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Comparison of LQG Controller with Reliable H Infinity Controller Designed for TRMS

Vidya S. Rao*, V.I. George, Surekha Kamath and Shreesha C.

Abstract: In this paper LQG controller for TRMS without and with sensor, actuator failure is designed. Implementation of LQG controller for TRMS is done under no failure of sensor, actuator. TRMS output with LQG controller and reliable H infinity controller are compared without and with sensor, actuator failure. The objective of the proposed technique is to prove the H infinity controller is reliable over LQG controller for TRMS with sensor, actuator failure which is validated.

Keywords: TRMS, LQG controller, reliable H infinity controller.

1. INTRODUCTION

Twin Rotor Multi input Multi Output System(TRMS) is a bench mark setup for validating new control methodologies. The TRMS setup shown in Fig. 1 consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical planes. A counter balance arm with a weight at its end is fixed to the beam at the pivot to have the TRMS stabilisable. At both ends of the beam, there are two propellers driven by two independent DC motors. For the control of TRMS, the output voltages of a controller are applied to the DC motors. A change in the voltage value result in a change of the speed of the propeller to adjust the corresponding position of the beam[1].

The system identification method is used to get a stochastic model of the system on performing experimentation on TRMS and 10th order model for TRMS is identified as given in [2],[3]. In [4],[5],[6], the authors discuss Linear Quadratic Regulator design for TRMS based on output feedback technique. The design of robust dead beat controller considering the cross coupling as disturbances is designed in [7]. In the paper mentioned the model obtained is decoupled into two SISO systems and two PID based deadbeat controllers are designed for which obtaining exact TRMS model is very essential. Linear Quadratic Gaussian(LQG) controller is designed for TRMS in [8],[9]. In these the authors have shown that the states of TRMS are estimated using Kalman observer and is fed through the LQR which is nothing but the LQG control of TRMS which is also explained in [11],[12]. Fault tolerance and fault isolation has become more prominence in these days which in other words means to provide reliability. However, due to sensors or actuators aging, external disturbance sensors or actuators may fail partially or completely which may degrade the TRMS performance or TRMS may lose its stability. Therefore a reliable controller is essential which gives the stable and best performance even under sensor, actuator failure. In [13] the authors have designed Kalman observer for TRMS and obtains all the estimated states. In this paper, the author describes how the fault identification and isolation is done using Kalman observer. Same concept is used in designing LQG controller and reliable controller for TRMS for fault identification and fault isolation. In the literature [8],[9],[10], and all the literatures mentioned above except [13] it has been assumed that all sensors and

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actuators are in good working conditions. But in the present work, the LQG controller is designed without failure of sensor, actuator as well as with failure of sensor, actuator. The design is implemented on the TRMS setup when sensors, actuators of TRMS are working fine without failure. The reliable controller is designed and simulated and results are shown in [16] . The reliable controller is implemented on real TRMS under both without and with sensor, actuator failure and the results are demonstrated in [17]. The results show that controller designed for TRMS under partial or complete failure of sensor, actuator is reliable. In this paper the results demonstrated in [16],[17] for TRMS with reliable controller is compared with the results obtained using LQG controller designed for TRMS.

Figure 1: TRMS setup

Referring to TRMS setup (Fig. 1), the 2 DOF model of TRMS in transfer function form is shown in Eq. (1) to Eq. (4). The details of obtaining 2 DOF model of TRMS using system identification technique is given in [4][16].

Main Pitch (pitch angle to the voltage supplied to main rotor):

$$\frac{1.9*10^{-6}s^3-0.000169s^2+0.015s+1.274}{s^3+1.193s^2+4.283s+3.514} \quad (1)$$

Main Yaw (yaw angle to the voltage supplied to tail rotor):

$$\frac{0.001602s^3+0.01689s^2+0.1299s+0.381}{s^3+1.251s^2+0.7983s+0.4075} \quad (2)$$

Cross Pitch (yaw angle to the voltage supplied to main rotor): $\frac{0.04858s + 0.02051}{s2 + 0.9204s + 3.152}$ (3)

