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From the Editor
This is the last paper submitted to the IRAMP
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RESEARCH NOTE

On the Polarization of Stark or Zeeman Components of Spectral Lines

V. P. GAVRILENKO[*, **]

[*] *Center for Surface and Vacuum Research, 40 Novatorov St., Moscow 119421, Russia*
[**] *National Research University "Moscow Institute of Physics and Technology",*
9 Institutsky per., Dolgoprudny, Moscow Region, 141700, Russia

ABSTRACT: Due to some ambiguity in the literature concerning the polarization of Stark or Zeeman components of spectral lines, we revisit this subject and derived the general expressions for the polarizations of π- and σ-components. We show that a uniquely defined intensity of the σ-component does not seem to exist because it depends on the specific choice out of infinite number of definitions of the σ-polarization. We also demonstrate that Cooper-Ringler's statement in their paper, published in Phys. Rev. **179** (1969) 226, that their Eq. (5) corresponds to the polarization parallel to the π-polarization (the statement repeated later in paper by Hicks, Hess, and Cooper, Phys. Rev. A **5** (1972) 490) is *incorrect*.

1. INTRODUCTION

There is some ambiguity in the literature concerning the polarization of Stark or Zeeman components of spectral lines. In the fundamental Cooper-Ringler's paper [1] (devoted to satellites of He spectral lines) , there are formulas (5) and (6) for the polarization of line components versus the angle of observation θ with respect to the direction of the oscillatory electric field **E**:

for the polarization parallel to **E** (π-components)

$$(1/2)(|x|^2 + |y|^2) \cos^2 \theta + |z|^2 \sin^2 \theta, \tag{1}$$

and for the polarization perpendicular to **E** (σ-components)

$$(1/2) (|x|^2 + |y|^2). \tag{2}$$

On the other hand, for the polarization parallel to **E**, e.g., for the observation along **E** (i.e., $\theta = 0$), the result should be zero because the emitted electromagnetic wave (the light), being a transverse wave, cannot have the polarization parallel to the direction of its wave vector (the direction of observation). However, at $\theta = 0$, the above formula (1) yields a non-zero result: $(1/2)(|x|^2 + |y|^2)$.

Cooper and Ringler [1] refer the derivation of the above formulas to the textbook "Quantum Mechanics" by Powell and Crasemann [2]. The corresponding result, given by Eq. (11-207) of Powell-Crasemann [2], shows the following angular dependence:

for π-components

$$|z|^2 \sin^2 \theta, \tag{3}$$

and for σ-components (for σ_+ and σ_-)

$$(1/4)(|x|^2 + |y|^2) \cos^2 \theta. \tag{4}$$

Equations (3) and (4), reproduced here from Powell-Crasemann's book [2], seem to make sense. However, it is questionable how and whether from these formulas one can obtain Eqs. (1) and (2), reproduced here from Cooper-Ringler's paper [1].

We note that formulas identical to (1) and (2) are also encountered in Hicks-Hess-Cooper' paper [3] as their formulas (38) and (39). The only difference is that the angle of observation θ is counted with respect to the direction of a magnetic field **B**.

Therefore in the present paper we will study this subject from the very beginning and then compare to the results in the literature.

2. DERIVATION OF THE POLARIZATION FORMULAS

We consider the following setup of the problem – see Fig. 1. We choose the z-axis along the direction of the static electric or magnetic field. Let **q** be the wave vector of the emitted photons, i.e., the direction of the observation as along vector **q**. Let this vector be contained in the xy-plane. We denote by $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ the unit vectors along the axis x, y, z, respectively. Let $\mathbf{e}_1$ be the unit vector within the xy-plane, $\mathbf{e}_1$ being perpendicular to **q**. We introduce also the unit vector $\mathbf{e}_2$ coinciding with vector $\mathbf{e}_y$. We denote by θ the angle between the vector **q** and the z-axis. Then the angle between vectors $\mathbf{e}_x$ and $\mathbf{e}_1$ is also equal to θ.

