***IJMS*, Vol. 15, Nos. 1-2, (January-June 2016), pp. 47-62**

ISSN: 0972-754X

STUDY OF FLOW THROUGH A STRAIGHT POROUS TUBE OF ARBITRARY CROSS-SECTION UNDER THE INFLUENCE OF UNIFORM TRANSVERSE MAGNETIC FIELD

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Abstract

The MHD steady flow of a viscous in-compressible, slightly conducting fluid through an infinite long straight porous tube of arbitrary cross-section in the presence of a constant pressure gradient, under the influence of a uniform transverse magnetic field has been studied. A numerical difference scheme is employed. The equilateral and rectangular cross-sections situations are considered as special cases. An appropriate numerical solution for the velocity have been investigated and discussed for different situations. The effect of the permeability of the porous medium together with the magnetic parameter on the flow of the fluid is examined at length.

1. INTRODUCTION

The study of flow through porous medium assumed importance because of its interesting applications in the diverse fields of science, engineering and technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter-disciplinary fields such as bio-medical engineering etc. The lung alveolar is an example that finds application in an animal body. The classical Darcy's law [1] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as,

$$\vec{V} = -\left(\frac{K}{\mu}\right) gradP \text{ . (with the usual notation)}$$

Received: 19.7.05

2000 Mathematics Subject Classifications: 76D, 76S.

Key words and phrases: Viscous fluid, Transverse Magnetic Field, Arbitrary cross-section, Permeability, Pressure Gradient.

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortions in the velocity field. In the case of highly porous media such as fiberglass, papus of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beverse and Joseph (2), Saffman (3) and others. A generalized Darcy's law proposed by Brinkman (4) is given by,

$$0 = -\nabla P - \left(\frac{\mu}{K}\right)\vec{V} + \mu\nabla^2\vec{V}$$

where μ and K are coefficients of viscous of the fluid and permeability of the porous medium.

The generalized equation of momentum for the flow through the porous medium is,

$$\rho\left[\frac{\partial\vec{V}}{\partial \in} + \left(\vec{V}\bullet\nabla\right)\vec{V}\right] = -\nabla P + \mu\nabla^2\vec{V} - \left(\frac{\mu}{K}\right)\vec{V}.$$

The flow problems through porous medium are studied by several investigators [10]. The present paper deals with the flow of a viscous liquid through a porous tube of an arbitrary cross-section under the influence of magnetic field. Closed form solutions for the velocity field of different cross-sections are obtained by Narasimha Charyulu and Pattabhi Rama Charyulu [5, 6, 7, 8] earlier.

In the present paper a numerical difference scheme is obtained to study the flow of an incompressible viscous fluid through straight porous tube of an arbitrary cross-section under the influence of magnetic field and it is applied to the cases of rectangular cross-section and equilateral cross-section.

The effect of the permeability coefficient of the porous medium and the magnetic parameter of transverse magnetic field on the velocity of the flow of the fluid is examined at length.

2. FORMULATION AND SOLUTION

The flow of an in-compressible, slightly conducting fluid in the presence of a constant pressure gradient under the influence of uniform magnetic field is considered through a straight porous tube of an arbitrary cross-section t perpendicular to the flow direction and enclosing the region Â. The rectangular co-ordinate system 0 (x, y, z) is taken such that the z-axis lies along the axis of the tube and x, y axis are perpendicular to the z-axis.

The velocity $\vec{V}$ [0, 0, w (x, y)] of the fluid satisfies the equation of continuity

$$\nabla \bullet \vec{V} = 0 \tag{2.1}$$

together with the equation of motion

$$\rho\left[\frac{\partial \vec{V}}{\partial \in} + \left(\vec{V} \bullet \nabla\right)\vec{V}\right] = -\nabla P + \mu \nabla^2 \vec{V} - \left(\frac{\mu}{K}\right)\vec{V} - \sigma \mu_e^{\,2} H_0^{\,2} \vec{V} \tag{2.2}$$

with the boundary condition

$$\vec{V} = 0 \text{ on t.} \tag{2.3}$$

where ρ is the fluid density, P is the fluid pressure, μ is the coefficient of viscosity, s is the electric conductivity, μ_e is the magnetic permeability, K is the permeability of the porous medium, H_0 is the uniform magnetic field intensity acting perpendicular to the axes to the tube.

The induced effect is neglected in comparison with applied magnetic field as the fluid is considered to be slightly conducting and the Reynolds's number of magnetic field will be very small [Sparrow and Cess (12)].

The electric force E given by the Ohm's law $\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B})$, Where $\vec{B}$ = $(H_0, 0, 0)$ is assumed to be a null vector for simplicity of the problem.

The equation of continuity (2.1) is satisfied by the choice of the velocity $\vec{V}$ [0,0,w (x, y)] and the equation of motion (2.2) becomes

$$\nabla^2 w - \alpha^2 w = -\frac{P}{\mu} \tag{2.4}$$

where, $P = -\partial P/\partial t$, $\alpha^2 = M^2 + K^2$,

$$M^2 = \sigma \mu_e^2 H_0^2/\mu, \quad \alpha = (K)^{-1/2} \tag{2.5}$$

where M is the magnetic parameter and K is the permeability parameter of the porous medium.

