KINEMATICS OF 2 × 3*R* ROBOTICS MODELLING: CLIMBING THE STAIRS FORM

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ABSTRACT: *A mechanical manipulator can be modelled as an open-loop articulated chain with several rigid bodies (links) connected in series by either revolute or prismatic joints driven by actuators. The action of climbing the stairs can be taken as a model of a robot to be defined. Robotics modelling has been given in the study. The related robotic modelling consisting of triple revolute joints are defined and motion matrices of model and its parameters have been achieved. We demonstrate the robot kinematics equations of 2 × 3R for man's climbing the stairs. The modelling of motion varies in every consecutive phases in terms of redescribing the fixed frame.*

Keywords: *Climbing the stairs motion, transforming frame, kinematics, link, joint, pole point.*

2000 AMS subject classification number: *70B10.*

1. INTRODUCTION

The position of one link relative to another in a kinematic chain is defined mathematically by a coordinate transformation between reference frames attached to each body. The link is rigid so this transformation must preserve the distances measured between points, and it is called a rigid transformation. Rigid transformations will consist simply of rotations and translations [7]. Rotations and translations in $\mathbb{R}^n$ are rigid transformations and used commonly in kinematic studies [2].

The position of one body relative to another, we attach coordinate frames to each. One is chosen as the ground with coordinate frame F, and the other, the moving body, has the coordinate frame M. We use the coordinate transformation

$$D : F \to M$$

which transforms coordinates measured in M to those measured in F, to represent the position of M relative to F. This transformation is given by

$$X = [A]\, x + d$$

where x is the coordinate vector of a point in M and X is coordinate vector of the same point but measured in F. If the moving body is of dimension n (usually $n = 2$ or 3), then

$[A]$ is an $n \times n$ matrix and d is an n-dimensional vector. This transformation's matrix form is [7].

$$D = \begin{bmatrix} A & d \\ 0 & 1 \end{bmatrix}$$

For a general planar displacement there is a point that does not move, which means that its coordinates are the same in both reference frames F and M. This point is called the pole. Let $D = (A, d)$ be the placement, then its pole p satisfies the equation, $Dp = p$, or $p = [A]\mathrm{p} + d$. Solving for p we obtain

$$p = -[A - I]^{-1} d,$$

or

$$p_1 = \frac{\left(\left(\frac{d_1}{2}\right)\sin\left(\frac{\theta}{2}\right) - \left(\frac{d_2}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)}{\sin\left(\frac{\theta}{2}\right)},$$

$$p_2 = \frac{\left(\left(\frac{d_1}{2}\right)\cos\left(\frac{\theta}{2}\right) + \left(\frac{d_2}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)}{\sin\left(\frac{\theta}{2}\right)}.$$

For some related papers or books on the topics, one can also refer to the works [4], [8], [9], [10] and the references mentioned therein.

As is known, in a robotics modelling, first it should be determined how joints will be settled and what kind of motion they will carry out. There are two types of joints used . These are revolute joints that can make rotation motion and prismatic joints that can make translation motion. By using these two joints a model-robot that can reach any point in three dimensional space can be structured. In each model the number and the term of revolute joints and prismatic joints are modelled according to the task the model will perform and the model is named according to the use of these revolute joints and prismatic joints. For example; cartesian co-ordinate robot (PPP), cylindrical coordinate robot (RPP), spherical robot (RRP), articulated robot (RRR) [12].

To describe the translational and rotational relations between adjacent links, Denavit and Hartenderg proposed a matrix method of systematically establishing a coordinate system (body-attached frame) to each link of an articulated chain. The D-H representation