Cross Yaw(pitch angle to the voltage supplied to tail rotor: $\frac{-0.01031s^2 - 0.007363s + 0.6054}{s2 + 1.676s + 1.161}$ (4)

$$A=\begin{bmatrix}-1.1930 & -2.1415 & -1.7570 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -0.9204 & -1.5760 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -1.6760 & -1.1610 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.2510 & -0.7983 & -0.8150\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0\end{bmatrix}$$

$$B=\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0.25 & 0\\ 0 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix} D=\begin{bmatrix}0 & -0.0103\\ 0 & 0.0016\end{bmatrix} \tag{5}$$

$$C=\begin{bmatrix}0.0002 & 0.0075 & 0.637 & 0 & 0 & 0.0099 & 0.6174 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.1943 & 0.0410 & 0 & 0 & -0.0189 & 0.1286 & 0.7607\end{bmatrix}$$

TRMS model in state space form is shown in Eq. (5)[17]. In Section 2 Linear Quadratic Gaussian (LQG) controller is designed and implemented for TRMS as per the design given in [8][9]. The simulation results show that the LQG controller is robust enough to handle uncertainties like modeling errors. In Section 2.1 the implementation of LQG controller for TRMS is demonstrated. The results show that LQG controller designed for TRMS is capable of handling cross coupling along with modeling errors. The main aim of this research work is to develop the controller which tolerates the sensor, actuator failure and gives the same results as that of without failure condition. This is achieved by designing reliable controller for TRMS with and without sensor actuator failure demonstrated in [16][17] which is discussed in Section 3. The Section 4 show the results of reliable controller as well as the results of LQG controller designed for TRMS with uncertainties like modeling errors, cross coupling with and without sensor, actuator failure. The results demonstrate that the LQG controller fails to handle uncertainties like sensor, actuator failure whereas the reliable controller works fine without and with the presence of the same uncertainties.

2. LQG CONTROLLER DESIGN FOR TRMS

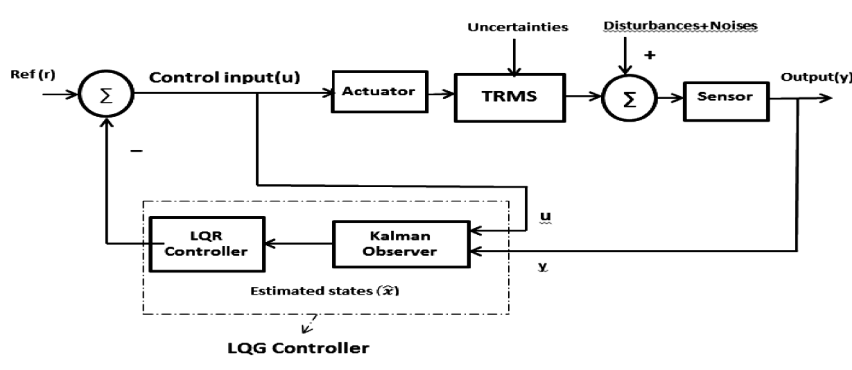

Figure 2: TRMS with LQG controller

Block diagram of TRMS with LQG controller is shown in Fig. 2. The objective of LQG controller which is the combination of Kalman observer and Linear Quadratic Regulator is to minimize the average energy over all frequencies captured by the closed loop transfer function from exogenous inputs to the error signal[12]. The plant output error is augmented with an integrator to achieve zero steady state tracking error. The goal of LQR is to find the control sequence which minimizes a quadratic cost on the states and inputs which is shown in Eq.(8)

Considering TRMS as a continuous time linear system, with as system matrix, as the input matrix, as the output matrix and as the direct transmission matrix. With process noise and measurement noise, the system is shown in Eq.(6) and Eq.(7)

$$\dot{x} = Ax + Bu + w_1 \tag{6}$$

$$y = Cx + Du + v_1 \tag{7}$$

$$J_{LQR} = lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} x(t)^T\, Qx(t) + u(t)^T Ru(t) \tag{8}$$

Where is the state vector, and are the positive definite matrices, which are weighting matrices on the states and yields optimal controller gain.[8][9]. The optimization of the cost function gives the optimal control signal as in Eq.(9)

$$u(\mathrm{t}) = r - Kx\,(\mathrm{t}) \tag{9}$$

$$\text{With } K = R^{-1}\, B^T P \tag{10}$$

and is a positive definite matrix which is solution for Riccati equation. It is found by solving the Riccati equation shown in Eq.(11)

$$A^T P + PA - PBR^{-1}\, B^T P + Q = 0 \tag{11}$$

The plant model has ten states with only two states being measurable, necessitating the inclusion of an observer which is done by Kalman observer. The observer gain matrix is computed in a similar manner as that of regulator ensuring that the estimator roots are faster than the closed loop control roots, so that the total system response is dominated by the control roots.