The most general linear partial differential equation [Jain (9)] is given by

$$A\frac{\partial^2 w}{\partial x^2} + C\frac{\partial^2 w}{\partial y^2} + D\frac{\partial w}{\partial x} + E\frac{\partial w}{\partial y} + Fw = F^* \tag{2.6}$$

where A, C and F assume values A>0, C>0, F>0 in R + ∂R and where ∂R is the boundary of the region so that the equation becomes elliptic for the flow problem under consideration.

Let us consider at (x_l , y_m) of the flow region,

$$A\frac{\partial^2 w}{\partial x^2} + C\frac{\partial^2 w}{\partial y^2} + D\frac{\partial w}{\partial x} + E\frac{\partial w}{\partial y} + Fw =$$

$$l_0 w(x_l, y_m) + l_1 w(x_l + h_1, y_m) + l_2 w(x_l, y_m + h_2) + l_3 w(x_l - h_3, y_m) + l_4 w(x_l, y_m - h_4) \tag{2.7}$$

Expanding each of the above terms by Taylor's series and comparing the various order derivatives on both sides, we get

$$l_0 + l_1 + l_2 + l_3 + l_4 = F(x_l, y_m) \tag{2.8}$$

$$h_1 l_1 - h_3 l_3 = D(x_l, y_m) \tag{2.9}$$

$$h_2 l_2 - h_4 l_4 = E(x_l, y_m) \tag{2.10}$$

$$h_1^2 l_1 + h_3^2 l_3 = 2A(x_l, y_m) \tag{2.11}$$

$$h_2^2 l_2 + h_4^2 l_4 = 2C(x_l, y_m)\,. \tag{2.12}$$

Solving these equations from (2.8) to (2.12), we get

$$l_0 = F^* - \left[\frac{2A}{h_1 h_3} + \frac{(h_3 - h_1)D}{h_1 h_3} + \frac{2C}{h_2 h_4} + \frac{(h_4 - h_2)}{h_3 h_4}\right];$$

$$l_1 = \frac{(2A + h_3 D)}{h_1(h_1 + h_3)} \quad ; \quad l_2 = \frac{(2C + h_4 D)}{h_2(h_2 + h_4)} \;;$$

$$l_3 = \frac{(2A - h_1 D)}{h_3(h_1 + h_3)} \quad ; \quad l_4 = \frac{(2C - h_2 D)}{h_4(h_2 + h_4)} \, . \tag{2.13}$$

Therefore the difference scheme for the arbitrary cross-section of the flow of the fluid is given by

$$F^*(x_l, y_m) = l_0 w(x_l, y_m) + l_1 w(x_l + h_1, y_m) + l_2 w(x_l, y_m + h_2) + l_3 w(x_l - h_3, y_m) + l_4 w(x_l, y_m - h_4) \, . \tag{2.14}$$

To apply the difference scheme given by equation (2.14) to the arbitrary cross-section (Fig.1). The cross-section area is divided into rectangular meshes of equal step size along x, y axis by using $h = h_1 = h_3 = a$ and $h^* = h_2 = h_4 = b$ along the axes respectively. The partial differential equation becomes elliptic if we choose A = C = 1, D = E = 0, $F = -\alpha^2$, $F^* = -P/m$ and reduces to the equation of motion (2.4) to be solved for the present situation.

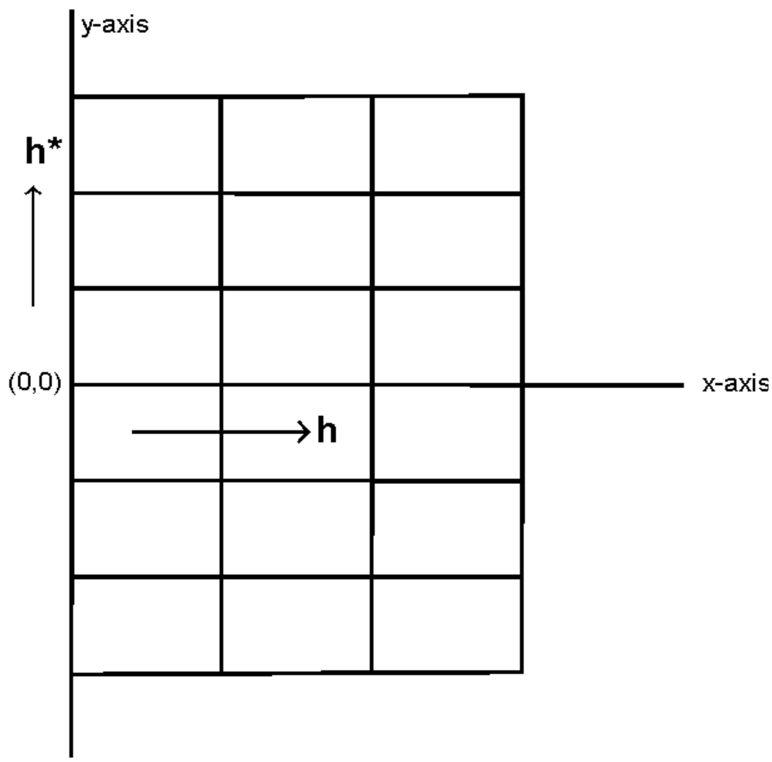

Fig -1. Flow through arbitrary cross-section
(**h**=a and **h***=b)