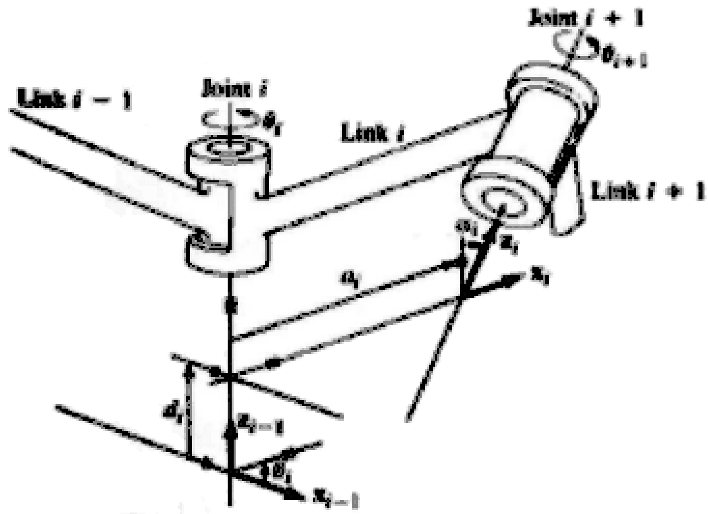


Figure 1

of a rigid link depends on four geometric parameters associated with each link. These four parameters completely describe any revolute or prismatic joint. Referring to Figure 1, these four parameters are defined as follows:

θ_i is the joint angle from the x_{i-1} axis to the x_i axis about the z_{i-1} axis (using the right-hand rule).

d_i is the distance from the origin of the i–1th coordinate frame to the intersection of the z_{i-1} axis with the x_i axis along the z_{i-1} axis.

a_i is the offset distance from the intersection of the z_{i-1} axis with the x_i axis to the origin of the ith frame along the x_i axis.

α_i is the offset angle from the z_{i-1} axis to the z_i axis about the x_i axis [3].

As was seen in studying the skeleton and its articulations, the body is composed of segments linked together at their articulations. Thus, the body is basically a system consisting of movable segments or links. The skeleton may be divided into minute divisions for precise description in which each bone, no matter how small, is considered a separate unit; or it may be divided into the largest possible units for simplicity. The more numerous the identified links, the more precise the description of what the body is doing; however, the description may become too intricate for practical purposes. The fewer the links, the simpler the description. If too few are included, however, identification of many important movements is lost to the observer. Thus, a compromise is in order. For our purposes, the body is divided into eleven functional segments, or links: head and cervical vertebrae, thorax and thoracic vertebrae, lumbar vertebrae, pelvis and sacrum, thigh, leg, foot, shoulder girdle, arm, forearm, and hand. Technically, some of these segments comprise more than one link [5].

In the present study, a robotics modelling similar to the triples foot-leg-thigh in human anatomy has been studied. It has been named $2 \times 3R$ robotics modelling because our system is two sided and joint motions are revolute. The phases of the motion have been modelled as 1-passing from walking to climbing the stairs, 2-first step climbing the stairs, 3-second step climbing and the repetition of the second and the third, *n*-passing from climbing the stairs to walking. Each phase has a structure which consists of rotations around a chosen pole. Thus, the chosen pole points are the points where the fixed frame of the motion is put in each phase. Determined frames at the beginning of the system keep indic made throughout the motion.

It should be noted that some papers, which are similar to our work, were performed in the literature on PRR manipulators, RRP, RPR and PRR serial chains and $3R$ manipulators can be found in [6], [11], [1].

2. MODELLING OF 2 × 3*R*

At the starting stage, the model parts are below (Figure 2).

Let's take 2-models, which are represented by Robot L and Robot R, with 3 revolute joint as in Figure 2. Let's coincide these two robots by joints J_{03}, $\overline{J_{03}}$. Let's reorder joints names of Robot R as $\overline{J_{02}} = J_{04}$, $\overline{J_{01}} = J_{05}$ and $\overline{J_{00}} = J_{06}$. New model which consists of two parts is:

