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Trajectory Tracking Control of an AMM Modelled TLFM using Backstepping Method

K. Lochan* and B. K. Roy*

Abstract : This paper addresses the problem of trajectory tracking control for a planner assumed modes modelled two-link flexible manipulator. A robust backstepping control technique is designed for this control problem. The effectiveness of the proposed trajectory tracking strategy is validated using numerical simulations in MATLAB environment. When compared with other existing control techniques used for trajectory tracking of a TLFM in the presence of uncertainties, it is found that the proposed technique tracks the desired trajectoriesbwith a small tracking error.

Keywords : Robust control, tracking control of TLFM, AMM, backstepping control.

Nomenclature :

Table 1

Symbol	*Explanation*	*Symbol*	*Explanation*
X_i, Y_i	Inertial frame axis	j	No. of modes for the link i
$\hat{X}_i$, $\hat{Y}_i$	Axis of rigid body moving frame	ϕ_{ji}	Assumed spatial mode shapes
M_p	Payload mass	δ_{ij}	Time varying variables associated with the ϕ_{ij}
q	Vector of generalised coordinate of the manipulator	y_i	Tip position of the i^{th} link
τ_i	Actuated torque input	y_{di}	Desired tip trajectory
l_i	Length of i^{th} link	u_i	Link deflection

1. INTRODUCTION

Flexible manipulators are more effective from the application point of view in various fields like space, industries, defence, hospitals, home appliances, etc., When compared with their rigid counter part [1,2]. The research in flexible manipulators is more concentrated on a two-link flexible manipulator (TLFM) [3,4]. A flexible manipulator (FM) has non-minimum phase, inherent nonlinearity, under actuation, noncolocation and unstablility kinds of control issues [5]. There are mainly three control problems for a FM: (*i*) trajectory tracking control, (*ii*) position control and (*iii*) deflection suppression [6]. The trajectory tracking control is the most challenging control problem.

The effectiveness of the tracking control problem depends upon the type of modelling methods used. The modelling of a TLFM can be obtained using lumped parameter method (LPM), finite element method (FEM) or assumed modes method (AMM) [2]. The AMM is more widely used. Many control techniques are reported for the trajectory tracking control of a TLFM like adaptive control [1,7], MPC [4], SMC [8–10], observer based [11], FLC [6], H_∞ control [12], LMI based PD control [13], etc. But, designing an

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appropriate controller for the trajectory tracking control for a TLFM is still a challenging task. Backstepping control is an efficient and effective control technique in the presence of uncertainties and disturbances [14]. The major advantages of the backstepping control is that it relaxes the matching condition of the uncertainties whereas other robust control techniques like SMC stabilises the uncertain system when the uncertainties satisfy the matching conditions [14]. Backstepping control is used for different control problems of a rigid manipulator [15–21]. Backstepping control for the trajectory tracking of a single link flexible joint manipulators is used in [22–28]. Backstepping with extended state observer for the trajectory tracking control of a two-link flexible space manipulator is used in [11]. But, designing of a robust control technique to achieve negligible steady state trajectory tracking error is still a challenging task. Considering the above discussions, the motivation behind this paper is to design of a robust controller for the trajectory tracking control of a TLFM by using a backstepping control technique.

In this paper, a backstepping controller is designed for the trajectory tracking control of a AMM modelled planner TLFM. It is shown that the fast trajectory tracking control for the TLFM is achieved effectively with negligible tracking error and small control efforts. To the best of our knowledge, very few efforts are taken for robust stability of TLFMs with negligible tracking error.

The rest of the paper is organised as follows: Section 2 describes the dynamic model of a TLFM. The designing of the proposed trajectory tracking control using backstepping control for a TLFM is given in Section 3. Section 4 consists of discussions along with the results. Finally, the conclusions of the paper are given in Section 5.

2. DYNAMICS OF A PLANNER TLFM

The schematic representation of a planner TLFM is shown in Fig. 1.