The values of l_0, l_1, l_2, l_3, l_4 from eq. (2.13) are given by

$$l_0 = -\alpha^2 - 2\left[\frac{a^2+b^2}{(ab)^2}\right];\ l_1 = l_3 = \frac{1}{a^2} \qquad ; l_2 = l_4 = \frac{1}{b^2} \tag{2.15}$$

The values at the nodal points which are on the cross-section t will be zero as the velocity on the boundary is to be zero. Hence

w (0, 0) = w (0, ±b) = w (0, ±2b) = w (0, ±3b) = w (a, ±3b)

= w (2a, ±3b)=w (3a, ±3b) = w (3a, ±2b) = w (3a, ±b) = w (3a, 0) = 0 on τ.

The difference scheme equ.(2.14) applied at other nodal points together with equation (2.15) will give,

$$-\left\{a^2 + 2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(a,0) + \frac{1}{a^2}w(2a,0) + \frac{1}{b^2}w(a,b) + \frac{1}{b^2}w(a,-b) = \frac{-P}{\mu} \tag{2.16}$$

$$-\left\{a^2 + 2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(2a,0) + \frac{1}{a^2}w(a,0) + \frac{1}{b^2}w(2a,b) + \frac{1}{b^2}w(2a,-b) = \frac{-P}{\mu} \tag{2.17}$$

$$-\left\{a^2 + 2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(a,b) + \frac{1}{a^2}w(2a,b) + \frac{1}{b^2}w(a,2b) + \frac{1}{b^2}w(a,0) = \frac{-P}{\mu} \tag{2.18}$$

$$-\left\{a^2 + 2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(2a,b) + \frac{1}{a^2}w(a,b) + \frac{1}{b^2}w(2a,2b) + \frac{1}{b^2}w(2a,0) = \frac{-P}{\mu} \tag{2.19}$$

$$-\left\{a^2 + 2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(a,2b) + \frac{1}{a^2}w(2a,2b) + \frac{1}{b^2}w(a,b) = \frac{-P}{\mu} \tag{2.20}$$

$$-\left\{a^2 + 2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(2a,2b) + \frac{1}{a^2}w(a,2b) + \frac{1}{b^2}w(2a,b) = \frac{-P}{\mu} \tag{2.21}$$

$$-\left\{a^2 + 2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(a,-b) + \frac{1}{a^2}w(2a,-b) + \frac{1}{b^2}w(a,-2b) + \frac{1}{b^2}w(a,0) = \frac{-P}{\mu} \tag{2.22}$$

$$-\left\{a^2+2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(2a,-b)+\frac{1}{a^2}w(a,-b)+\frac{1}{b^2}w(2a,-2b)+\frac{1}{b^2}w(2a,0)=\frac{-P}{\mu} \tag{2.23}$$

$$-\left\{a^2+2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(a,-2b)+\frac{1}{a^2}w(2a,-2b)+\frac{1}{b^2}w(a,-b)=\frac{-P}{\mu} \tag{2.24}$$

$$-\left\{a^2+2\left[\frac{a^2+b^2}{(ab)^2}\right]\right\}w(2a,-2b)+\frac{1}{a^2}w(a,-2b)+\frac{1}{b^2}w(2a,-b)=\frac{-P}{\mu}. \tag{2.25}$$

The smaller values on *a* and *b* make the division of the mesh fine. This division of the mesh with decrease in the *a* and *b* values becomes more laborious to solve the equations for the nodal values. The process of division is to be stopped when the desired accuracy is obtained. Therefore instead of fine division further a simple method of iteration is given to get the values at the nodal points by employing the difference scheme.

3. APPLICATION OF THE DIFFERENCE SCHEME

CaseI. Equilateral Triangle Cross-Section

The difference scheme for the arbitrary cross-section τ is to be applied to the equilateral cross-section (Fig.2). Taking h = 2 and $h^* = 2/\sqrt{3}$ along x, y axes respectively with $h = h_1 = h_3 = 2$ and $h^* = h_2 = h_4 = 2/\sqrt{3}$.

The values of l_0, l_1, l_2, l_3, l_4 from eq. (2.15) are given by

$$l_0 = -(\alpha^2+2)\ ;\ l_1 = l_3 = \frac{1}{4};\ l_2 = l_4 = \frac{3}{4}. \tag{3.1}$$

The values at the nodal points which are on the cross-section t will be zero as the velocity on the boundary is to be zero. Hence

$$\begin{aligned} w(0,0) &= w(2, \pm 2/\sqrt{3}) = w(4, \pm 4/\sqrt{3}) = w(6, \pm 6/\sqrt{3}) = w(6,0) \\ &= w(6, \pm 4/\sqrt{3}) = w(6, \pm 2/\sqrt{3}) = 0 \text{ on } \tau. \end{aligned}$$

From the equations (2.16) to (2.25), we have

$$-4(a^2+2)\,w(4,0) + 3w(4, 2/\sqrt{3}) + w(2,0) + 3w(4, -2/\sqrt{3}) = -4\,P/\mu \tag{3.2}$$

$$-4(\alpha^2+2)\,w(4, 2/\sqrt{3}) + 3w(4,0) = -4\,P/\mu \tag{3.3}$$

$- 4(\alpha^2 + 2)\, w(2, 0) + w(4, 0) = - 4\, P/\mu$ (3.4)

$- 4(\alpha^2 + 2)\, w(4, -2/\sqrt{3}) + 3w(4, 0) = - 4\, P/\mu.$ (3.5)

From (3.3) and (3.5), It is observed that $w(4, 2/\sqrt{3}) = w(4, -2/\sqrt{3})$ which shows the symmetry of the flow about x-axis.