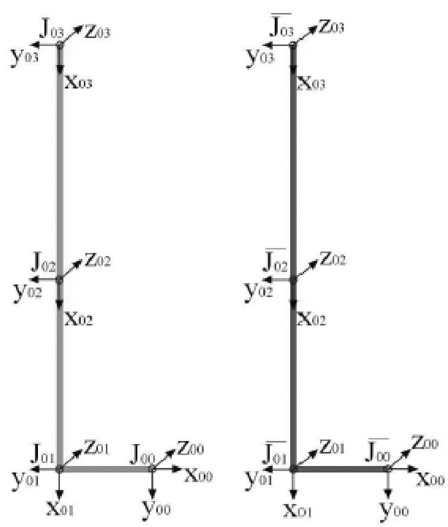


Figure 2: First Position

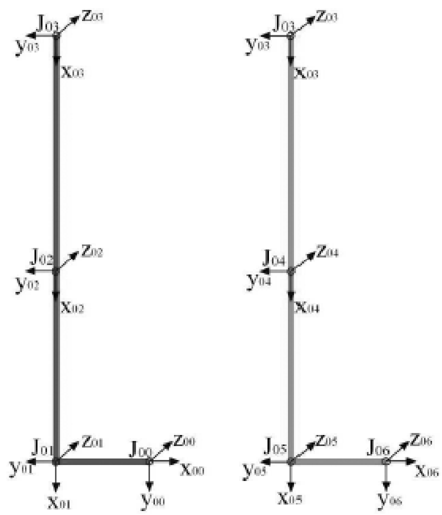


Figure 3: 2 × 3*R* Frame Attachment

Part $L = \{J_{00}, J_{01}, J_{02}, J_{03}\}$ and Part $R = \{J_{06}, J_{05}, J_{04}, J_{03}\}$.We will name 2 × 3*R* robotics modelling as a model which consists of Part *L* and Part *R* . Thus, the Figure for 2 × 3*R* can be taken as in Figure 3.

When *L* (left leg) and *R* (right leg) has been put upper and upper, frame and joints matchings for the start stage is in Figure 4.

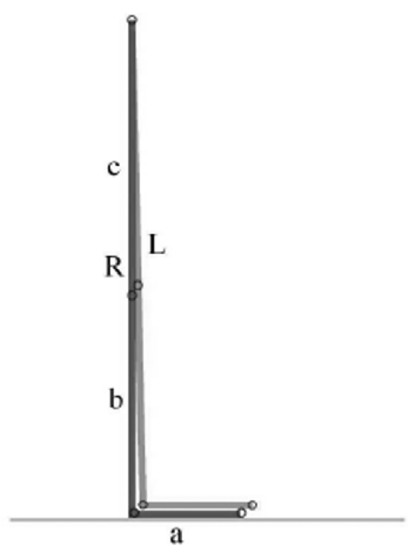


Figure 4: Right and Left Parts

3. MATRICES OF THE MOTION

In order to be able to write the matrices of the motion, initially, let's construct a mechanical system and replace a frame to every joint:

The frame at J_{00} is F_{00} (fixed frame),

the frame at J_{01} is M_{01} (moving frame),

the frame at J_{02} is M_{02} (moving frame),

the frame at J_{03} is M_{03} (moving frame),

the frame at J_{04} is M_{04}(moving frame),

the frame at J_{05} is M_{05} (moving frame),

the frame at J_{06} is M_{06} (moving frame). This mechanism is like Figure 3.

The diagram of the passing from walking to climbing has been given at Figure 5.

Let's show the motion with WS belonging to the first step from walking to climbing the stair.

Matrix of the motion which takes ${}_1P(0, 0)$ as pole point:

$$\left[{}_{1p}WS\right]=\begin{bmatrix} C_{13}\gamma & -S_{13}\gamma & 0 & 0 \\ S_{13}\gamma & C_{13}\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Similarly, matrix of the motion which takes ${}_2P(cC_{14}\gamma, cC_{14}\gamma)$ as pole point:

$$\left[{}_{2p}WS\right]=\begin{bmatrix} C_{14}\gamma & S_{14}\gamma & 0 & cC_{12}\gamma - cC\left({}_{13}\gamma - {}_{14}\gamma\right) \\ -S_{14}\gamma & C_{14}\gamma & 0 & cS_{12}\gamma - cS\left({}_{13}\gamma - {}_{14}\gamma\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where, we used C as cos and S as sin, for the sake of brevity.

