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SQUARE DIFFERENCE PRIME LABELING OF WHEEL GRAPH, FAN GRAPH, FRIENDSHIP GRAPH, GEAR GRAPH, HELM GRAPH & UMBRELLA GRAPH

Sunoj B.S.[*] and Mathew Varkey T.K.[**]

Abstract

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The graph for which every edge (uv), the labels assigned to u and v are whole numbers and for each vertex of degree at least 2, the g c d of the labels of the incident edges is 1. Here we characterize wheel graph, fan graph, helm graph, gear graph, friendship graph, umbrella graph for square difference prime labeling.

Keywords: *Graph labeling, prime labeling prime graphs, wheel graph.*

1. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p, q)-graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1], [2], [3] and [4]. Some basic concepts are taken from Frank Harary [1]. In this paper we investigated square difference prime labeling of gear graph, wheel graph, fan graph, helm graph, friendship graph and umbrella graph.

2. MAIN RESULTS

Definition 2.1: Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. Define a bijection $f : V(G) \rightarrow \{0, 1, 2, ..., p - 1\}$ by $f(v_i) = i - 1$, for every i from 1 to p

* Department of Mathematics, Government Polytechnic College, Attingal, India. *Email: spalazhi@yahoo.com*

** Department of Mathematics, T.K.M. College of Engineering, Kollam, India. *Email: mathewvarkeytk@gmail.com*

and define a 1 – 1 mapping f^*_{sdp} : E(G) → set of natural numbers N by $f^*_{sdp}(uv) = |f(u)^2 - f(v)^2|$. The induced function f^*_{sdp} is said to be a square difference prime labeling, if for each vertex of degree at least 2, the *g c d* of the labels of the incident edges is 1.

Definition 2.2: A graph which admits square difference prime labeling is called a square difference prime graph.

Definition 2.3: A wheel graph is defined as $W_n = C_n + K_1$, where n is the number of vertices in the cycle.

Definition 2.4: Helm graphs are extension of wheel graphs. To form a helm graph, denoted here as H_n, we take the wheel graph W_n and append a pendant edge to each vertex of the n–cycle.

Definition 2.5: Closed helms are denoted here as CH_n, and they are formed by taking the helm graph, H_n, and placing an edge between each pendant vertex to form a second cycle in the graph.

Definition 2.6: The friendship graph T_n can be constructed by joining n copies of the cycle C_3 with a common vertex.

Definition 2.7: The graph obtained by joining a path P_n to the Fan F_m is denoted by U(m, n) and is called umbrella graph.

Definition 2.8: Gear graph is a Wheel Graph with a vertex added between each pair of adjacent vertices.

Definition 2.9: A fan graph F_n is obtained by joining all vertices of a path p_n to a further vertex called the centre.

Theorem 2.1: Wheel graph W_n admits square difference prime labeling when n is a positive even integer greater than 4.

Proof: Let G = W_n and let $v_1, v_2, ..., v_{n+1}$ are the vertices of G.

Here | V(G) | = n + 1 and | E(G) | = 2n.

Define a function f : V → {0, 1, 2, ..., n} by

$$f(v_i) = i - 1,\ i = 1, 2, ..., n + 1$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows

$$f^*_{sdp}(v_i, v_{i+1}) = 2i - 1, \qquad i = 1, 2, ..., n.$$

$$f^*_{sdp}(v_1, v_n) = (n - 1)^2.$$

$$f^*_{sdp}(v_i, v_{n+1}) = (n + i - 1)(n - i + 1), \qquad i = 1, 2, ..., n - 1.$$

According to this pattern, W_n admits square difference prime labeling.

Theorem 2.2: Fan graph F_n admits square difference prime labeling.

Proof: Let $G = F_n$ and let $v_1, v_2, ..., v_{n+1}$ are the vertices of G.

Here $|V(G)| = n + 1$ and $|E(G)| = 2n - 1$.

Define a function $f : V \to \{0, 1, 2, ..., n\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, ..., n + 1.$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(v_i v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., n$$

$$f^*_{sdp}(v_i v_{n+1}) = (n + i - 1)(n - i + 1), \quad i = 1, 2, ..., n - 1$$

According to this pattern F_n, admits square difference prime labeling.

Theorem 2.3: The helm graph H_n admits square difference prime labeling when n is a positive even integer greater than 4.

Proof: Let $G = H_n$ and let $v_1, v_2, ..., v_{2n+1}$ are the vertices of G.