Solving the above equations, we get

$$w(2,0) = \left(\frac{4P}{\mu}\right)\left[\frac{(4\alpha^4 + 17\alpha^2 + 15)}{(\alpha^2 + 2)(16\alpha^4 + 64\alpha^2 + 45)}\right] \tag{3.6}$$

$$w(4,0) = \left(\frac{4P}{\mu}\right)\left[\frac{(4\alpha^2 + 15)}{(16\alpha^4 + 64\alpha^2 + 45)}\right] \tag{3.7}$$

$$w(4, \pm 2/\sqrt{3}) = \left(\frac{P}{\mu}\right)\left[\frac{(16\alpha^4 + 76\alpha^2 + 90)}{(\alpha^2 + 2)(16\alpha^4 + 64\alpha^2 + 45)}\right]. \tag{3.8}$$

The smaller values on h and h^* make the division of the mesh fine. This division of the mesh with decrease in the h and h^* values becomes more laborious to solve the equations for the nodal values. The process of division is to be stopped when the desired accuracy is obtained. Therefore instead of fine division further a simple method of iteration is given to get the values at the nodal points by employing the difference scheme. [Appendix]

(a) Flow through Porous Medium Under no Magnetic Field

In this, M = 0, α = K. Equations (3.6) to (3.8) become,

$$w(2,0) = \left(\frac{4P}{\mu}\right)\left[\frac{(4K^4 + 17K^2 + 15)}{(K^2 + 2)(16K^4 + 64K^2 + 45)}\right]$$

$$w(4,0) = \left(\frac{4P}{\mu}\right)\left[\frac{(4K^2 + 15)}{(16K^4 + 64K^2 + 45)}\right]$$

$$w(4, \pm 2/\sqrt{3}) = \left(\frac{P}{\mu}\right)\left[\frac{(16K^4 + 76K^2 + 90)}{(K^2 + 2)(16K^4 + 64K^2 + 45)}\right].$$

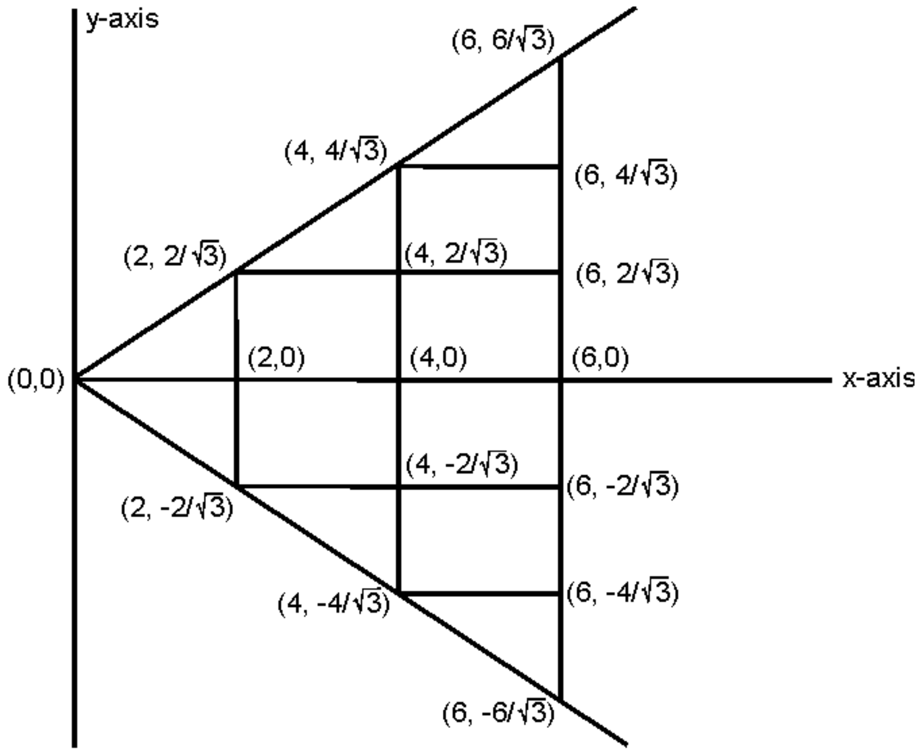

Fig -2. Flow through equilateral triangular cross-section

(b) Navier- Stokes Flow Under Magnetic Field

In this, K = 0, α = M. Equations (3.6) to (3.8) become,

$$w(2,0) = \left(\frac{4P}{\mu}\right)\left[\frac{(4M^4 + 17M^2 + 15)}{(M^2 + 2)(16M^4 + 64M^2 + 45)}\right]$$

$$w(4,0) = \left(\frac{4P}{\mu}\right)\left[\frac{(4M^2 + 15)}{(16M^4 + 64M^2 + 45)}\right]$$

$$w(4,\pm 2/\sqrt{3}) = \left(\frac{P}{\mu}\right)\left[\frac{(16M^4 + 76M^2 + 90)}{(M^2 + 2)(16K^4 + 64M^2 + 45)}\right].$$