Combining the state feedback with the estimation problems, the LQG control signal is obtained as in Eq.(12).

$$u = r - K\hat{x}. \tag{12}$$

Where $\hat{x}$ is estimated by Kalman observer [11][14] which is shown in Eq. (13) to Eq. (16).

$$P_1 = P_0 \tag{13}$$

$$\dot{P}_1 = AP_1 + P_1\, A^T + Q - K_{01} CP_1 \tag{14}$$

$$K_{01} = P_1 C^T\, (CP_1\, C^T + R)^{-1} \tag{15}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + K_{o1}(y - C\hat{x}) \tag{16}$$

In Kalman observer the choice of adjusting and matrix elements does not exist which is explained in [11]. The and are given by Eq.(17) and Eq.(18).

$$Q = \text{var(process noise)} \tag{17}$$

$$R = \text{var(measurement noise)} \tag{18}$$

3. IMPLEMENTATION OF LQG CONTROLLER FOR TRMS

The experiments are carried on 2 DOF TRMS using MATLAB real time toolbox and Advantech PCI 1171 card. Once all the configuration parameters are set and Simulink blocks are made then the MATLAB software has to be linked to the TRMS module to run it in real time.[17]. Fig. 3 shows the interfacing of TRMS with LQG controller in which the subsystems implementing LQG controller Eq. (6) to Eq. (18) are embedded.

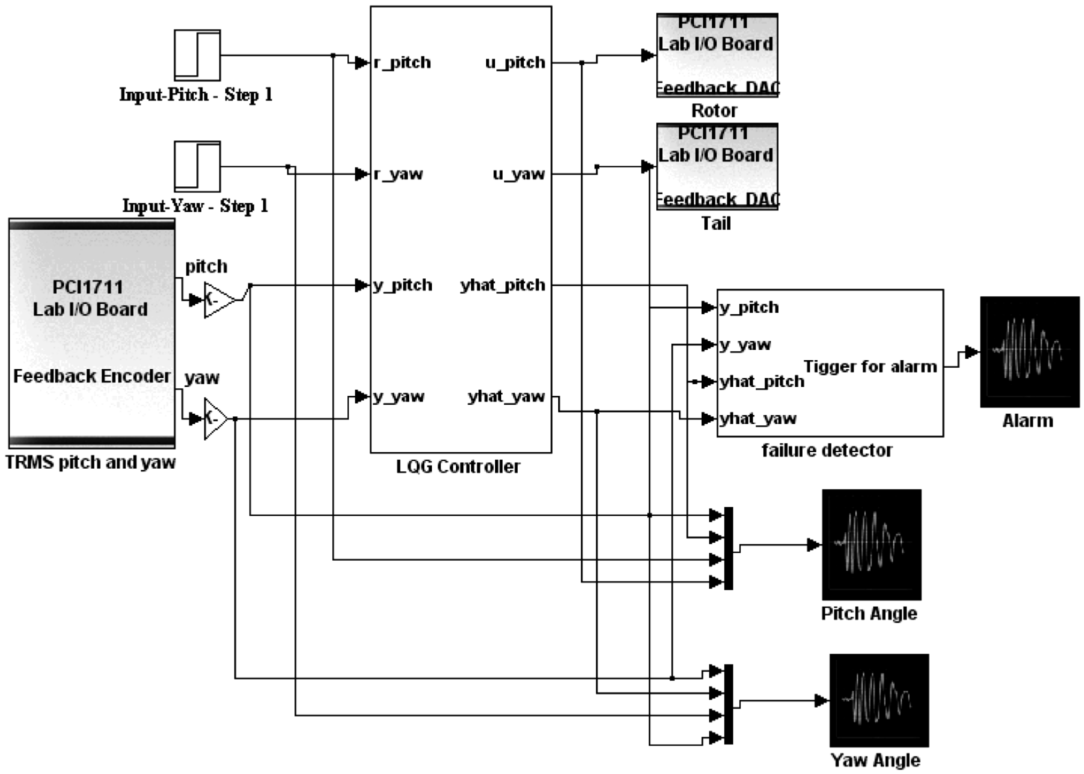

Figure 3: Implementation of LQG controller under no sensor, actuator failure

4. RELIABLE H-INFINITY CONTROLLER

H_∞ methods are used in control theory primarily to synthesize controllers achieving stabilization with guaranteed performance. These controllers are Robust and hence they can handle uncertainties reasonably well. H_∞ techniques have the advantage over classical control techniques in that they are readily applicable to problems involving multivariate systems with cross-coupling between channels. Hence H_∞ controller with H_∞ observer is designed for controlling TRMS since the TRMS system demands high reliability and robustness[18][22].