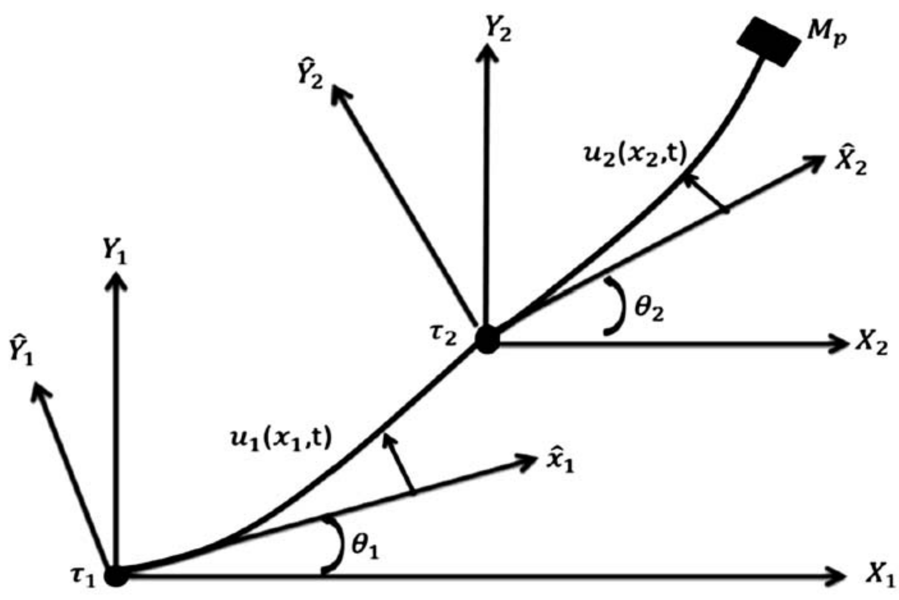

Figure 1: The coordinates of a planner TLFM

The link deflection is represented by $u_i(x_i, t)$ and is expressed as [29,30]

$$u_i(x_i, t) = \sum_{j=1}^{n} \varnothing_{ij}(x_i)\delta_{ij}(t) \tag{1}$$

The tip position of i^{th} link can be written as [29, 30].

$$y_i = \frac{1}{l_i}[u_i(x_i, t)] \tag{2}$$

Considering Lagrangian generalised coordinate as $q = (\theta_i, \delta_{ij}(t)) \in R^{6X1}$ and using the Lagrange's assumed modes method (AMM), the dynamic equation of a planner TLFM [5,30] is described as

$$\mathrm{N}(q)\ddot{q} + h(q, \ddot{q}) + \mathrm{K}q + \mathrm{D}\dot{q} \;=\; \tau + \tau_d \tag{3}$$

where $\mathrm{N}(q) \in \mathrm{R}^{6X6} = \mathrm{N}_0(q) + \Delta\mathrm{N}(q)$ is the mass inertia matrix, $h(q) \in \mathrm{R}^{6X1} = h_0(q) + \Delta h(q)$ is the centrifugal and coriolis force vector, $\mathrm{K} \in \mathrm{R}^{6X6} = \mathrm{K}_0 + \Delta\mathrm{K}$ is the positive definite stiffness matrix, $b \in \mathrm{R}^{6X2}$ is the input scaling factor, $\mathrm{D} \in \mathrm{R}^{6X6} = \mathrm{D}_0 + \Delta\mathrm{D}$ is the positive definite damping matrix and $\tau \in \mathrm{R}^{2X1}$, $\tau_d \in \mathrm{R}^{2X1}$ are the joint input and disturbance torque vectors. Here, $\mathrm{N}_0(q)$, $h_0(q)$, K_0, D_0 and $\Delta\mathrm{N}(q)$, $\Delta h(q)$, $\Delta\mathrm{K}$, $\Delta\mathrm{D}$ represent the nominal and bounded perturbations in the system description. The dynamic equation with nominal parameters is given as.

$$\mathrm{N}_0(q)\ddot{q} + h_0(q, \dot{q}) + \mathrm{K}_0 q + \mathrm{D}_0\dot{q} \;=\; b(\tau + \tau_d) + \mathrm{R}(q, \dot{q}, \ddot{q}) \tag{4}$$

where $\mathrm{R}(q, \dot{q}, \ddot{q}) = -\Delta\mathrm{N}(q)\ddot{q} - \Delta\mathrm{K}q - \Delta h(q, \dot{q}) - \Delta\mathrm{D}\dot{q}$ are the system uncertainties and are bounded as

$$\|\mathrm{R}(q, \dot{q}, \ddot{q})\| \;=\; \alpha_0 + \alpha_1\|q\| + \alpha_2\|\dot{q}\|^2 \tag{5}$$

where α_0, α_1 and α_2 are some positive constants.