Case II. Rectangular cross-section

The difference scheme for the arbitrary cross-section τ is to be applied to the rectangular cross-section (Fig.3). Taking h = 2 and h^* = 2 along x, y axes respectively with h = h_1 = h_3 = 2 and h^* = h_2 = h_4 = 2.

The values of l_0, l_1, l_2, l_3, l_4 from eq. (2.15) are given by

$$l_0 = -(\alpha^2 + 1) \ ; \ l_1 = l_3 = \frac{1}{4} \quad ; \ l_2 = l_4 = \frac{1}{4}. \tag{3.9}$$

The values at the nodal points which are on the cross-section t will be zero as the velocity on the boundary is to be zero. Hence

w (0, 0) = w (0, ±2) = w (0, ±4) = w (0, ±6) = w (2, ±6) = w (4, ±6)

= w (6, ±6) = w (6, ±4) = w (6, ±2) = w (6, 0) = 0 on t.

Equations (2.16) to (2.25) will give,

$$-4(\alpha^2 + 1)w(2,0) + w(4,0) + w(2,2) + w(2,-2) = \frac{-4P}{\mu} \tag{3.10}$$

$$-4(\alpha^2 + 1)w(4,0) + w(2,0) + w(4,2) + w(4,-2) = \frac{-4P}{\mu} \tag{3.11}$$

$$-4(\alpha^2 + 1)w(2,2) + w(2,0) + w(4,2) + w(2,4) = \frac{-4P}{\mu} \tag{3.12}$$

$$-4(\alpha^2 + 1)w(4,2) + w(2,2) + w(4,4) + w(4,0) = \frac{-4P}{\mu} \tag{3.13}$$

$$-4(\alpha^2 + 1)w(2,4) + w(2,2) + w(4,4) = \frac{-4P}{\mu} \tag{3.14}$$

$$-4(\alpha^2 + 1)w(4,4) + w(2,4) + w(4,2) = \frac{-4P}{\mu} \tag{3.15}$$

$$-4(\alpha^2 + 1)w(2,-2) + w(2,0) + w(4,-2) + w(2,-4) = \frac{-4P}{\mu} \tag{3.16}$$

$$-4(\alpha^2+1)w(4,-2)+w(2,-2)+w(4,-4)+w(4,0)=\frac{-4P}{\mu} \tag{3.17}$$

$$-4(\alpha^2+1)w(2,-4)+w(2,-2)+w(4,-4)=\frac{-4P}{\mu} \tag{3.18}$$

$$-4(\alpha^2+1)w(4,-4)+w(2,-4)+w(4,-2)=\frac{-4P}{\mu} \tag{3.19}$$

From the equations (3.12) to (3.19), It is observed that w (2, 2) = w (2, -2), w (4, 2) = w (4, -2), w (2, 4) = w (2, -4) and w (4, 4) = w (4, -4)) which shows the symmetry of the flow about x-axis.

Solving the above equations, we get

$$w(2,0)=\frac{4P}{\mu}\frac{(14\alpha^8+7\alpha^6+12\alpha^4+9\alpha^2+45)}{(32\alpha^{10}+24\alpha^8+22\alpha^6+25\alpha^4+128\alpha^2+90)} \tag{3.20}$$

$$w(4,0)=\frac{4P}{\mu}\frac{(22\alpha^{10}+27\alpha^8+21\alpha^6+27\alpha^4+49\alpha^2+52)}{(\alpha^2+1)(32\alpha^{10}+24\alpha^8+22\alpha^6+25\alpha^4+128\alpha^2+90)} \tag{3.21}$$

$$w(2,\pm 2)=\frac{P}{\mu}\frac{(36\alpha^{10}+32\alpha^8+22\alpha^6+78\alpha^4+102\alpha^2+144)}{(\alpha^2+1)(32\alpha^{10}+24\alpha^8+22\alpha^6+25\alpha^4+128\alpha^2+90)} \tag{3.22}$$

$$w(4,\pm 2)=\frac{P}{\mu}\frac{(49\alpha^{10}+48\alpha^8+46\alpha^6+84\alpha^4+106\alpha^2+172)}{(\alpha^2+1)(32\alpha^{10}+24\alpha^8+22\alpha^6+25\alpha^4+128\alpha^2+90)} \tag{3.23}$$

$$w(2,\pm 4)=\frac{P}{\mu}\frac{(20\alpha^{10}+12\alpha^8+22\alpha^6+32\alpha^4+98\alpha^2+132)}{(\alpha^2+1)(32\alpha^{10}+24\alpha^8+22\alpha^6+25\alpha^4+128\alpha^2+90)} \tag{3.24}$$

$$w(4,\pm 4)=\frac{P}{\mu}\frac{(42\alpha^{10}+32\alpha^8+23\alpha^6+41\alpha^4+104\alpha^2+142)}{(\alpha^2+1)(32\alpha^{10}+24\alpha^8+22\alpha^6+25\alpha^4+128\alpha^2+90)}. \tag{3.25}$$

