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A New Method for Order Reduction of High order Multi variable Systems using Modified Routh Approximation Technique

G.V.K.R. Sastry*, G. Surya Kalyan and K. Tejeswar Rao*****

ABSTRACT

A novel Procedure is presented for modeling of higher order MIMO systems based on matching the time responses of the original and reduced order systems. Theflexibility of method is shown through familiar example.

Key words: MIMO Systems, Order reduction, large scale systems

1. INTRODUCTION

Familiar methods are available for the Modelling of SISOcontinuous time systemswhereas thereduction methods for MIMO systems are seldom available. Some of the familiar methods available in the literature for the reduction of high order MIMO systems are viz., Multivariable Systems Reduction using Model Method and Pade Type Approximation, Matrix Continued Fraction Method, Multivariable Systems Reductions Using Polynomial Derivatives, Pade-Routh Approximation Method for Multivariable Systems, Simplified Model-Pade Approximation Method Model-Continued Fraction Method and Impulse Response Gramian and Markov Parameters Method [1-5, 14].This paper presents a computationally simplemodel reduction method for high order MIMO systems. The proposed method is based on the Modified Routh Approximation Technique. The proposed method always generates stable low-order model for stable high order system.

2. PROPOSED REDUCTION METHOD

Consider an n^{th} order linear dynamic MIMO System, described by the form:

$$G_{j,n}(s) = \frac{\begin{bmatrix} g_{11(s)} & \cdots & g_{1n-1(s)} \\ \vdots & \vdots & \vdots \\ g_{j1(s)} & \cdots & g_{jn-1(s)} \end{bmatrix}}{D_n(s)} \tag{1}$$

Where g_{j1}, g_{j2}…… are the outputs of the MIMO system. and the $D(s)$ polynomial is the input of the MIMO system.

Therefore now the original system can be written as:

* EEE Dept, GIT, GITAM University Visakhapatnam, India, *Email: profsastrygvkr@yahoo.com*

** EEE Dept, Chaitanya Engg. College, JNT Univ, India, *Email: suryakalyangarimella@yahoo.com*

*** EEE Dept, Avanthi Engg College, Visakhapatnam, India, *Email: tejeshk222@gmail.com*

$$G_{j,n}(s) = \frac{\begin{bmatrix} g_1(s) \\ \vdots \\ g_j(s) \end{bmatrix}}{D_n(s)}$$

Where

$$g_1(s) = g_{11}(s) + g_{12}(s) + \ldots\ldots + g_{1n}(s)$$
$$g_2(s) = g_{21}(s) + g_{22}(s) + \ldots\ldots + g_{2n}(s)$$
$$\ldots\ldots$$
$$\ldots\ldots$$
$$g_n(s) = g_{n1}(s) + g_{n2}(s) + \ldots\ldots + g_{nn}(s)$$

The transfer function for the 1st output is:

$$G_1(s) = \frac{g_1(s)}{D(s)}$$

Similarly for the nth output it is:

$$G_n(s) = \frac{g_n(s)}{D(s)}$$

For each output, the corresponding nth order linear dynamic SISO system, can be described by the transfer function as

$$G(s) = \frac{N(S)}{D(S)} = \frac{d_0 + d_1 s + d_2 s^2 + \cdots\ldots\ldots + d_{n-1}s^{n-1}}{e_0 + e_1 s + e_2 s^2 + \cdots\ldots\ldots + e_n s^n}$$

Where e_i; $0 \le i \le n$-1 and d_i; $0 \le i \le n$ are scalar constants.

Then the corresponding k^{th} ($k < n$) order reduced model is defined as

$$G_k(s) = \frac{N_k(S)}{D_k(S)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots\ldots\ldots + a_{k-1}s^{k-1}}{E_0 + E_1 s + E_2 s^2 + \cdots\ldots\ldots + E_k s^k}$$

Where a_i; $0 \le i \le k$-1 and E_i; $0 \le i \le n$ are scalar constants.

The reduced form of the transfer functions of SISO systems are to be combined to get the reduced order model of the considered MIMO system as:

$$R_{j,k}(s) = \frac{\begin{bmatrix} N_{1,k}(s) \\ \vdots \\ N_{j,k}(s) \end{bmatrix}}{D_k(s)}$$

The coefficients of $D_k(s)$ and N_j, $k(s)$ are determined as follows .