3. BACKSTEPPING CONTROLLER DESIGN

In this section, backstepping controller design method for a TLFM is presented.

3.1. Transformation of variables

Define two new variables $\theta_n = (\theta_1, \theta_2)^{\mathrm{T}}$, $q_n = (\delta_{11}, \delta_{12}, \delta_{21}, \delta_{22})^{\mathrm{T}}$. Using these variables the system matrices can be written as:

$$\mathrm{N}_0 \;=\; \begin{bmatrix} \mathrm{N}_{11} & \mathrm{N}_{12} \\ \mathrm{N}_{21} & \mathrm{N}_{22} \end{bmatrix}, \mathrm{D}_0 = \begin{bmatrix} \mathrm{D}_{11} & \mathrm{D}_{12} \\ \mathrm{D}_{21} & \mathrm{D}_{22} \end{bmatrix}, \mathrm{H}_0 = \begin{bmatrix} \mathrm{H}_1 \\ \mathrm{H}_2 \end{bmatrix}, \mathrm{K}_0 = \begin{bmatrix} \mathrm{K}_{11} & \mathrm{K}_{12} \\ \mathrm{K}_{21} & \mathrm{K}_{22} \end{bmatrix} \tag{6}$$

Using the matrices defined in (6), the dynamics in (4) can be written as

$$\mathrm{N}_{11}\ddot{\theta}_n + \mathrm{N}_{12}\,\dot{q}_n + \mathrm{D}_{11}\ddot{\theta}_n + \mathrm{D}_{12}\,\dot{q}_n + \mathrm{H}_1 + \mathrm{K}_{11}\theta_n + \mathrm{K}_{12}q_n \;=\; \tau \tag{7}$$

$$\mathrm{N}_{21}\ddot{\theta}_n + \mathrm{N}_{22}\,\dot{q}_n + \mathrm{D}_{21}\ddot{\theta}_n + \mathrm{D}_{22}\,\dot{q}_n + \mathrm{H}_2 + \mathrm{K}_{21}\theta_n + \mathrm{K}_{22}q_n \;=\; 0 \tag{8}$$

From (8), we can write the following transformation equation

$$\ddot{q}_n \;=\; -\mathrm{N}_{22}^{-1}(\mathrm{N}_{21}\,\ddot{\theta}_n + \mathrm{D}_{21}\,\dot{\theta}_n + \mathrm{D}_{21} + \mathrm{D}_{22}\,\dot{q}_n + \mathrm{H}_2 + \mathrm{K}_{21}\,\theta_n + \mathrm{K}_{22}q_n) \tag{9}$$

Using (9) we can write (7) as

$$\mathrm{A}\ddot{\theta}_n + \mathrm{B}\ddot{\theta}_n + \mathrm{C} \;=\; \tau \tag{10}$$

$$\mathrm{A} \;=\; \mathrm{N}_{11} - \mathrm{N}_{12}\mathrm{N}_{22}^{-1}\mathrm{N}_{21},$$

$$\mathrm{B} \;=\; \mathrm{D}_{11} - \mathrm{N}_{12}\mathrm{N}_{22}^{-1}\mathrm{D}_{21} \text{ and}$$

$$\mathrm{C} \;=\; (\mathrm{D}_{12} - \mathrm{N}_{12}\mathrm{N}_{22}^{-1}\mathrm{D}_{22})\dot{q}_n - \mathrm{N}_{12}\mathrm{N}_{22}^{-1}\mathrm{H}_2 - \mathrm{N}_{12}\mathrm{N}_{22}^{-1}\mathrm{K}_{21}\,\theta_n - \mathrm{N}_{12}\mathrm{N}_{22}^{-1}\mathrm{K}_{22}q_n + \mathrm{H}_1 + \mathrm{K}_{11}\theta_n + \mathrm{K}_{12}q_n \tag{11}$$