This work uses a method based on the algebraic Riccati equation for designing robust reliable H_∞ control laws for plants with structured uncertainty. The design method consists of incorporating information on the plant uncertainty into the Algebraic Riccati Equations (ARE) used for nominal H_∞ disturbance-rejection designs. The development of the reliable H_∞ controller assumes that the sensor failures can be detected and the observer dynamics accordingly adjusted [13][15].

The design of reliable H_∞ observer and controller with and without failure of sensor, actuator and both sensor and actuator is presented in [16]. The implementation of reliable observer controller with and without sensor, actuator and both sensor and actuator are presented in [17]. The design technique is taken from [19],[20],[21] and is applied for TRMS. The results given in [16][17] are compared with LQG controller in section 4. The H_∞ controller is named as reliable because it is robust to the uncertainties and takes up external disturbances with and without sensor, actuator failure. In this controller design the designer has the choice of adjusting and matrices to obtain good disturbance rejection, high damping and a bandwidth that provides fast response without saturating the control[21]. By trial and error method the values are chosen[11] for Q and R are $Q = I$ and $= \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$.

5. RESULTS

5.1 Results of Reliable controller

5.1.1 Under no sensor, actuator failure

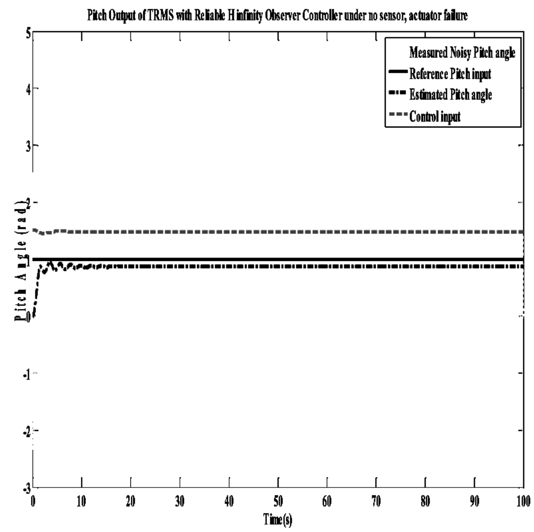

Figure 4: Pitch angle

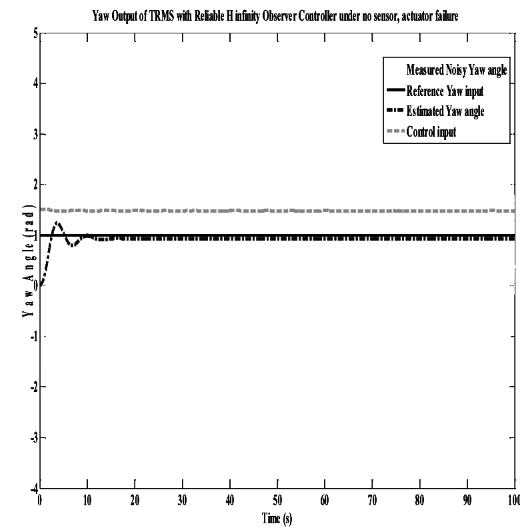

Figure 5: Yaw angle

Fig. 4 and Fig. 5 show TRMS pitch output and Yaw output with reliable controller. Here the sensors and actuators are working perfectly fine.

