, for the Beta and Gamma techniques, are tangent for a switch point. An intersection of wage curves at a rate of profits of zero is another fluke case. An intersection of the wage curves for the Beta and Gamma techniques at a rate of profits for which the wage is zero for the Alpha technique is a third fluke case.

These fluke cases partition the parameter space into regions among which the order of fertility varies. To the southwest in Figure 2, one switch point between the wage curves for Beta and Gamma has vanished over the wage axis, and the other is at a rate of profits exceeding the maximum rate of profits for Alpha. The order of fertility matches the order of rentability. The northwest and southeast illustrate parameters for which the order of fertility varies with the rate of profits. The order of fertility varies from the order of rentability at one or the other extreme, as indicated.

A PARAMETRIZATION WITH THE ORDER OF FERTILITY INVARIANT

The reswitching of the order of rentability arises for parameters well to the northeast of those used in drawing Figure 2. Consider Figure 3. The rent curve for Type 1 land has varied such that it intersects the rent curve for Type 2 land twice. The order of fertility is Type 2, Type 1, and Type 3 lands, and this order does not vary with either the wage of the rate of profits. The order of fertility is the same as the order of rentability at low and high rate of profits. The order of rentability between the intersections of rent curves is, however, Type 1, Type 2, and Type 3. Unlike in the range of parameters considered in the previous section, the order of rentability varies here with the rate of profits.

Figure 3: Wage Curves and Rent for Another Example of Extensive Rent

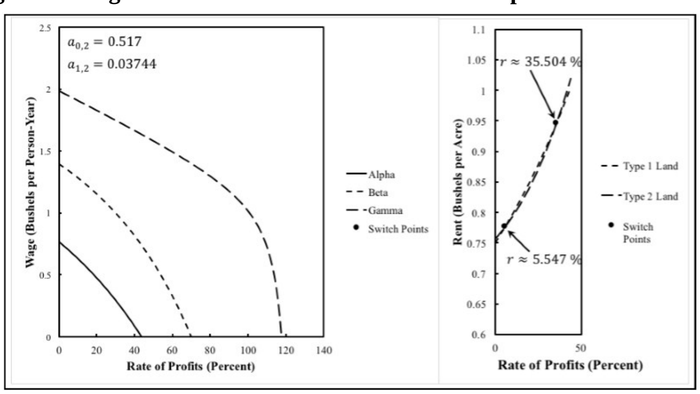
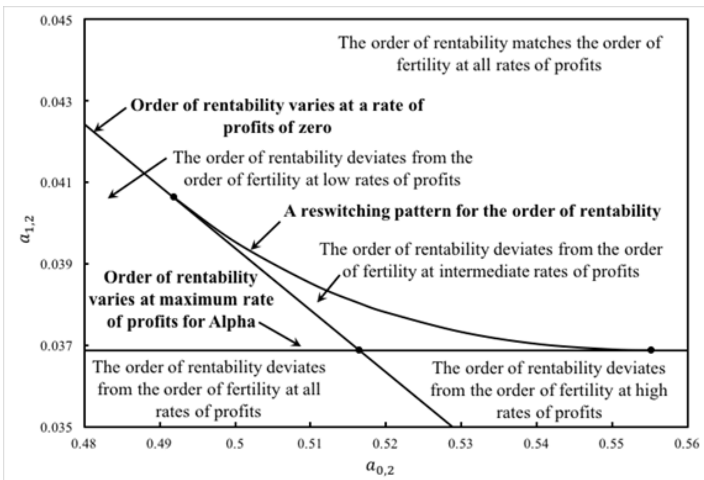


Figure 4 shows a partition of part of the parameter space around the example illustrated in Figure 3. No switch points exist for any regions in this small two-dimensional slice of the parameter space. The wage curves do not intersect, and the order of fertility is such that Type 2 land is more fertile than Type 1 land for any income distribution. Type 3 land pays no rent.

The example illustrated is for parameters in the central region in Figure 4. Here, the order of rentability varies from the order of fertility at intermediate rates of profits, but not for low and high rates of profits. This intermediate region disappears at the partition between this central region and the region to the northeast. The two curves for rent on the right in Figure 4 become tangent at a single rate of profits for points along this partition. Consequently, the order of rentability is identical to the order of fertility for any parameters in the region to the northeast.

Figure 4: The Parameter Space for a Second Example of Extensive Rent



The two curves for rent intersect at a single point for parameters in the regions to the west and the southeast in Figure 4. The order of rentability deviates from the order of fertility for low rates of profits in the region to the west. In the region to the southeast, the order of rentability differs from the order of fertility for high rates of profits.

The two curves for rent do not intersect at all for parameters in the region to the southwest. The order of rentability differs from the order of rentability for any feasible rate of profits in this region. This exploration of the parameter space is only a local perturbation. Results for the analysis of the choice of technique are qualitatively quite different in other parts of the parameter space.