3.2. Design of controller

Considering $\theta_n = z_1$ and $\dot{\theta}_n = \dot{z}_1 = z_2$, $\dot{z}_2 = z_3 + \mathrm{A}^{-1}\tau$ then $z_3 = -\mathrm{A}^{-1}(\mathrm{B}z_1 + \mathrm{C})$. Suppose θ_d is a twice differentiable desired trajectory tracking signal and u is a virtual control variable. The trajectory tracking error can be defined as

$$e_1 \;=\; \theta_d - z_1 \tag{12}$$

$$e_2 \;=\; u - z_2 \tag{13}$$

The derivative of the error variables (12) and (13) can be written as:

$$\dot{e}_1 \;=\; \dot{\theta}_d = z_2 \tag{14}$$

$$\dot{e}_2 \;=\; \dot{u} - z_3 - \mathrm{A}^{-1}\tau \tag{15}$$

Theorem 1: Consider the backstepping controller defined in (16) for the error dynamics in (12) and (13). The joint angles of the flexible manipulator (4) follow the desired trajectories.

$$\tau = A(-z_3 \dot{u} + c_2 e_2 + e_1) \quad (16)$$

where c_2 is a positive definite matrix.

Proof : The designing of a backstepping controller for a TLFM is achieved using the following steps:

Step 1: Consider a Lyapunov function as

$$v_1 = \frac{1}{2} e_1^2 \quad (17)$$

Then

$$\dot{v}_1 = e_1(\dot{\theta}_d + e_2 - u) = e_1\dot{\theta}_d - e_1 u + e_1 e_2 \quad (18)$$

Now defining the virtual control variable u as

$$u = \dot{\theta}_d + c_1 e_1 \quad (19)$$

where c_1 is a positive definite matrix. We get

$$= -c_1 e_1^2 + e_1 e_2 \quad (20)$$

If $e_2 = 0$, then the first joint angle of the manipulator follows the desired trajectory.

Step 2: Considering another Lyapunov function as

$$v_2 = v_1 + \frac{1}{2} e_2^2 \quad (21)$$

The derivative of (21) can be written as

$$\dot{v}_2 = -c_1 e_1^2 + e_1 e_2 + e_2(\dot{u} - z_3 - A^{-1}\tau) \quad (22)$$

From (22), we can obtain the actual torque input as

$$\tau = A(-z_3 + \dot{u} + e_1 - c_2 e_2) \quad (23)$$

Using (23) the derivative of the second Lyapunov function (22) is written as

$$\dot{v}_2 = (-c_1 e_1^2 - c_2 e_2^2) \quad (24)$$

Since (24) is negative definite, the error variables e_1 and e_2 asymptotically converge to origin with suitable choice of constant matrices c_1 and c_2. Thus, the joint angles follows the desired trajectories.

4. RESULTS AND DISCUSSIONS

The parameters of a physical TLFM used for simulating the flexible manipulator dynamics (2) are given in Table 2.

The expressions of the desired trajectories used for link-1 and link-2 are given as.

$$\theta_{d1} = \frac{\pi}{4} - \frac{7}{6} e^{-\frac{3}{2}t} + \frac{7}{19} e^{-2t}$$

$$\theta_{d2} = \frac{\pi}{6} - \frac{7}{6} e^{-\frac{3}{2}t} + \frac{7}{11} e^{-2t} \quad (25)$$

In this paper, all the simulations are carried out using ode-45 simulation method in MATLAB simulation environment. The initial condition for simulating TFLM dynamics (2) is considered as

$$q(0) = (0.01, 0.01, 0, 0.0001, 0, 0.0001)^T,$$

$$\dot{q}(0) = (0,0,0,0,0,0)^T \quad (26)$$

The value of some other constants used for simulating backstepping controller is

$$c_1 = \begin{bmatrix} 2 & 0 \\ 0 & 40 \end{bmatrix}, c_2 = \begin{bmatrix} 2 & 0 \\ 0 & 40 \end{bmatrix} \quad (27)$$

Table 2

Parameters of a physical TLFM [31]