5.1.2 Under Sensor, Actuator Failure

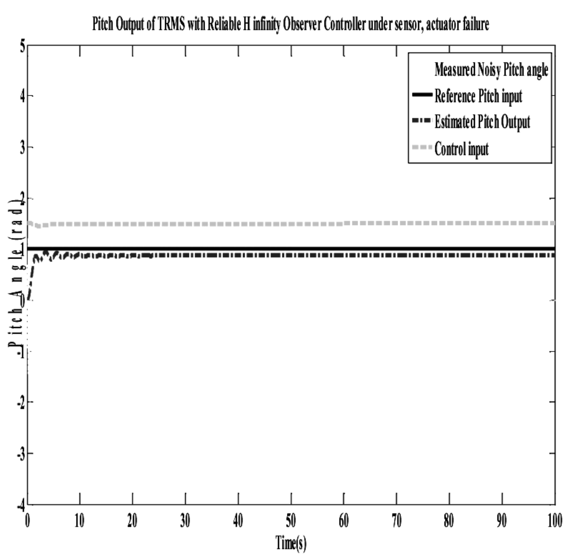

Figure 6: Pitch angle

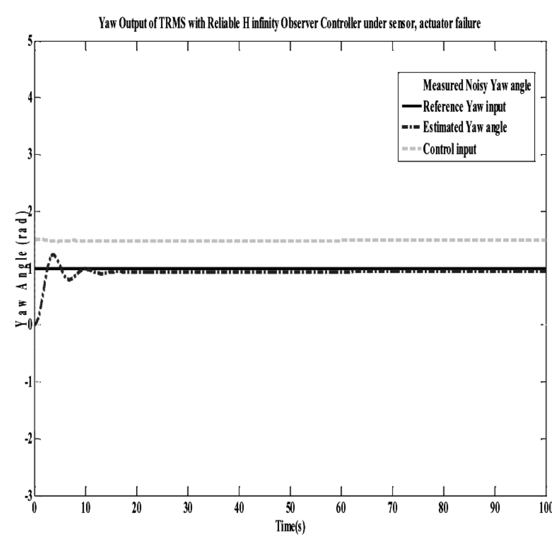

Figure 7: Yaw angle

The sensors are failed at 50s and actuators are failed at 60s as shown in Fig. 6 and Fig. 7 the pitch output and yaw output are same as that of without failure case. Thus reliable H_∞ controller output is same before and after the failure of sensor, actuator.

5.2 RESULTS OF LQG CONTROLLER

5.2.1 Under No Sensor, Actuator Failure

A. Simulation Results

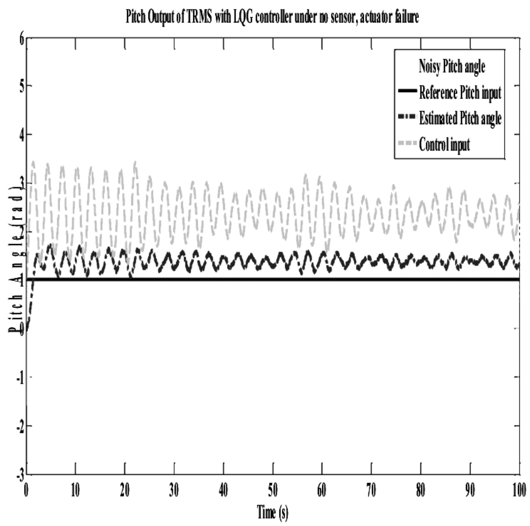

Figure 8: Pitch angle

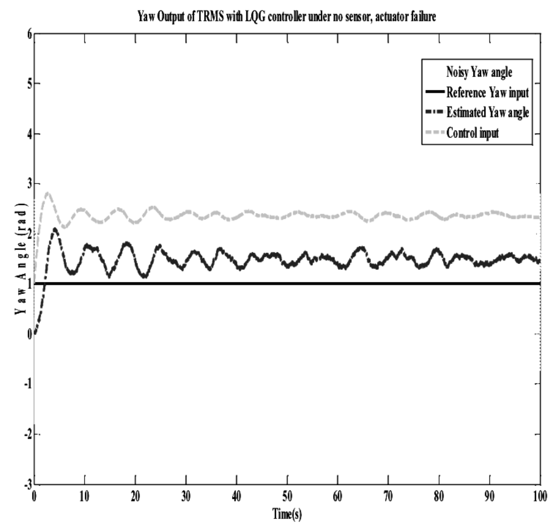

Figure 9: Yaw angle

From Fig. 8 and Fig 9 it is seen that when sensor and actuator working fine the LQG controller gives the similar result as that of Fig. 4 and Fig. 5 even though the performance is lower. That is in presence of uncertainties like modeling errors, cross coupling reliable H_∞ controller performs better than LQG controller for TRMS. Control signal is also oscillating with LQG controller which shows that the control algorithm is marginally stable. That is roots of observer and controller are very closer.