CONCLUSION

Whether or not the order of fertility differs from the order of rentability can vary with the wage or the rate of profits. The order of fertility may vary with distribution, while the order of rentability stays constant. Or the order of rentability may vary with distribution, while the order of fertility stays constant. And both of these variations with distribution may reverse themselves, as in the reswitching of techniques. This article has presented a numeric example in which, with one parametrization, the reswitching of the order of fertility arises. The reswitching of the order of rentability arises with another parametrization.

The reswitchings of techniques, of the order of fertility, and of the order of rentability are not fluke cases. In models of extensive rent, these reswitchings of the order of fertility and of the order of rentability are contrasted with genuine fluke cases, such as when wage curves or rent curves are tangent at a single point. The exploration of the results of perturbing parameters, such as coefficients of production, reveals a variety of fluke cases and other phenomena in models of prices of production.

Appendix A: Prices of Production with Extensive Rent

The technology in Table 1 can be specified by a four-element row vector $\mathbf{a}_0$, the 2x4 input matrix **A**, the 3x4 matrix **C** of coefficients of production for land, and the 2x4 output matrix **B**:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \qquad \text{[A-1]}$$

One also needs a three-element column vector **t** of the quantities of each type of land available. Requirements for use are specified in the two-element column vector **d**. The vectors and matrices $\mathbf{a}_0$, **A, B, C, t** and **d** can be defined for any number of produced commodities and for any number of types of land for producing an agricultural commodity.

In this model of extensive rent, only one process can be operated on each type of land, and each of these processes produces the same agricultural commodity. Only one process is known for producing each non-agricultural commodity. Aside from land, no joint production exists in this model. These assumptions have implications about the structure of the sparse matrices **B** and **C**. A square input matrix is associated with each type of land, formed by combining the processes for producing the non-agricultural commodities with the process operating on that type of land. Each of these input matrices is assumed to be a Sraffa matrix. A Leontief matrix for a viable technique is a Sraffa matrix when at least one commodity is basic – that is, enters directly or indirectly into the production of all commodities – and the maximum rate of profits for the submatrix of non-basic commodities, so to speak, exceeds the maximum rate of profits for the submatrix for basic commodities (Kurz and Salvadori, 1995). Furthermore, the agricultural commodity, called corn, is assumed to be a basic commodity in each one of these Sraffa matrices. The choice of technique, in this model, is a choice of which lands to farm.

A number of conditions must be satisfied by quantity flows and prices to allow the economy to undergo smooth reproduction. The net output of each produced commodity meets the requirements for use:

$$(\mathbf{B} - \mathbf{A}) \cdot \mathbf{q} = \mathbf{d}, \quad \text{[A-2]}$$

where the elements of the column vector **q** are the levels of operation of each process. The amount of each quality of land that is farmed cannot exceed the available quantity:

$$\mathbf{C} \cdot \mathbf{q} \leq \mathbf{t} \quad \text{[A-3]}$$

A vector is greater than or equal to another if each element of the vector is greater or equal to the corresponding element of the other vector. The level of operation of each process is non-negative:

$$q_i \geq 0, i = 1, 2, \ldots, m, \quad \text{[A-4]}$$

where m is the number of processes.

The costs of each process cannot fall below the revenues for that process:

$$\mathbf{p} \cdot \mathbf{A} \cdot (1 + r) + \boldsymbol{\rho} \cdot \mathbf{C} + w \cdot \mathbf{a}_0 \geq \mathbf{p} \cdot \mathbf{B} \quad \text{[A-5]}$$

where **p** is the row vector of prices, ρ is the row vector of rents, w is the wage, and r is the rate of profits. In other words, no process returns supernormal profits. The required net output is the numeraire:

$$\mathbf{p} \cdot \mathbf{d} = 1 \quad \text{[A-6]}$$

The price of each produced commodity is non-negative:

$$p_j \geq 0, j = 1, 2, \ldots, n, \quad \text{[A-7]}$$

where n is the number of produced commodities. The rent on each type of land is non-negative:

$$\rho_j \geq 0, j = 1, 2, \ldots, k, \quad \text{[A-8]}$$

where k is the number of types of land.

The rule of free goods applies to prices of production. Display [A-9] specifies the rule of free goods for land:

$$\boldsymbol{\rho} \cdot (\mathbf{C} \cdot \mathbf{q} - \mathbf{t}) = 0 \quad \text{[A-9]}$$

According to Display [A-3], the elements in the column vector in the parenthesis are non-positive. Since rents cannot be negative, types of lands that are not fully farmed receive a rent of zero.

Display [A-10] specifies the rule of non-operated processes:

$$[\mathbf{p} \cdot \mathbf{B} - \mathbf{p} \cdot \mathbf{A} \cdot (1 + r) - \boldsymbol{\rho} \cdot \mathbf{C} - w \cdot \mathbf{a}_0] \cdot \mathbf{q} = 0 \quad \text{[A-10]}$$

If costs exceed revenues in a process, that process is not operated.

Displays [A-2], [A-3], and [A-4] specify the quantity system. Displays

[A-5] through [A-8] specify the price system. Displays [A-9] and [A-10] are duality conditions. Consider a non-trivial solution to the price and quantity systems that also satisfies the rule of free goods and the rule of non-operated processes. If the rate of profits is given, such a solution consists of a cost-minimizing technique, the wage, and prices of production. The prices of production include the rents on each type of land. Kurz and Salvadori (1995) prove an existence theorem for this model of extensive rent.