Mass of link 1, $m_1 = 0.15268$ Kg	Coefficients of viscous damping, $B_{eq1} = 4$ Nms/rad, $B_{eq2} = 1.5$ Nms/rad	Link-1 MI, $J_{arm1} = 0.002035$ Kgm^2
Mass of link 2, $m_2 = 0.0535$ Kg	Efficiency of gear boxes, $\eta_{g1} = 0.85$, $\eta_{g2} = 0.9$	Link-2 MI, $J_{arm2} = 0.0007204$ Kgm^2
Length of link 1, $L_1 = 0.201$ *m*	Efficiency of motors, $\eta_{m1} = 0.85$, $\eta_{m2} = 0.85$	
Length of link 2, $L_2 = 0.201$ *m*	Constants of back emf, $K_{m1} = 0.119$ *v*/rad, $K_{m2} = 0.0234$ *v*/rad	
Resistance of Armatures, $R_{m1} = 11.5\ \Omega$, $R_{m2} = 2.32\ \Omega$	Gear ratio, $K_{g1} = 100$, $K_{g2} = 50$	
Equivalent MI at load, $J_{eq1} = 0.17043$ Kgm^2	Motor torque constants $K_{t1} = 0.119$ Nm/A, $K_{t2} = 0.0234$ Nm/A	
Equivalent MI at load, $J_{eq2} = 0.0064387$ Kgm^2	Stiffness of the links, $K_{s1} = 22$ Nm/rad, $K_{s2} = 2.5$ Nm/rad	

Remark 1: The constant matrices c_1 and c_2 are chosen in a manner to achieve a good tracking performance with low control effort.

The trajectory tracking responses for both the links are shown in Fig. 2. Trajectory tracking error for both the links are shown in Fig. 3. The modes of the first and second flexible link with 0.145 kg payload are shown in Fig. 4. The responses of the tip deflection of the links are shown in Fig. 5. The required control torque inputs used for trajectory tracking are shown in Fig. 6.

It is seen from Fig. 2 that the trajectory tracking for both the links are achieved within $2s$. It is apparent from Fig. 3 that the steady state tracking error for first joint angle is 10^{-5} *mm*, the same for second joint angle is 10^{-12} *mm*. Thus, it is seen that the tracking errors are negligible. The second modes of both the links in the Fig. 4 are suppressed properly and in the range of 10^{-3} *rad*. It is noted from the Fig. 6 that the required control torque using backstepping for both the links are in the range of [–0.5,0.5] Nm. Therefore, the desired trajectory tracking is achieved with almost no steady state error and minimum control efforts.

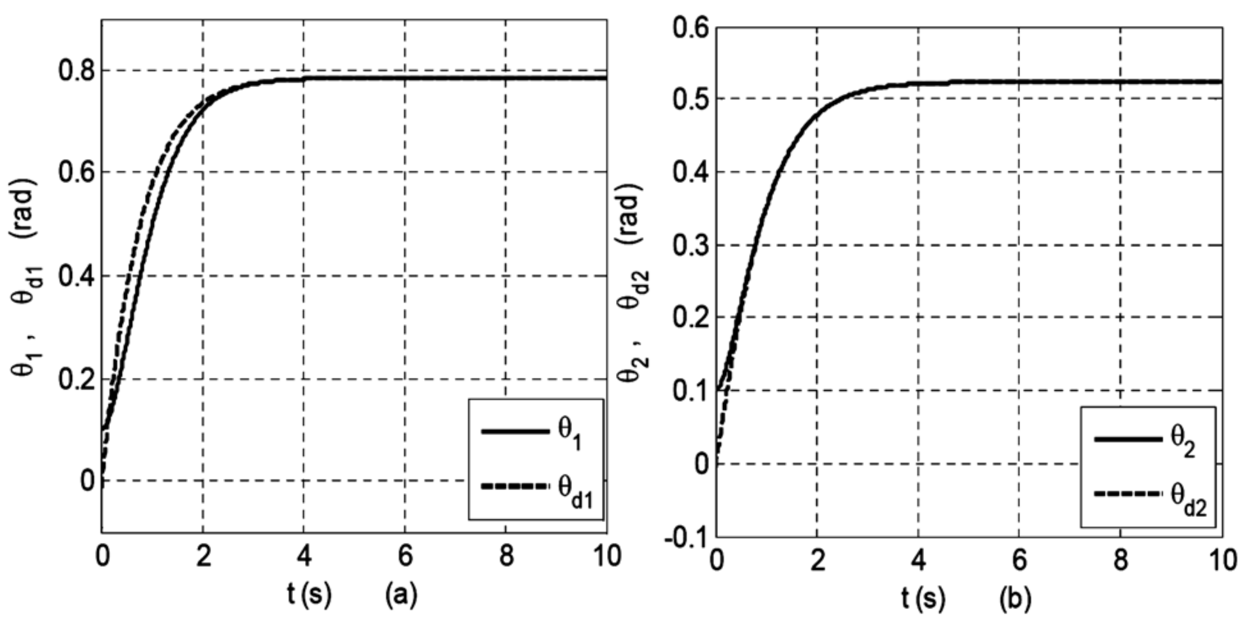

Figure 2: Responses of trajectory tracking for: (*a*) link-1 and (*b*) link-2.