B. Real Time Implementation Results

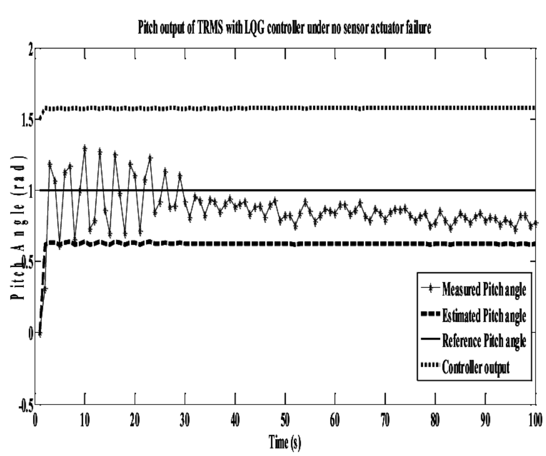

Figure 10: Pitch angle

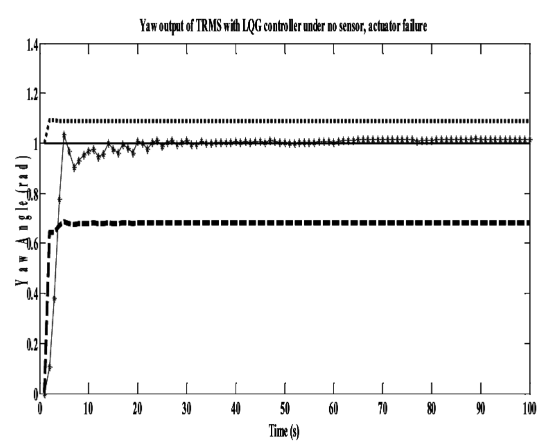

Figure 11: Yaw angle

TRMS with LQG controller with no failure of sensors, actuators is implemented and Pitch output and Yaw output are obtained which are shown in Fig. 10 and Fig. 11.

5.2.2 Under Sensor, Actuator Failure

A. Simulation Results

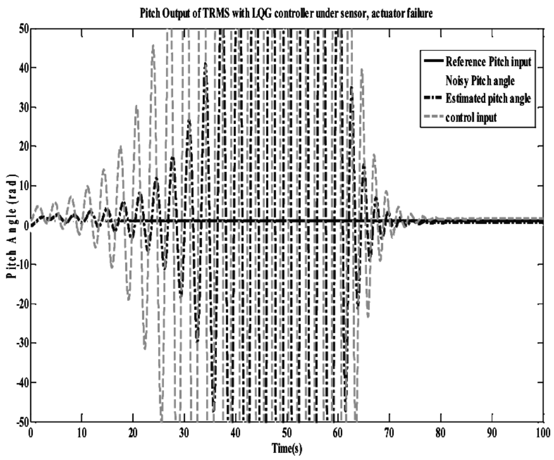

Figure 12: Pitch angle

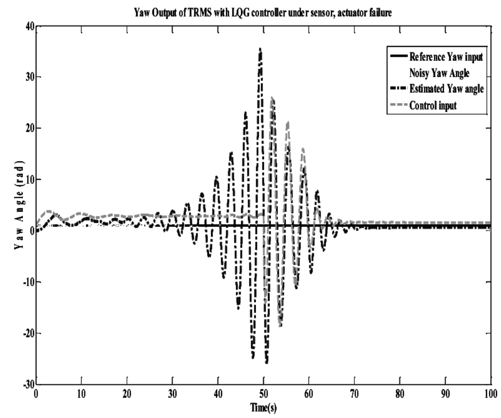

Figure 13: Yaw angle

At 40s the pitch sensor and yaw sensor of TRMS are failed. At 50s main rotor actuator and tail rotor actuator are failed. It is seen from Fig. 12 and Fig. 13, the estimated pitch angle and the estimated Yaw angle of TRMS go out of control after sensor, actuator of TRMS have failed. Also control signal after 50s is erratic. Therefore LQG controller is unreliable for uncertainties like sensor, actuator failure.

6. CONCLUSION

This paper contributes to control of TRMS using LQG controller. Similarly for TRMS reliable H_∞ controller is also designed with and without sensor actuator failure. Without the sensor, actuator failure of the system the LQG controller gives stable result. But performance is poor compared to reliable H_∞ controller due to uncertainties like modeling error and cross coupling. The combination of Kalman observer and LQR designed for TRMS is not sufficient to remove the effect of these uncertainties whereas the same uncertainties are well handled and a very good performance is obtained in case of reliable H_∞ controller. But when the uncertainties like sensor actuator failure occur LQG controller completely fails whereas reliable controller gives the same output as that in case of no TRMS sensor, actuator failure.

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