Appendix B: Solutions of Price Equations for the Numeric Example

This appendix presents a solution for the price equations for the Alpha technique and notes a couple of properties of the solution. Displays [1] and [4], with $\rho_3 = 0$, provide a system of equations for the Alpha technique. The solution, with the wage and price of iron indexed for the technique, is:

$$w_\alpha(r) = \frac{(a_{1,1} \cdot a_{2,4} - a_{1,4} \cdot a_{2,1}) \cdot (1+r)^2 - (a_{1,1} + a_{2,4}) \cdot (1+r) + 1}{a_{0,4} + (a_{0,1} \cdot a_{1,4} - a_{0,4} \cdot a_{1,1}) \cdot (1+r)} \qquad \text{[B-1]}$$

$$p_\alpha(r) = \frac{a_{0,1} - (a_{0,1} \cdot a_{2,4} - a_{0,4} \cdot a_{2,1}) \cdot (1+r)}{a_{0,4} + (a_{0,1} \cdot a_{1,4} - a_{0,4} \cdot a_{1,1}) \cdot (1+r)} \qquad \text{[B-2]}$$

For a technique in which all coefficients of production are positive and in which a surplus product can be produced, w_α must be a decreasing function of the rate of profits, intersecting the axis for the rate of profits at a finite, positive value. The rent per acre on land of Type 1 is found from Display 2 to be:

$$\rho_1^\alpha(r) = \left(\frac{1}{c_{1,2}}\right) \cdot \left[1 - (a_{1,2} \cdot p_\alpha + a_{2,2}) \cdot (1+r) - a_{0,2} \cdot w_\alpha\right] \qquad \text{[B-3]}$$

The rent per acre on land of Type 2 is found from Display 3 to be:

$$\rho_2^\alpha(r) = \left(\frac{1}{c_{2,3}}\right) \cdot \left[1 - (a_{1,3} \cdot p_\alpha + a_{2,3}) \cdot (1+r) - a_{0,3} \cdot w_\alpha\right] \qquad \text{[B-4]}$$

Similarly, the wage, the price of iron, and the rents per acre for scarce land can be found, as functions of the rate of profits, for the Beta and Gamma techniques.

For the numeric example and the range of the parameters $a_{0,2}$ and $a_{1,2}$ considered in the main text, the wage curve for Alpha, as defined in Display B-1, is always on the inner frontier of the three wage curves for the three techniques. Thus, the rent per acre for land of Type 1 and Type 2 is always positive for the Alpha technique, for any non-negative rate of profits not exceeding the maximum rate.

Wage curves can likewise be derived for the Beta and Gamma techniques. The intersections of the wage curves in Figure 1 are found by solving the equation for $w_\beta(r) = w_\gamma(r)$ for the rate of profits. Points in the parameter space for which solutions to this equation are repeated roots, at a rate of profits of zero, or at the maximum rate of profits for Alpha are found by numeric methods.

The intersections of the rent curves in Figure 3 are found by solving the equation $\rho_1^{\alpha}(r) = \rho_2^{\alpha}(r)$ for the rate of profits. Here too points in the parameter space for which solutions are repeated roots, at a rate of profits of zero, or at the maximum rate of profits for Alpha are found by numeric methods.

References

Baranzini, M. L., Rotondi C., and Scazzieri R. (eds.) (2015) *Resources, Production and Structural Dynamics*, Cambridge: Cambridge University Press.

Bidard, C. and Klimovsky E. (2004) 'Switches and fake switches in methods of production', *Cambridge Journal of Economics* 28 (1): 89-97.

D'Agata, A. (1983) 'The existence and unicity of cost-minimizing systems in intensive rent theory', *Metroeconomica* 35: 147-158.

Harcourt, G. C. (1972) *Some Cambridge Controversies in the Theory of Capital*, Cambridge: Cambridge University Press.

Kaldor, N. (1966) 'Marginal productivity and the macroeconomic theories of distribution: comment on Samuelson and Modigliani', *Review of Economic Studies* 33 (4): 309-319.

Kurz, H. D. and Salvadori N. (1995) *Theory of Production: A Long-Period Analysis*, Cambridge: Cambridge University Press.

Lazzarini, A. (2011) *Revisiting the Cambridge Capital Theory Controversies: A Historical and Analytical Study*, Pavia: Pavia University Press.

Quadrio-Curzio, A. (1980) 'Rent, income distribution, and orders of efficiency and rentability' in Pasinetti, L. L. (ed.) *Essays on the Theory of Joint Production*, New York: Columbia University Press.

Schefold, B. (1989) *Mr. Sraffa on Joint Production and other Essays*, London: Unwin-Hyman.

Vienneau, R. L. (2021) 'Fluke switch points in pure fixed capital systems', *Centro Sraffa Working Papers*, n. 48.

Vienneau, R. L. (2023) 'Characteristics of labor markets varying with perturbations of relative markups', *Review of Political Economy*, forthcoming.

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