5. CONCLUSIONS

In this paper a backstepping control technique is designed for the trajectory tracking of a planner TLFM. It is shown through simulation that the tracking is achieved with negligible steady state tracking error and minimum control inputs. The assumed modes method (AMM) with two modes for each link is used for modelling the TLFM. Simulation results confirm the successful achievement of the objective of the paper. Experimental verification of the proposed trajectory tracking strategy, is kept as the future scope of this work.

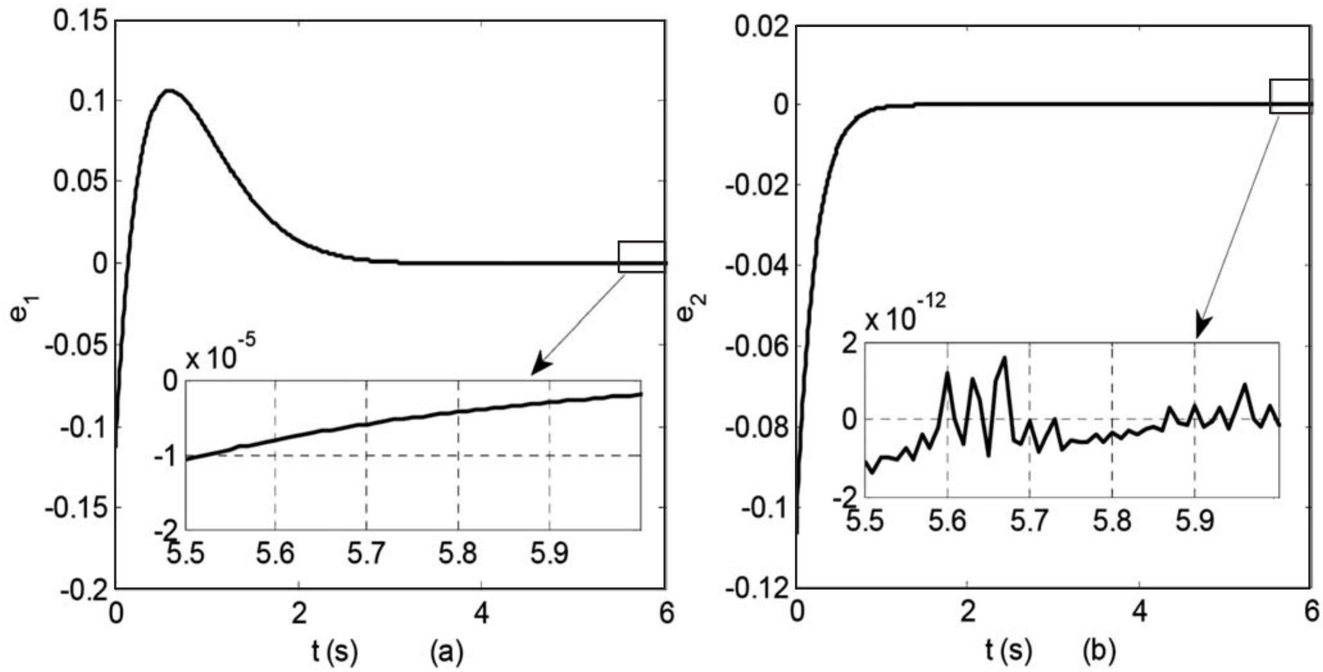

Figure 3: Trajectory tracking error for: (*a*) link-1 and (*b*) link-2

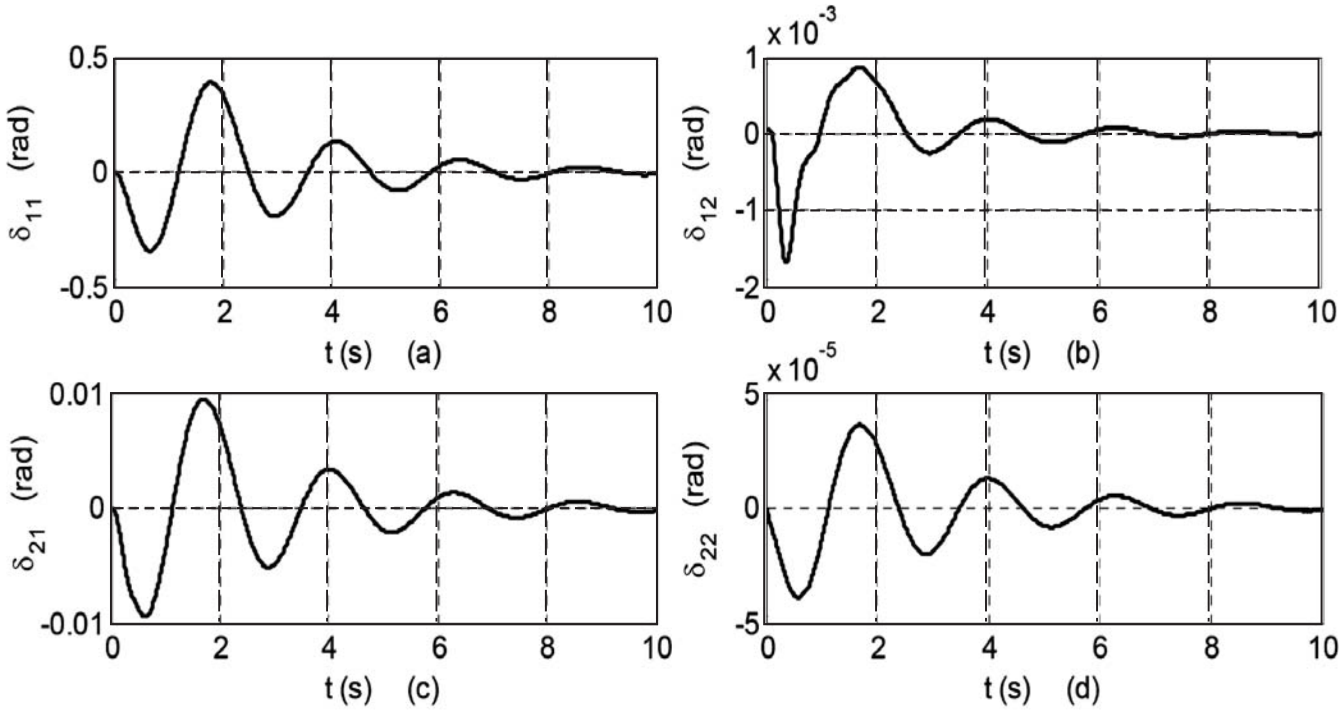

Figure 4: Modes of the links of the TLFM during trajectory tracking for: (*a*) link-1 and (*b*) link-2