2.1. Reduced Denominator $D_k(s)$

The Routhtable is obtainedusing the denominator $D(s)$ of eqn. (3) as given below:

$e_n e_{n-2} \ldots\ldots e_0$
$e_{n-1} e_{n-3} \ldots e_1$
$\vdots$
$\vdots$
$\vdots$
e_0

Table 1: Routh Table

the algorithms for $D_k(s)$ are:

$$\text{for } k = 1;\ D_1(s) = E_0 + E_1 s$$

$$\text{for } k = 2;\ D_2(s) = E_0 + E_1 s + E_2 s^2$$

$$\vdots$$

$$\vdots$$

and in general,

$$D_k(s) = E_0 + E_1 s + \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. + E_k s^k$$

Where $E_0 = e_n$, $E_1 = e_{n-1}$ and so on from the first column of the Routh table.

2.2. Proposed Numerator $N_k(s)$

For each output, the unknown coefficients of $N_j, k(s)$ are obtained either by matching initial time moments and Markov parameters or retaining only initial time moments of the original system in its low order model with $k = t + m$.

Let "*t*" be the no. of initial time moments to be retained and "m" be the number of Markov parameters to be retained.

$$R_k(s) = \frac{a_0 + a_1 s + a_2 s^2 + \ldots + a_{k-1} s^{k-1}}{E_0 + E_1 s + E_2 s^2 + \ldots + E_k s^k} \quad \text{with } k = t + m;\ t = k;\ m = 0,$$

Where,

$$a_0 = E_0 \times \left(\frac{d_0}{e_0}\right); a_1 = \frac{\left(\left[\left(\frac{e_0 d_1 - d_0 e_1}{e_0^2}\right) \times E_0^2\right] + a_0 E_1\right)}{E_0};$$

$$\vdots$$

$$\vdots$$

$$a_k = \frac{\left(\left[\left(\frac{e_{n-1} d_n - d_{n-1} e_n}{e_{n-1}^2}\right) \times E_{k-1}^2\right] + a_{k-1} E_k\right)}{E_{k-1}}$$

3. ILLUSTRATIVE EXAMPLE

Consider the fourth order original transfer function is given by

$$G(s)=\frac{\begin{bmatrix} g_{11}(s)g_{12}(s) \\ g_{21}(s)g_{22}(s)\end{bmatrix}}{D_4(s)}$$

Where

$g_{11}(s)$ =14.90s^2 + 1560.472s + 2543.2

$g_{12}(s)$ = 95150s^2 + 11132094s + 1805947

$g_{21}(s)$ = 85.2s^2 + 8642.888s + 4320

$g_{22}(s)$ = 124000s2 + 1492588s + 252880

$D_4(s)$ = s^4 + 113.225s^3 + 1357.275s^2 + 3499.75s +2525

The original MIMO system can be written as

$$G'(s)=\frac{\begin{bmatrix} g_{11}(s) \\ g_{21}(s)\end{bmatrix}}{D_4(s)}$$

Where

$G_{11}(s)$ = $g_{11}(s)$ + $g_{12}(s)$

= 95164.90s^2 + 11133654.47s + 1808490.2

$G_{21}(s)$ = $g_{21}(s)$ + $g_{22}(s)$

= 124085.2s^2 + 1501230.888s + 265148.80

TRANSFER FUNCTION(1ST Output)

$$G_{11}(s)=\frac{95164.90s^2+11133654.47s+1808490.2}{s^4+113.2s^3+1357s^2+3500s+2525}$$
$$(Original)$$

TRANSFER FUNCTION(2nd Output)

$$G_{12}(s)=\frac{124085.2s^2+1501230.888s+265148.80}{s^4+113.2s^3+1357s^2+3500s+2525}$$
$$(Original)$$

Then the 2nd order model using proposed method is defined as

$$R_2(s)=\frac{\begin{bmatrix} N_{11}(s) \\ N_{21}(s)\end{bmatrix}}{D_2(s)}$$

Where the common denominator $D_2(s) = E_0 + E_1 s + E_2 s^2$

Routh table to obtain $E_{0,} E_{1,...}$ is given below:

s^4	2525	1357.275	1
s^3	3499.75	113.225	
s^2	1275.585	1	
s^1	140928.36		
s^0	1		

Table 2: Implementation of Routh table for above example

$\therefore D_2(s) = s^2 + 2.7436s + 1.97948$

The 2^{nd}order model with $t = 2$; $m = 0$; using the proposed method is obtained as

Numerator for first output

$N_{11}(s) = 10598.318s + 1417.7703$

Numerator for second output

$N_{12}(s) = 1176.9414s + 207.864$

The transfer function for the reduced order model of second order can therefore be expressed as

$$R_2(s) = \frac{\begin{bmatrix} N_{11}(s) \\ N_{21}(s) \end{bmatrix}}{D_2(s)}$$

$N_{11}(s) = 10598.318s + 1417.7703$

$N_{12}(s) = 1176.9414s + 207.864$

$D_2(s) = s^2 + 2.7436s + 1.97948$

The responses of $G_n(s)$ and $R_2(s)$ for first output are compared in Fig. 1.