(a) Flow through Porous Medium under no Magnetic Field

In this, M = 0, a = K. Equations (3.20) to (3.25) become,

$$w(2,0)=\frac{4P}{\mu}\frac{(14K^8+7K^6+12K^4+9K^2+45)}{(32K^{10}+24K^8+22K^6+25K^4+128K^2+90)}$$

$$w(4,0)=\frac{4P}{\mu}\frac{(22K^{10}+27K^{8}+21K^{6}+27K^{4}+49K^{2}+52)}{(K^{2}+1)(32K^{10}+24K^{8}+22K^{6}+25K^{4}+128K^{2}+90)}$$

$$w(2,\pm 2)=\frac{P}{\mu}\frac{(36K^{10}+32K^{8}+22K^{6}+78K^{4}+102K^{2}+144)}{(K^{2}+1)(32K^{10}+24K^{8}+22K^{6}+25K^{4}+128K^{2}+90)}$$

$$w(4,\pm 2)=\frac{P}{\mu}\frac{(49K^{10}+48K^{8}+46K^{6}+84K^{4}+106K^{2}+172)}{(K^{2}+1)(32K^{10}+24K^{8}+22K^{6}+25K^{4}+128K^{2}+90)}$$

$$w(2,\pm 4)=\frac{P}{\mu}\frac{(20K^{10}+12K^{8}+22K^{6}+32K^{4}+98K^{2}+132)}{(K^{2}+1)(32K^{10}+24K^{8}+22K^{6}+25K^{4}+128K^{2}+90)}$$

$$w(4,\pm 4)=\frac{P}{\mu}\frac{(42K^{10}+32K^{8}+23K^{6}+41K^{4}+104K^{2}+142)}{(K^{2}+1)(32K^{10}+24K^{8}+22K^{6}+25K^{4}+128K^{2}+90)}.$$

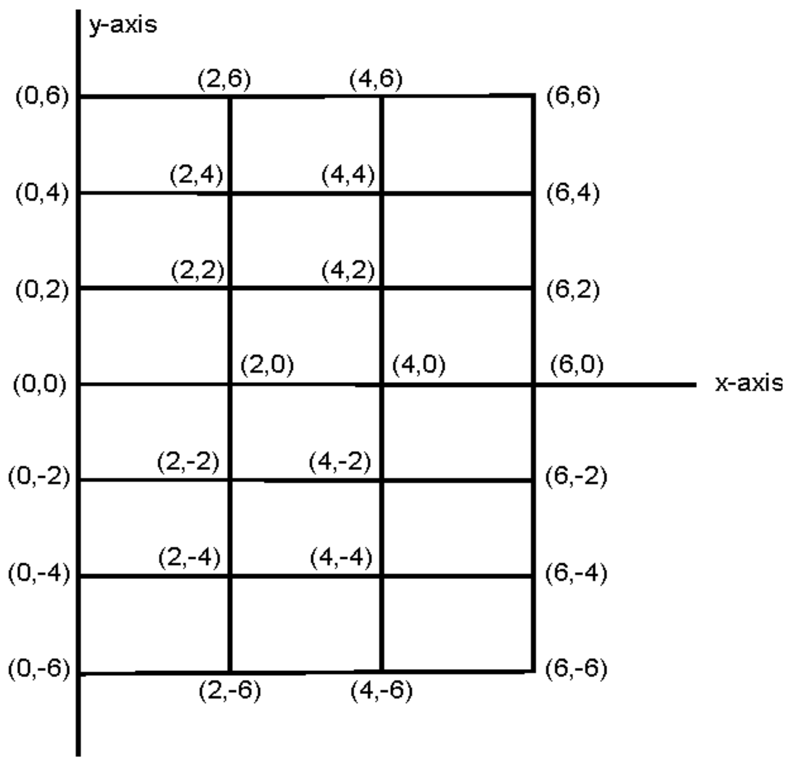

Fig -3. Flow through rectangular cross-section

(b) Navier-Stokes Flow under Magnetic Field

In this, K = 0, a = M. Equations (3.20) to (3.25) become,

$$w(4,0) = \frac{4P}{\mu}\frac{(22M^{10}+27M^8+21M^6+27M^4+49M^2+52)}{(M^2+1)(32M^{10}+24M^8+22M^6+25M^4+128M^2+90)}$$

$$w(2,\pm 2) = \frac{P}{\mu}\frac{(36M^{10}+32M^8+22M^6+78M^4+102M^2+144)}{(M^2+1)(32M^{10}+24M^8+22M^6+25M^4+128M^2+90)}$$

$$w(4,\pm 2) = \frac{P}{\mu}\frac{(49M^{10}+48M^8+46M^6+84M^4+106M^2+172)}{(M^2+1)(32M^{10}+24M^8+22M^6+25M^4+128M^2+90)}$$

$$w(2,\pm 4) = \frac{P}{\mu}\frac{(20M^{10}+12M^8+22M^6+32M^4+98M^2+132)}{(M^2+1)(32M^{10}+24M^8+22M^6+25M^4+128M^2+90)}$$

$$w(4,\pm 4) = \frac{P}{\mu}\frac{(42M^{10}+32M^8+23M^6+41M^4+104M^2+142)}{(M^2+1)(32M^{10}+24M^8+22M^6+25M^4+128M^2+90)}.$$