Finally, the matrix of all the motion is:

$$[WS] = [{}_{2p}WS][{}_{1p}WS].$$

At the $2 \times 3R$ modelling, motion ordering of the parts of the mechanism is important. The indic can not give it a priority according to the time parameter. A suitable ordering

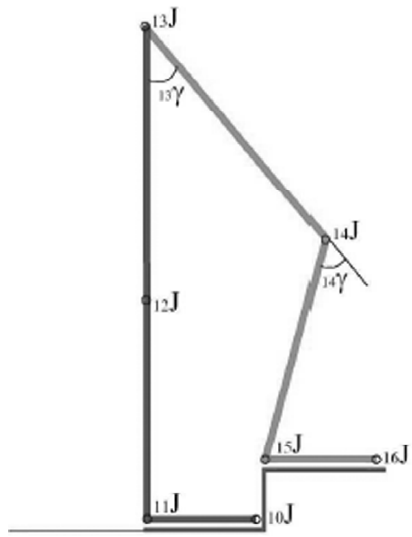


Figure 5: Motion of WS

of indic of the links, together with motion which contains t for the time parameter t has been given.

Let $I = \{1, 2, 3, 4, ..., n\}$ and $J = \{1, 2\}$. Let's give a relation on

$$I \times J = \{(1, 1), (1, 2), (2, 1), (2, 2), ..., (n, 1), (n, 2)\}$$

For all $(i, j), (b, k) \in I \in J$; let

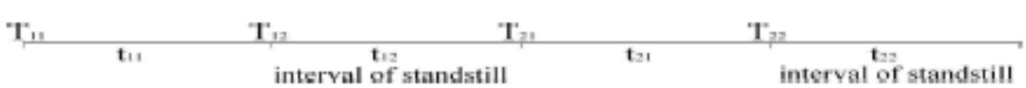
$$\left.\begin{array}{l} i < b \\ i = b \Rightarrow j \leq k \end{array}\right\} \Leftrightarrow (i,\, j) \leq (b, k).$$ This is an order relation. Then, the order is:

$$(1, 1)\ (1, 2)\ (2, 1)\ (2, 2), ..., (n, 1)\ (n, 2)$$

and in addition to this relation, T_{ij} are interval end points. As it is $T_{11} = 0$, and if $[0, t]$ interval is a

and t_{ij} has shown the parameters at this interval, it is $t_{ij} \in [T_{ij}, T_{ij+1}]$.

And so the axes is



Then, time distribution which belongs to *WS* could be written as below:

$$_{13}\gamma(t_{11}),$$

$$_{14}\gamma(t_{11}).$$

The diagram of the first climbing the stair step has been given at Figure 6. Let's show the motion with S_1.

Then, the matrices of the motion which take p_1, p_2, p_3, p_4 and p_5 as pole points are as below, respectively:

$$\left[S_{1p_1}\right] = \begin{bmatrix} C\varphi_{25} & S\varphi_{25} & 0 & d_1 \\ -S\varphi_{25} & C\varphi_{25} & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[S_{1p_2}\right] = \begin{bmatrix} C\varphi_{25} & -S\varphi_{25} & 0 & d_1' \\ S\varphi_{25} & C\varphi_{25} & 0 & d_2' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