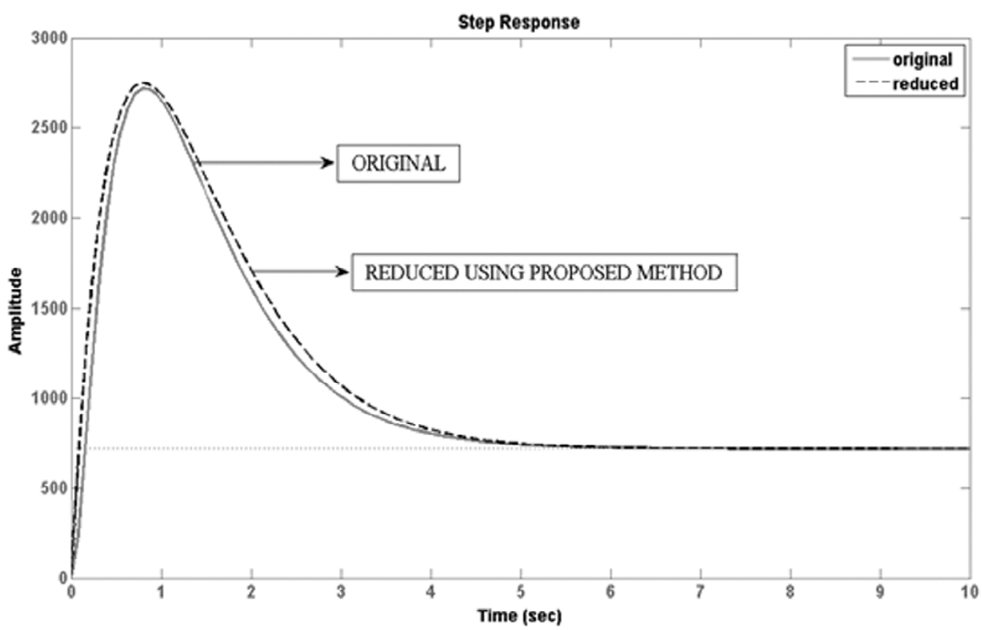

Figure 1: Comparison of responses of original system and reduced order models obtained by proposed methodfor first output

The responses of $G_n(s)$ and $R_2(s)$ for second output are compared in Fig. 2.

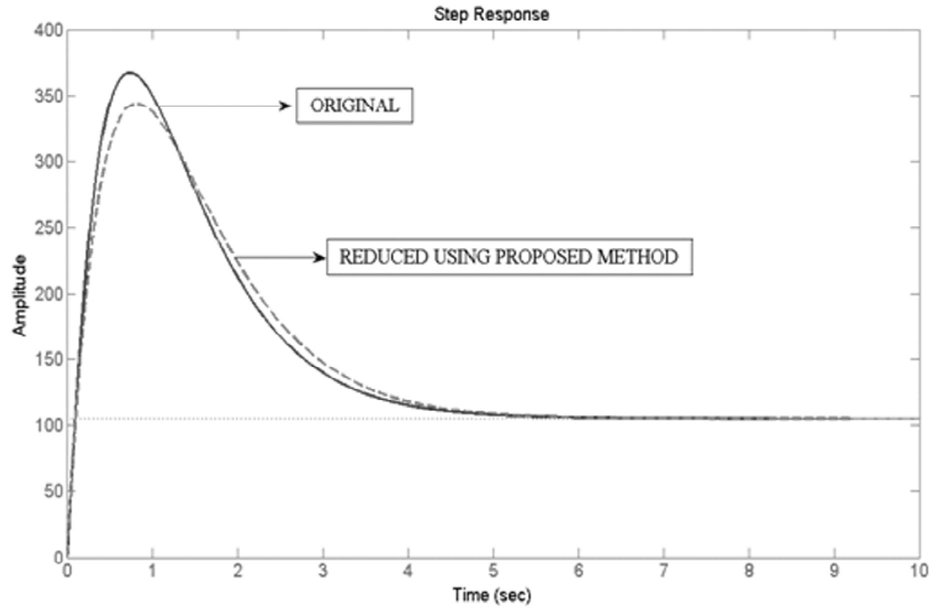

Figure 2: Comparison of responses of original system and reduced order models obtained by proposed method for second output

For comparison, the second order reduced model is obtained using the method mentioned in reference [3] as:

$$R_{11}^{1}(s) = \frac{942.744s + 15.09119}{s^2 + 2.918s + 2.107} \text{ (First output)}$$

$$R_{12}^{1}(s) = \frac{1248.207s + 2117.993}{s^2 + 2.918s + 2.107} \text{ (Second output)}$$

The responses of $G_n(s)$ and $R_2(s)$ and $R_{11}^{1}(s)$ compared in Fig. 3.