4. RESULTS AND DISCUSSIONS

The values of the flow of the fluid of the nodal points for $\alpha = 0$ in equilateral and rectangular cross-sections respectively given by,

$$w(2,0) = \frac{2P}{3\mu}; \quad w(4,0) = \frac{4P}{3\mu}; \quad w(4,\pm 2/\sqrt{3}) = \frac{P}{\mu}; \text{ and}$$

$$w(2,0) = \frac{2P}{\mu}; \quad w(4,0) = \frac{2.31P}{\mu}; \quad w(2,\pm 2) = \frac{1.6P}{\mu};$$

$$w(4,\pm 2) = \frac{1.91P}{\mu}; \quad w(2,\pm 4) = \frac{1.46P}{\mu}; \quad w(4,\pm 4) = \frac{1.57P}{\mu}.$$

Which correspond to the those values with the classical flow of Navier Stokes in absence of any resistance i.e., non-porous medium, with no magnetic field.

Tables-I and III shows, If the permeability of the medium is small i.e., $K \rightarrow 0$, $\alpha \rightarrow \infty$, the flow decreases at the nodal points. Also it is observed that the non-

Darcian effect is felt more near the boundary of the tube, i.e., the flow decreases with increasing distance from the axis of the tube and along the axis flow is increasing with the increase in h values.

Tables II and IV shows, the flow decreases with increases in magnetic parameter M for given value of K. As $K \to \infty$, the flow becomes classical flow under the effect of magnetic field.

The result of magnetic parameter and permeability of the medium is to decrease the flow field with the increase in either M or K.

APPENDIX

Difference scheme for equilateral triangular cross-section.

I. Iteration

Using the values w (4, 0), w (2, ±2/√3) and w (0, 0) from the difference scheme, we get

$$w^{(1)}(2,0) = \frac{4P}{\mu(4\alpha^2 + 7)}.$$

By employing the values $w^{(1)}$ (2, 0), w (6, 0) and w (4, ±4/Ö3), we get

$$w^{(1)}(4,0) = \frac{32P(\alpha^2 + 2)}{\mu(4\alpha^2 + 7)(8\alpha^2 + 7)}.$$

The symmetrical flow gives w (4, 2/√3) = w (4, -2/√3) and using the values w (6, 2/√3), w (2, 2/√3), w (4, 4/√3) and $w^{(1)}$ (4, 0), we get

$$w^{(1)}(4,\pm 2/\sqrt{3}) = \frac{P(32\alpha^4 + 108\alpha^2 + 97)}{\mu(\alpha^2 + 2)(4\alpha^2 + 7)(8\alpha^2 + 7)}.$$

Employing the values $w^{(1)}$ (2, 0), $w^{(1)}$ (4, 0) and w (4, ±2/√3), the second iteration can be obtained.

II. Iteration

$$w^{(2)}(2,0) = \frac{P(32\alpha^4 + 92\alpha^2 + 65)}{\mu(\alpha^2 + 2)(4\alpha^2 + 7)(8\alpha^2 + 7)}$$

$$w^{(2)}(4,0) = \frac{P(128\alpha^6 + 816\alpha^4 + 1525\alpha^2 + 1035)}{\mu 4(\alpha^2 + 2)^2(4\alpha^2 + 7)(8\alpha^2 + 7)}$$

$$w^{(2)}(4,\pm 2/\sqrt{3}) = \frac{P(512\alpha^8 + 3904\alpha^6 + 10656\alpha^4 + 13087\alpha^2 + 6253)}{\mu 16(\alpha^2+2)^3(4\alpha^2+7)(8\alpha^2+7)}.$$

Iteration scheme for rectangular cross-section can be developed in similar way.

REFERENCES

[1] M. Musakat, Flow of Homogeneous Fluid Through Porous Medium, McGraw Hill Inc., New York, 1937.

[2] S.G. Beavers and D.D. Joseph, Boundary Conditions at a Natural Permeable Wall, Jr. of Fluid Mechanics, 1967, 30, pp. 197-207.

[3] P. G. Saffman, On the Boundary Condition at the Surface of a Porous Medium, Studies of Applied Math., 1971, 50, pp. 93-101.

[4] H. C. Brinkman, The Calculation of Viscous Force Exerted by a Flowing Fluid on a Dense Swerf of Particles, *Jr. of Applied Science Research*, 1947, 27, A1, pp. 27-34.

[5] N. Ch. Pattabhi Ramacharyulu, International Centre for Th. Physics Trieste International Report, IC/76/122.

[6] V. Narsimha Charyulu and Pattabhi Ramacharyulu, N.Ch., Steady flow through a Porous Region Contained between two Cylinders, *Journal of the Indian Institute of Sciences*, 1978, 60, No.2, pp. 37-42.