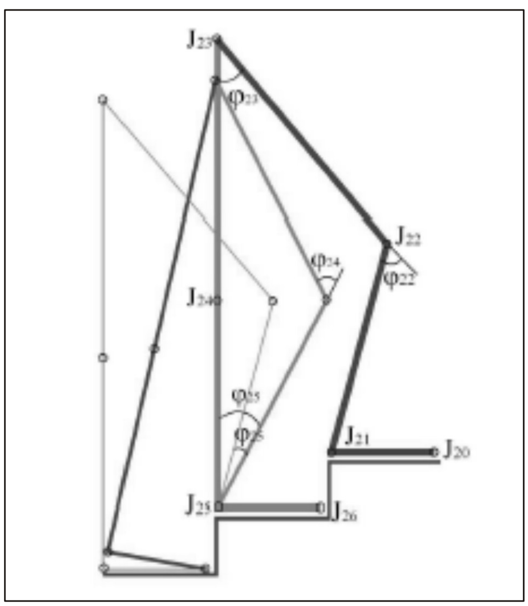


Figure 6: Motion of S_1

$$\left[S_{1_{p_3}}\right]=\begin{bmatrix} C\varphi_{24} & -S\varphi_{24} & 0 & d_1'' \\ S\varphi_{24} & C\varphi_{24} & 0 & d_2'' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[S_{1_{p_4}}\right]=\begin{bmatrix} C\varphi_{23} & -S\varphi_{23} & 0 & d_1''' \\ S\varphi_{23} & C\varphi_{23} & 0 & d_2''' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[S_{1_{p_5}}\right]=\begin{bmatrix} C\varphi_{22} & S\varphi_{22} & 0 & d_1'''' \\ -S\varphi_{22} & C\varphi_{22} & 0 & d_2'''' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix of S_1 is:

$$\left[S_1\right]=\left[S_{1_{p5}}\right]\left[S_{1_{p4}}\right]\left[S_{1_{p3}}\right]\left[S_{1_{p2}}\right]\left[S_{1_{p1}}\right].$$

Time distribution of $S1$ could be written as below:

$$\varphi_{25}(t_{21})$$
$$\varphi_{25}(t_{22})$$
$$\varphi_{24}(t_{22})$$
$$\varphi_{23}(t_{23})$$
$$\varphi_{22}(t_{23})$$

The diagram of second climbing step to the stairs has been given at Figure 7. Let's show the motion with $_2S$.

Then, the matrices of the motion which take $_1p$, $_2p$, $_3p$, $_4p$, $_5p$ as pole points are as below:

$$\left[{}_{_{1p}2}S\right]=\begin{bmatrix} C_{31}\varphi & S_{31}\varphi & 0 & d_1 \\ -S_{31}\varphi & C_{31}\varphi & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}_{2p}{}^{2}S\right]=\begin{bmatrix} C_{31}\varphi & -S_{31}\varphi & 0 & d_1' \\ S_{31}\varphi & C_{31}\varphi & 0 & d_2' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}_{3p}{}^{2}S\right]=\begin{bmatrix} C_{32}\varphi & S_{32}\varphi & 0 & d_1'' \\ -S_{32}\varphi & C_{32}\varphi & 0 & d_2'' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}_{4p}{}^{2}S\right]=\begin{bmatrix} C_{33}\varphi & -S_{33}\varphi & 0 & d_1''' \\ S_{33}\varphi & C_{33}\varphi & 0 & d_2''' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}_{5p}{}^{2}S\right]=\begin{bmatrix} C_{34}\varphi & S_{34}\varphi & 0 & d_1'''' \\ -S_{34}\varphi & C_{34}\varphi & 0 & d_2'''' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix of ${}_2S$ is:

$$\left[{}_2S\right]=\left[{}_{5p}{}^{2}S\right]\left[{}_{4p}{}^{2}S\right]\left[{}_{3p}{}^{2}S\right]\left[{}_{2p}{}^{2}S\right]\left[{}_{1p}{}^{2}S\right].$$

Time distribution of 2S could be written as below:

$$_{31}\varphi(t_{31})$$
$$_{31}\varphi(t_{32})$$
$$_{32}\varphi(t_{32})$$
$$_{33}\varphi(t_{33})$$
$$_{34}\varphi(t_{33}).$$

The diagram of passing from climbing the stairs to walking has been given at Figure 8. Let’s show the motion with SW.