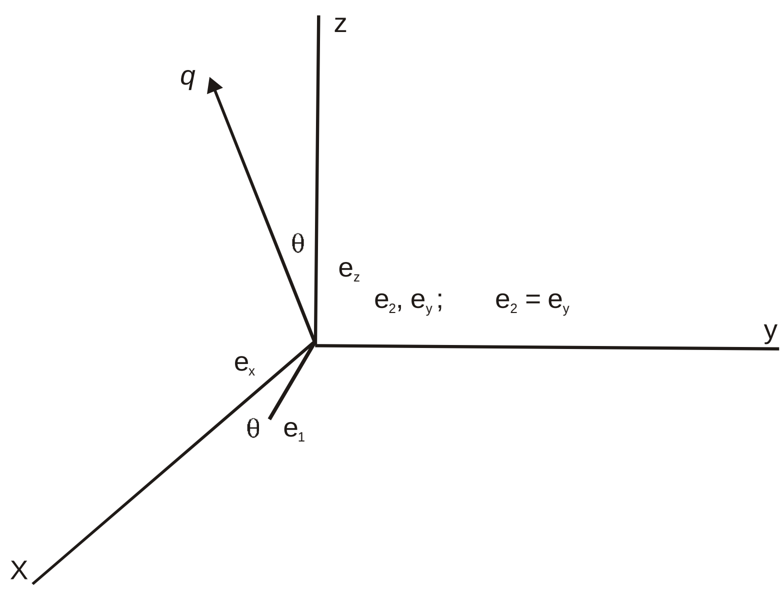

Figure 1: Geometry of the polarization setup. The z-axis is chosen along an electric or magnetic field. The observation is along vector q**. The xz-plane is chosen such as to contain vector** q**.**

To begin with, let us try deriving Eqs. (1) and (2), which reproduce Cooper-Ringler's Eqs. (5) and (6), and see whether or not Eq.(1) corresponds to the polarization parallel to **E** and Eq. (2) – to the polarization perpendicular to **E**. Vector $\mathbf{e}_1$ can be represented in the form

$$\mathbf{e}_1 = \mathbf{e}_x \cos\theta - \mathbf{e}_y \sin\theta. \quad (5)$$

We calculate here the intensity of the radiation, having the wave vector **q** and corresponding to the transition between states j and k, for two linear polarizations of the emitted photons: $\mathbf{e}_1$ and $\mathbf{e}_2$. For the polarization $\mathbf{e}_1$ we get

$$I_{jk}(\mathbf{e}_1) = |< j|\mathbf{r}\mathbf{e}_1|k>|^2 = |< j|x \cos\theta - z \sin\theta|k>|^2, \quad (6)$$

where we took into account Eq. (5). Here and below, for any two vectors **a** and **b**, the notation **ab** stands for the scalar product of **a** and **b**.

It should be emphasized that matrix elements $<j|x|k>$ and $<j|z|k>$ cannot be simultaneously different from zero. Therefore Eq. (6) can be rewritten as follows:

$$I_{jk}(\mathbf{e}_1) = |< j|\mathbf{r}\mathbf{e}_1|\, k>|^2 = |< j|x \cos\theta - z \sin\theta|k>|^2 = |x_{jk}|^2 \cos^2\theta + |z_{jk}|^2 \sin^2\theta. \quad (7)$$

It is easy to see that

$$|x_{jk}|^2 = |y_{jk}|^2. \quad (8)$$

Therefore from Eq. (7) we obtain

$$I_{jk}(\mathbf{e}_1) = (1/2)\,(|x_{jk}|^2 + |y_{jk}|^2) \cos^2\theta + |z_{jk}|^2 \sin^2\theta. \quad (9)$$

For the intensity of the radiation with the polarization $\mathbf{e}_2$, in the analogous way we get:

$$I_{jk}(\mathbf{e}_2) = |<j|\mathbf{r}\mathbf{e}_2|k>|^2 = |< j|y|\, k>|^2 = (1/2)\,(|x_{jk}|^2 + |y_{jk}|^2). \quad (10)$$

Thus, we obtained the same formulas (9) and (10) as given by Eqs. (5) and (6) from Cooper-Ringler's paper [1], respectively. Our derivation shows the following. Cooper-Ringler's statement that their Eq. (6), which is our Eq. (10), corresponds to the polarization perpendicular to **E** is correct. However, Cooper-Ringler's statement that their Eq. (5), which is our Eq. (9), corresponds to the polarization parallel to **E** is *incorrect*.

Now let us derive more general results for the polarization of spectral line components. For definiteness we consider now the polarization of Zeeman components: the z-axis is along the magnetic field **B**. The radiative transition is between the states $j = (n'', l'', m'')$ and $k = (n', l', m')$. The intensity of the spontaneous emission, polarization of which is determined by some unit vector **e**, is given as follows

$$I(e) = \left(\sum_{m',m''} |< j\,|\,\mathbf{re}\,|\,k >|^2 \right) / (2l''+1). \quad (11)$$

Equation (11) is valid under the assumptions that the Zeeman sublevels of the state j are equally populated.

The intensity of the π-component (**e**||z), according to Eq. (11), is then the following:

$$I_m = \left(\sum_{m',m''} |< j\,|\,z\,|\,k >|^2 \right) / (2l''+1). \quad (12)$$

As for the intensity of the σ-component, let us define it as the intensity of the radiation polarized perpendicular to the magnetic field, i.e., perpendicular to the z-axis. The unit vector of the polarization perpendicular to the magnetic field can be represented in the form

$$\mathbf{e}_{perp} = a_x \mathbf{e}_x + a_y \mathbf{e}_y, \quad (13)$$

where a_x and a_y are, generally speaking, complex numbers satisfying the normalization condition: $|a_x|^2 + |a_y|^2 = 1$. Substituting Eq. (13) in Eq. (11), we get

$$I_\sigma = \left(\sum_{m',m''} |< j\,|\,a_x x + a_y y\,|\,k >|^2 \right) / (2l''+1). \quad (14)$$

It is seen that the intensity of the σ-component generally depends on the specific polarization registered in a particular experiment, i.e., depends on the coefficients a_x and a_y.

