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A BICRITERIA TWO MACHINE FLOW SHOP SCHEDULING WITH SEQUENCE DEPENDENT SETUP TIME

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Abstract: Bicriteria flow shop scheduling problems with sequence dependent setup time have been an escalating attention of researchers and managers in recent years. In this paper, a bicriteria scheduling problem with sequence dependent setup time (SDST) on two machines is considered. The processing times of the attributes on these machines are associated with probabilities with an objective to minimize the rental cost of machines with minimum makespan under a specified rental policy. A heuristic approach to find optimal or near optimal sequence has been discussed. The proposed method is easy to understand and provide an important tool for decision makers. A numerical illustration is also given to substantiate the algorithm.

Keywords: Bicriteria scheduling, Flowshop, Idle time, Makespan, Sequence dependent setup time, Rental cost, Utilization time.

1. INTRODUCTION

Scheduling is a decision making process that concerns the distribution of limited resources to a set of tasks with the view of optimizing one or more objectives. Scheduling in manufacturing systems is classically associated with scheduling a set of jobs on a set of machines in order to maximize the profits. In a general flowshop scheduling problem, n jobs are to be scheduled on m machines in order to optimize some measures of performance. All jobs have the same processing requirements so they need to be processed on all machines in same order. Two machine flowshop scheduling problem has been considered as a major sub problem due to its application in real-life. There are cases where setup times are negligible and therefore could be included in the jobs processing times. However, in some applications, setup has major impact on the performance measure considered for scheduling problem so they need to be considered separately. Scheduling problems involving setup times can be divided into two classes: the first class is sequence-independent and second is sequence-dependent setup times.

In this paper, we address a sequence dependent flowshop scheduling problem. The term "sequence-dependent" implies that the setup time depends on the sequence in which

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the jobs are processed on the machines. Each job J_i is characterized by some attributes. The processing time of attribute of job J_i on machine k is denoted by $a_{i,k}$. If job J_i is processed immediately after job J_j, a setup time $s_{ij,k}$ is required on machine k. Scheduling with sequence dependent setup time has received significant attention in recent years. Corwin and Esogbue [3] minimized makespan considering sequence dependent setup time. Gupta [8] proposed a branch and bound algorithm to minimize setup cost in n jobs and m machines flowshop with sequence dependent set up time. Noteworthy approaches are due to Lee and Jung [16], Pugazhendhi *et al*., [23], Gajpal *et al*., [9] and Wang and Cheng [27].

Also, most of research on sequence dependent setup time flowshop scheduling problems has been concentrated on single criterion problems. The scheduling literature also reveals that the research on bi-criteria is mainly focused on the single-machine or two machine problems without sequence dependent setup time. Toktas *et al*., [24] considered the two machine flow scheduling by minimizing makespan and maximum earliness simultaneously. Rahimi-Vaheda *et al*., [19] considered a bicriteria no-wait flowshop scheduling problem in which weighted mean completion time and weighted mean tardiness are minimized. Some of the noteworthy heuristic approaches are due to Smith [22], Van Wassenhove and Gelders [25], Sen and Gupta [20], Panwalker [18], Bagga and Bhambi [1], Chenj and Wang [27], Blazewicz *et al*., [2], Gupta and Sharma [11, 12].

Bicriteria scheduling problems are commonly divided into two classes. In the first class, one of the functions is considered as the objective to be optimized while the other considered as the constraints. In the second class, both the functions are weighted differently or equally and an overall objective function is defined as the weighted sum of individual functions where sum of the individual weight coefficient is unity. In the present work, the problem considered belongs to first class. A heuristic algorithm is proposed to optimize the bicriteria when the processing times of attributes on the machines are associated with probabilities under sequence dependent setup times. The two criteria of minimizing the maximum utilization of machines or rental cost and minimizing the maximum makespan are one of the combinations of our objective function reflecting the performance measure.

2. PRACTICAL SITUATION

Sequence dependent setup times are usually found in the situation where the facility is a multipurpose machine. Some examples of sequence dependent setup time flowshop scheduling problem include:

(a) Textile industry, where setup times are significant as fabric types are assigned to loom equipped with wrap chains, when the fabric type is changed on a machine, the wrap chain must be replaced and the time it takes depends on the previous and current fabric type;

(b) Stamping plants used by most auto-makers, in such plants, sequence dependent setup time exists between manufacturing parts involves the changing of heavy dies;

(c) Chemical compounds manufacturing, where the extent of the cleansing depends on both the chemical most recently processed and the chemical about to be processed;

(d) Printing industry, where the cleaning and setting of the press for processing the next job depend on its difference from the colour of ink, size of paper and types used in the previous job;

The case of sequence dependent setups can be found in numerous other industrial systems also, like pharmaceutical, die changing, automobile industry and roll slitting in the paper industry.