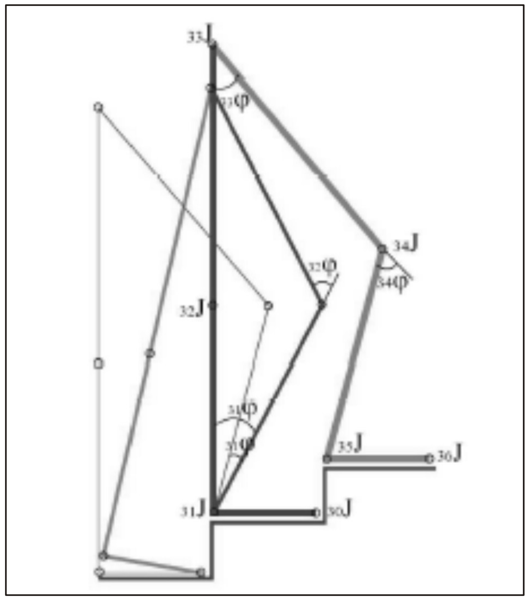


Figure 7: Motion of $_2S$

Then, the matrices of the motion which take p_1, p_2, p_3, p_4, p_5 as pole points are as below, respectively:

$$\left[SW_{p_1}\right]=\begin{bmatrix} C\beta_{45} & S\beta_{45} & 0 & d_1 \\ -S\beta_{45} & C\beta_{45} & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[SW_{p_2}\right]=\begin{bmatrix} C\beta_{45} & -S\beta_{45} & 0 & d_1' \\ S\beta_{45} & C\beta_{45} & 0 & d_2' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[SW_{p_3}\right]=\begin{bmatrix} C\beta_{44} & S\beta_{44} & 0 & d_1'' \\ -S\beta_{44} & C\beta_{44} & 0 & d_2'' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

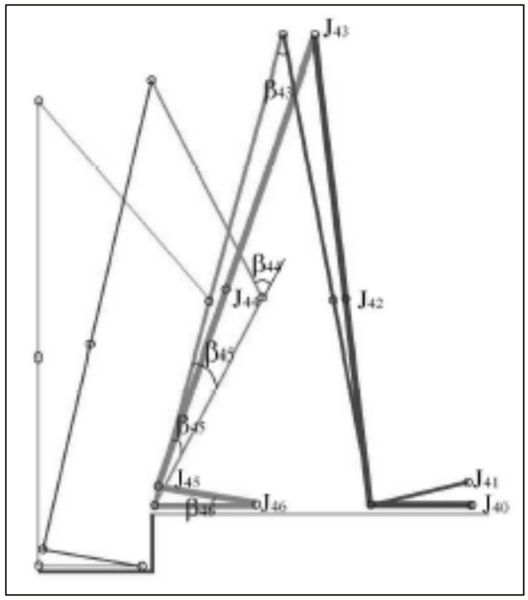


Figure 8: Motion of *SW*

$$\left[SW_{p_4}\right]=\begin{bmatrix} C\beta_{43} & -S\beta_{43} & 0 & d_1''' \\ S\beta_{43} & C\beta_{43} & 0 & d_2''' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[SW_{p_5}\right]=\begin{bmatrix} C\beta_{46} & S\beta_{46} & 0 & d_1'''' \\ -S\beta_{46} & C\beta_{46} & 0 & d_2'''' \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix of *SW* is:

$$\left[SW\right]=\left[SW_{p_5}\right]\left[SW_{p_4}\right]\left[SW_{p_3}\right]\left[SW_{p_2}\right]\left[SW_{p_1}\right].$$

Time distribution of *SW* could be written as below:

$$\beta_{45}(t_{41})$$
$$\beta_{45}(t_{42})$$
$$\beta_{44}(t_{42})$$
$$\beta_{43}(t_{43})$$
$$\beta_{46}(t_{44})$$

4. CONCLUSION

In this study, matrices of the the climbing the stairs motion of the model, which is defined as $2 \times 3R$ robotics modelling, have been obtained. The chain of motions belonging to the model is important because the fixed frame's placement could be re-fixed in all stages. The motion is independent from the order of the indic in many stages. Because of this, the order relation, which is well-matched with the given indic, was defined on the time line. Consequently, the matrices of the motion of WS, S_1, ${}_2S$ and SW are obtained.

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