Equation (12) determines the intensity of the π-component only if the direction of the observation is along x- or y-axis. More generally, if the direction of the observation is along vector **q**, one should take into account only the component of the electric field of the emitted wave perpendicular to **q**. Therefore, in the general case instead of Eq. (12) we get:

$$I_\pi = \left(\sum_{m',m''} |< j \,|\, z \sin\theta \,|\, k >|^2 \right) /(2l''+1) = \sin^2\theta \sum_{m',m''} |< j \,|\, z \,|\, k >|^2) /(2l''+1). \quad (15)$$

As for Eq. (14), it determines the intensity of the σ-components with the polarization defined by vector $\mathbf{e}_{perp}$ from Eq. (13) only if the direction of the observation is along the z-axis. If the direction of the observation would be along the x-axis, then one could register only the part of the emission related to the projection of the electric field of the wave on the y-axis. In this case instead of Eq. (14), one would get:

$$I_\sigma = \left(\sum_{m',m''} |< j \,|\, a_y y \,|\, k >|^2 \right) /(2l''+1) = |a_y|^2 \left(\sum_{m',m''} |< j \,|\, a_y y \,|\, k >|^2 \right) /(2l''+1). \quad (16)$$

In the general case of the observation along vector **q**, one should modify Eq. (14) for the intensity of the σ-component as follows:

$$I_{\sigma,q} = \left(\sum_{m',m''} |< j \,|\, a_x x \cos\theta + a_y y \,|\, k >|^2 \right) /(2l''+1). \quad (17)$$

Thus, a uniquely defined intensity of the σ-component does not seem to exist. Rather, it depends on the specific choice of the polarization from the general formula (13). For example, if we would choose $\mathbf{e}_{perp} = \mathbf{e}_x$, we would obtain one result; if we would choose $\mathbf{e}_{perp} = \mathbf{e}_y$, we would obtain another result; if we would choose $\mathbf{e}_{perp} = (\mathbf{e}_x + i\mathbf{e}_y)/2^{1/2}$, we would obtain yet another result. The invariant quantity, i.e., the quantity independent on the specific choice of the polarization, is the sum of the intensities of the two σ-components characterized by two mutually orthogonal polarizations $\mathbf{e}_{perp}$ and $\mathbf{e'}_{perp}$:

$$I_\sigma(\mathbf{e}_{perp}) + I_\sigma(\mathbf{e'}_{perp}) = \text{const}. \quad (18)$$

The above reasoning relates also to circular polarizations. For example, if we would rewrite Eq. (17) for one specific choice of the circular polarization $\mathbf{e}_{circ} = (\mathbf{e}_x + i\mathbf{e}_y)/2^{1/2}$, then for the observation along vector **q** we would *not* get the sum of intensities of the two mutually orthogonal polarizations $\mathbf{e}_x$ and $\mathbf{e}_y$: the result would be essentially (not just by a factor) different from $I_{\acute{o},\mathbf{q}}(\mathbf{e}_x) + I_{\acute{o},\mathbf{q}}(\mathbf{e}_y)$. However, if we would consider also another circular polarization $e'_{circ} = (\mathbf{e}_x - i\mathbf{e}_y)/2^{1/2}$, which is orthogonal to $\mathbf{e}_{circ} = (\mathbf{e}_x + i\mathbf{e}_y)/2^{1/2}$, then we would get:

$$I_{\sigma,\mathbf{q}}(\mathbf{e}_{perp}) + I_{\sigma,\mathbf{q}}(\mathbf{e'}_{perp}) = I_{\sigma,\mathbf{q}}(\mathbf{e}_x) + I_{\sigma,\mathbf{q}}(\mathbf{e}_y) \quad (19)$$

in conformance to Eq. (18).

3. CONCLUSIONS

Being motivated by some ambiguity in the literature concerning the polarization of Stark or Zeeman components of spectral lines, we revisited this subject and derived the general expressions for the polarizations of π- and σ-components. We demonstrated that a uniquely defined intensity of the σ-component does not seem to exist because it depends on the specific choice out of infinite number of definitions of the σ-polarization.

We also showed that Cooper-Ringler's statement in paper [1] that their Eq. (5) corresponds to the polarization parallel to the π-polarization (the statement repeated later in paper [3]) is *incorrect*.

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