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. Renting of machines is an affordable and quick solution for an industrial setup, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows up gradation to new technology.

3. ASSUMPTIONS

1. All the jobs and machines are available at the beginning of the processing.
2. Pre-emption of jobs is not allowed.
3. Machines never breakdown and are available throughout the scheduling process.
4. Processing time on the machines are deterministic, finite and independent of sequence of the jobs to be processed.
5. Each job is processed through each of the machine once and only once. A job is not available to the next machine until and unless processing on the current machine is completed.

4. NOTATIONS

S : Sequence of jobs 1, 2, 3 … n

S_l : Sequence obtained by applying Johnson's procedure, l = 1, 2 , 3, …

M_k : Machine k, $k = 1, 2$

M : Minimum makespan

$a_{i,k}$: Processing time of i^{th} attribute on machine M_k

$p_{i,k}$: Probability associated to the processing time $a_{i,k}$

$A_{i,k}$: Expected processing time of i^{th} attribute on machine M_k

J_i : i^{th} job, $i = 1, 2, 3 \ldots n$

$S_{ij,k}$: Setup time if job i is processed immediately after job j on k^{th} machine

$L_k(S)$: The latest time when machine M_k is taken on rent for sequence S

$t_{ij,k}(S)$: Completion time of i^{th} job processed immediately after j^{th} job for sequence S on machine M_k

$t'_{ij,k}$: Completion time of i^{th} job processed immediately after j^{th} job for sequence S on machine M_k when machine M_k start processing jobs at time $L_k(S)$

$I_{i,k}(S)$: Idle time of machine M_k for job i in the sequence S

$U_k(S)$: Utilization time for which machine M_k is required, when M_k starts processing jobs at time $L_k(S)$

$R(S)$: Total rental cost for the sequence S_j of all machine

C_i : Rental cost of i^{th} machine

5. RENTAL POLICY

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e., the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on 1^{st} machine and is in ready mode for processing 1^{st} job.

Definition 5.1: Completion time of i^{th} job processed immediately after j^{th} job for sequence S on machine M_k is defined as:

$$t_{ij,k} = \max(t_{i-1,k}, t_{i,k-1}) + a_{i,k} \times p_{i,k} + S_{ij,k} \quad \text{for } k \geq 2.$$

$$= \max(t_{i-1,k}, t_{i,k-1}) + A_{i,k} + S_{ij,k}, \quad \text{where,}$$

$A_{i,k}$ = Expected processing time of i^{th} attribute on k^{th} machine for a particular job say J_n.

$S_{ij,k}$ = Setup time if i^{th} job processed immediately after j^{th} job on machine M_k

Definition 5.2: Completion time if i^{th} job processed immediately after j^{th} job on machine M_k at time L_k is defined as

$$t'_{i,k} = L_k + \sum_{q=1}^{i} A_{q,k} + \sum_{r=1}^{i-1} S_{rj,k} = \sum_{q=1}^{i} I_{q,k} + \sum_{q=1}^{i} A_{q,k} + \sum_{r=1}^{i-1} S_{rj,k} \,,$$

Also, $t'_{i,k} = \max(t_{i-1,k}, t'_{i,k-1}) + A_{i,k} + S_{ij,k}$.

6. PROBLEM FORMULATION

Let some job J_i ($i = 1, 2 \ldots n$) are to be processed on two machines M_k ($k = 1, 2$) under the specified rental policy P. Let there are n attributes of jobs on Machine M_1 and m attributes of jobs are there on machine M_2. Let $a_{j,k}$ be the processing time of j^{th} attribute on k^{th} machine with probabilities $p_{j,k}$. Let $A_{j,k}$ be the expected processing time and $S_{ij,k}$ be the setup time if job i is processed immediately after job j on machine k. Our aim is to find the sequence $\{S\}$ of the jobs which minimize the rental cost of the machines while minimizing total elapsed time.