Here $|V(G)| = 2n + 1$ and $|E(G)| = 3n$.

Define a function $f : V \to \{0, 1, 2, ..., 2n\}$ by

$$f(v_i) = i - 1, \quad i = 1, 2, ..., 2n + 1$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(v_i v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., n - 1$$

$$f^*_{sdp}(v_i v_{n+1}) = (n + i - 1)(n - i + 1), \quad i = 1, 2, ..., n$$

$$f^*_{sdp}(v_i v_{n+i+1}) = (n + 2i - 1)(n + 1), \quad i = 1, 2, ..., n$$

$$f^*_{sdp}(v_1 v_n) = (n - 1)^2.$$

According to this pattern H_n, admits square difference prime labeling.

Theorem 2.4: The closed helm graph CH_n admits square difference prime labeling.

Proof: Let $G = CH_n$ and let $v_1, v_2, ..., v_{2n+1}$ are the vertices of G.

Here $|V(G)| = 2n + 1$ and $|E(G)| = 4n$.

Define a function $f : V \to \{0, 1, 2, ..., 2n\}$ by $f(v_i) = i - 1, \quad i = 1, 2, ..., 2n + 1$.

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(v_i v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., 2n$$

$$f^*_{sdp}(v_{n+i}v_{2n+1}) = (3n + i - 1)(n - i + 1), \quad i = 1, 2, ..., n - 1$$

$$f^*_{sdp}(v_i v_{2n-i+1}) = (2n - 1)(2n - 2i + 1), \quad i = 1, 2, ..., n - 1$$

$$f^*_{sdp}(v_1 v_n) = (n - 1)^2$$

$$f^*_{sdp}(v_{n+1}v_{2n}) = (3n - 1)(n - 1)$$

According to this pattern CH_n, admits square difference prime labeling.

Theorem 2.5: The Friendship graph admits square difference prime labeling.

Proof: Let $G = T_n$ and let $v_1, v_2, ..., v_{2n+1}$ are the vertices of G.

Here $|V(G)| = 2n + 1$ and $|E(G)| = 3n$

Define a function $f: V \to \{0, 1, 2, ..., 2n\}$ by

$$f(u_0) = 0$$

$$f(v_i) = i,\ i = 1, 2, ..., 2n$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(u_0 v_{2i-1}) = (2i - 1)^2, \quad i = 1, 2, ..., n$$

$$f^*_{sdp}(u_0 v_{2i}) = (2\mathrm{i})^2, \quad i = 1, 2, ..., n$$

$$f^*_{sdp}(v_{2i-1}v_{2i}) = 4i - 1, \quad i = 1, 2, ..., n$$

According to this pattern Fn admits, square difference prime labeling.

Theorem 2.6: The umbrella graph $U(n, n)$, admits square difference prime labeling, when $n > 3$.

Proof: Let $G = U(n, n)$ and let $v_1, v_2, ..., v_{2n}$ are the vertices of G.

Here $|V(G)| = 2n$ and $|E(G)| = 3n - 2$

Define a function $f: V \to \{0, 1, 2, ..., 2n - 1\}$ by

$$f(v_i) = i - 1,\ i = 1, 2, ..., 2n$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(v_i v_{i-1}) = 2i - 1, \quad i = 1, 2, ..., 2n - 1$$

$$f^*_{sdp}(v_{n+1}v_i) = (n + i - 1)(n - i + 1), \quad i = 1, 2, ..., n - 1$$

According to this pattern $U(n, n)$, admits square difference prime labeling.

Theorem 2.7: Gear graph G_n admits square difference prime labeling.

Proof: Let $G = G_n$ and let $v_1, v_2, ..., v_{2n+1}$ are the vertices of G.

Here $|V(G)| = 2n + 1$ and $|E(G)| = 3n$

Define a function $f : V \to \{0, 1, 2, ..., 2n\}$ by

$$f(v_i) = i - 1,\ i = 1, 2, ..., 2n + 1$$

For the vertex labeling f, the induced edge labeling f^*_{sdp} is defined as follows:

$$f^*_{sdp}(v_i v_{i+1}) = 2i - 1, \quad i = 1, 2, ..., 2n - 1$$

$$f^*_{sdp}(v_{2n+1} v_{2i}) = (2n + 2i - 1)(2n - 2i + 1), \quad i = 1, 2, ..., n$$

$$f^*_{sdp}(v_1 v_{2n}) = (2n - 1)^2$$

According to this pattern G_n admits, square difference prime labeling.

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