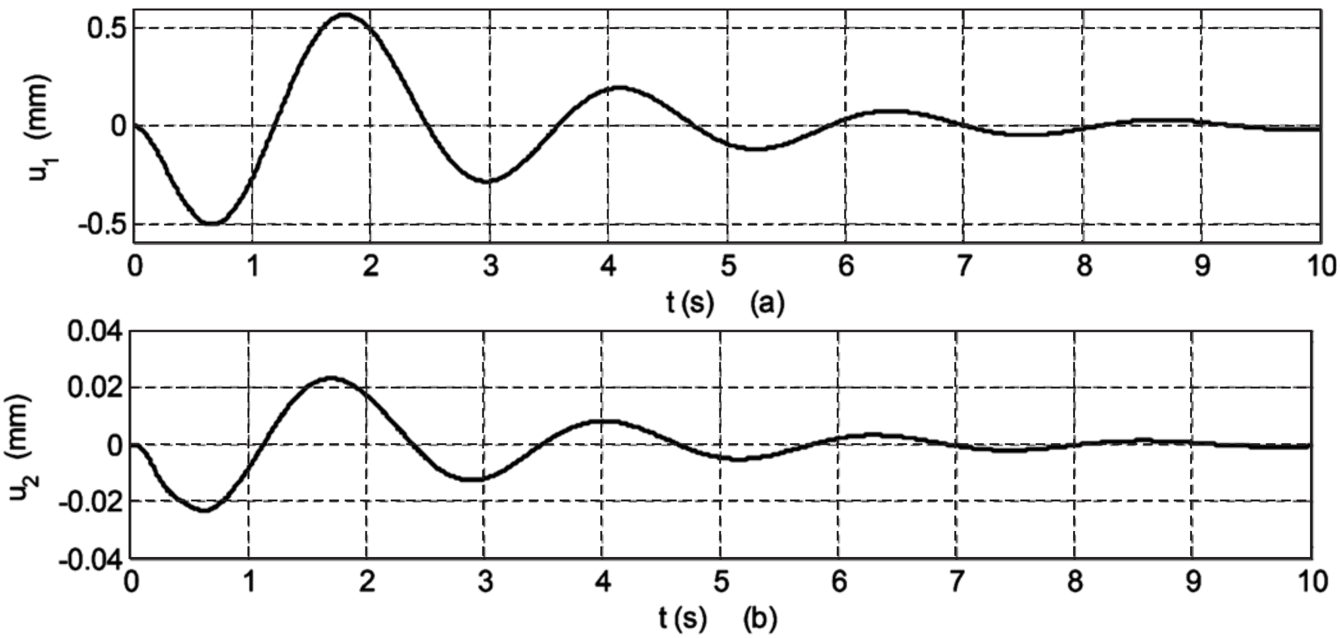

Figure 5: Response of tip deflection during trajectory tracking for: (*a*) link-1 and (*b*) link-2

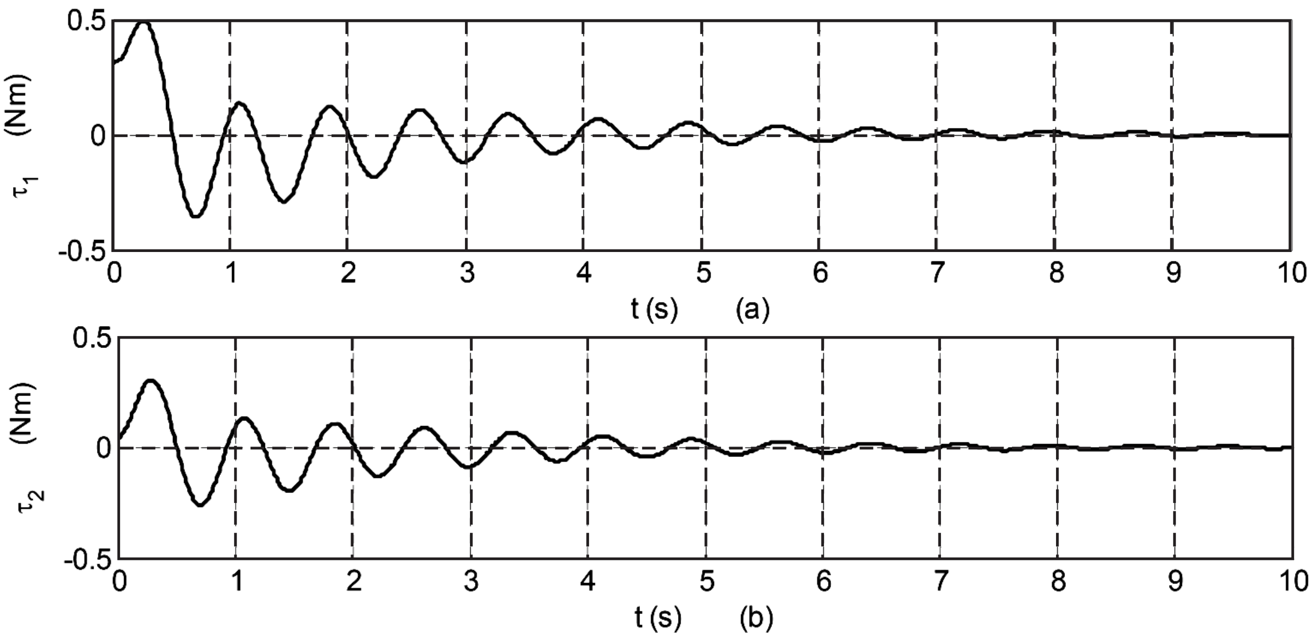

Figure 6: Required control torque inputs for trajectory tracking control for: (a) joint angle-1 and (b) joint angle-2.

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