[7] V. Narsimha Charyulu and Pattabhi Ramacharyulu, N.Ch., Th. Physics, 1981.

[8] V. Narsimha Charyulu and Pattabhi Ramacharyulu, N.Ch., Steady flow of a Viscous liquid in a Porous Elliptic tube, Proc. Indian Acad. Sci., 1978, 87A, No.2, pp. 79-83.

[9] M.K. Jain, Numerical Solution of Differential Equations, Wiley-Eastern Ltd., New Delhi, 1979.

[10] B. Sudhakar and Venkatramana, *Journal of Energy Heat and Mass Transfer*, **2**, No. 4, 1990.

[11] Goutham Chakraborthy and P.R. Senu Gupta, MHD flow of viscoelastic Maxwell fluids in a porous duct, *Indian Journal of Mathematics*, **36**, No. 3, (1994) 267.

[12] E.M. Sparrow and R.D. Cess, Tran. ASME journal of Applied Mechanics, 29 (1962) 181.

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Table I
Variation of Velocity w for Different values of K in Equilateral Cross-Section.
(P = 1.0; m = 1.0)

K	$W^{(1)}(2,0)$	$W^{(1)}(4,0)$	$W^{(1)}(2,\pm 2/\sqrt{3})$	$W^{(2)}(2,0)$	$W^{(2)}(4,0)$	$W^{(2)}(2,\pm 2/\sqrt{3})$	$W(2,0)$	$W(4,0)$	$W(2,\pm 2/\sqrt{3})$
0.00	0.571429	1.306122	0.989796	0.663265	1.320153	0.996971	0.666667	1.333333	1.000000
0.20	0.558659	1.245535	0.948113	0.642835	1.257727	0.954282	0.646365	1.274335	0.958702
0.40	0.523560	1.092647	0.842354	0.589427	1.101515	0.846661	0.593076	1.124177	0.853302
0.60	0.473934	0.905655	0.711543	0.519667	0.912793	0.714836	0.523083	0.937906	0.721792
0.80	0.418410	0.729111	0.585922	0.447832	0.736423	0.589334	0.450686	0.759240	0.594481
1.00	0.363636	0.581818	0.478788	0.381818	0.589899	0.482772	0.384000	0.608000	0.485333

Table II
Variation of Velocity w for Different Values of M in Equilateral Cross-Section
(P = 1.0; m = 1.0)

M	$W^{(1)}(2,0)$	$W^{(1)}(4,0)$	$W^{(1)}(2,\pm 2/\sqrt{3})$	$W^{(2)}(2,0)$	$W^{(2)}(4,0)$	$W^{(2)}(2,\pm 2/\sqrt{3})$	$W(2,0)$	$W(4,0)$	$W(2,\pm 2/\sqrt{3})$
0.00	0.571429	1.306122	0.989796	0.663265	1.320153	0.996971	0.666667	1.333333	1.000000
0.50	0.500000	1.000000	0.777778	0.555556	1.007716	0.781417	0.559140	1.032258	0.788530
1.00	0.363636	0.581818	0.478788	0.381818	0.589899	0.482772	0.384000	0.608000	0.485333
1.50	0.250000	0.340000	0.295294	0.255294	0.347933	0.299686	0.256209	0.355556	0.298039
2.00	0.173913	0.214047	0.193423	0.175585	0.219737	0.196780	0.175943	0.222621	0.194494

Table III
Variation of w for Different Values of K in Rectangular Cross-Secton
(P= 1.0; m = 1.0)

K	$w(2,0)$	$w(4,0)$	$w(2,\pm 2)$	$w(4,\pm 2)$	$w(2,\pm 4)$	$w(4,\pm 4)$
0.00	2.00000	2.31111	1.60000	1.91111	1.46667	1.57778
0.20	1.90748	2.18271	1.49752	1.80399	1.37391	1.47752
0.40	1.68247	1.87985	1.25891	1.54989	1.15170	1.23858
0.60	1.42914	1.55894	1.00484	1.27156	0.90137	0.97358
0.80	1.22543	1.33337	0.79962	1.04288	0.67955	0.75291
1.00	1.08411	1.23364	0.64486	0.87072	0.49221	0.59813

Table IV
Variation of w for Different Values of M in Rectangular Cross-Secton
(P = 1.0; m = 1.0)

M	$w(2,0)$	$w(4,0)$	$w(2,\pm 2)$	$w(4,\pm 2)$	$w(2,\pm 4)$	$w(4,\pm 4)$
0.00	2.00000	2.31111	1.60000	1.91111	1.46667	1.57778
0.0	1.55329	1.71293	1.12797	1.40741	1.02497	1.10351
1.00	1.08411	1.23364	0.64486	0.87072	0.49221	0.59813
1.50	0.70227	0.95642	0.37463	0.52983	0.21300	0.39114
2.00	0.41673	0.60915	0.23360	0.32701	0.12582	0.26081
2.50	0.27093	0.40711	0.15861	0.21981	0.08552	0.18075
3.00	0.18981	0.28925	0.11417	0.15730	0.06191	0.13121