The responses of $G_n(s)$ and $R_2(s)$ and $R_{12}^{1}(s)$ compared in Fig. 4

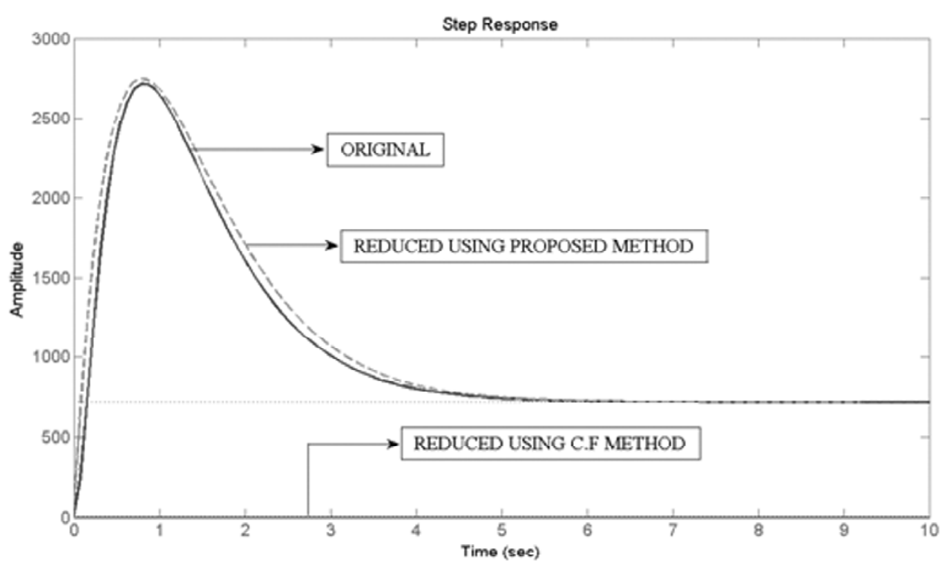

Figure 3: Comparison of Step responses of original system and reduced order models obtained by proposed methodand C.F method for first output

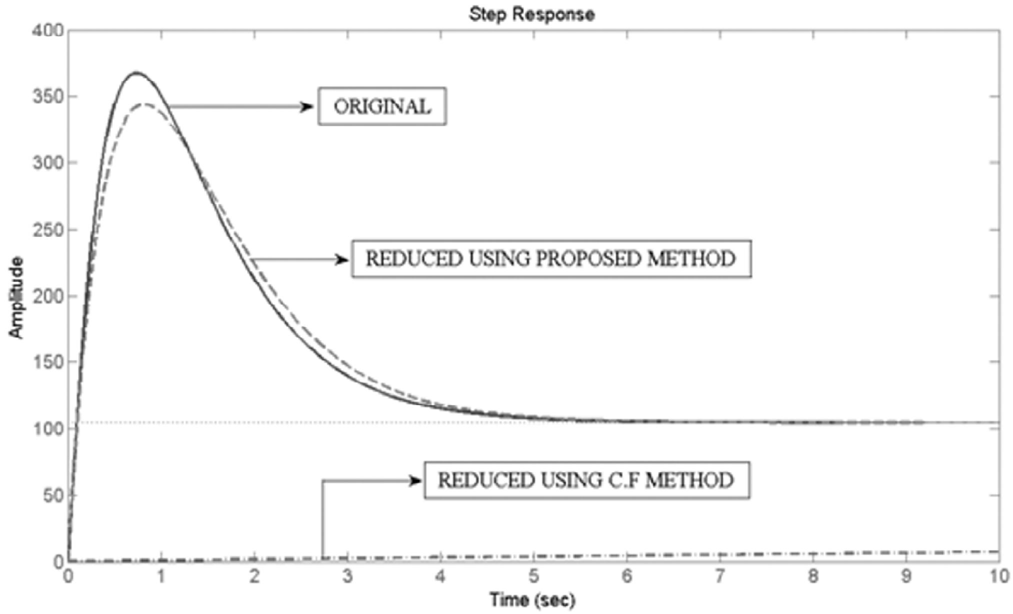

Figure 4: Comparison of Step responses of original system and reduced order models obtained by proposed methodand C.F method for second output

4. CONCLUSIONS

A new, computationally very simple procedure for order reduction of higher order MIMO system is proposed using modified Routh Approximation technique with initial Time moments and Markov parameters matching technique. The method is illustrated by considering familiar example.

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