The mathematical model of the problem in matrix form can be stated as:

Table 1
Attributes of Jobs

		Machine M_2						
		1	2	3	–	j	–	m
	1	J_1	–	J_2	–	J_3	–	–
	2	–	J_4	–	–	–	–	J_5
	3	–	–	J_6	–	–	–	–
Machine M_1	–	–	–	–	–	–	–	–
	i	–	–	–	–	J_i	–	–
	–	–	–	–	–	–	–	–
	n	J_{n-1}	–	–	–	–	–	J_n

Each job is characterized by its first attribute (row) on the first machine and second attribute (column) on the second machine.

The processing time of attributes with their corresponding probabilities on two machines $M_1$and M_2 are as shown in Table 2.

Table 2
Processing Times of Attributes

		Machine M_1		*Machine* M_2	
	1	$a_{1,1}$	$p_{1,1}$	$a_{2,1}$	$p_{2,1}$
	2	$a_{1,2}$	$p_{1,2}$	$a_{2,2}$	$p_{2,2}$
	3	$a_{1,3}$	$p_{1,3}$	$a_{3,2}$	$p_{3,2}$
Attributes	–	–	–	–	–
	m	$a_{1,m}$	$p_{1,m}$	$a_{m,2}$	$p_{m,2}$
	–	–	–	–	–
	n	$a_{1,n}$	$p_{1,n}$	–	–

The setup times for various attributes on machine M_1 is as shown in Table 3.

Table 3
Setup Times on Machine M_1

		Attributes						
		1	2	3	–	j	–	n
	1	–	$S_{12,1}$	$S_{13,1}$	–	$S_{1j,1}$	–	$S_{1n,1}$
	2	$S_{21,1}$	–	$S_{23,1}$	–	$S_{2j,1}$	–	$S_{2n,1}$
	3	$S_{31,1}$	$S_{32,1}$	–	–	$S_{3j,1}$	–	$S_{3n,1}$
Attributes	–	–	–	–	–	–	–	–
	i	$S_{i1,1}$	$S_{i2,1}$	$S_{i3,1}$	–	–	–	$S_{in,1}$
	–	–	–	–	–	–	–	–
	n	$S_{n1,1}$	$S_{n2,1}$	–	–	$S_{nj,1}$	–	–

(If the attribute in row i is processed immediately after the attribute in column j)
The setup times for various attributes on machine M_2 is as shown in Table 4.

Table 4
Setup Times on Machine M_2

		Attributes						
		1	2	3	–	j	–	m
	1	–	$S_{12,2}$	$S_{13,2}$	–	$S_{1j,2}$	–	$S_{1m,2}$
	2	$S_{21,2}$	–	$S_{23,2}$	–	$S_{2j,2}$	–	$S_{2m,2}$
	3	$S_{31,2}$	$S_{32,2}$	–	–	$S_{3j,2}$	–	$S_{3m,2}$
Attributes	–	–	–	–	–	–	–	–
	i	$S_{i1,2}$	$S_{i2,2}$	$S_{i3,2}$	–	–	–	$S_{im,2}$
	–	–	–	–	–	–	–	–
	m	$S_{m1,2}$	$S_{m2,2}$	–	–	$S_{mj,2}$	–	–

(If the attribute in row i is processed immediately after the attribute in column j)

Mathematically, the problem can be stated as

Minimize $U_k(S)$ and

Minimize $R(S_i) = t_{n,1} \times C_1 + U_k(S_i) \times C_2$

Subject to constraint: Rental Policy (P)

7. THEOREM

The processing of jobs on M_2 at time $L_2 = \sum_{i=1}^{n} I_{i,2}$ keeps $t_{n,2}$ unaltered:

Proof: Let $t'_{nj,2}$ be the completion time of n^{th} job processed immediately after j^{th} job when M_2 starts processing of jobs at L_2. We shall prove the theorem with the help of mathematical induction.

Let $P(n) : t'_{nj,2} = t_{nj,2}$.

Basic step: For $n = 1$

$$t'_{1j,2} = L_2 + \sum_{i=1}^{1} A_{i,2} + \sum_{i=1}^{1-1} S_{ij,2} = \sum_{i=1}^{1} I_{i,2} + \sum_{i=1}^{1} A_{i,2} + \sum_{i=1}^{1-1} S_{ij,2}$$

$$= \sum_{i=1}^{1} I_{i,2} + A_{1,2} = I_{1,2} + A_{1,2} = A_{1,1} + A_{1,2} = t_{1j,2}$$

$\therefore$ $P(1)$ is true.

Induction Step: Let $P(m)$ be true, i.e., $t'_{mj,2} = t_{mj,2}$.

Now, we shall show that $P(m+1)$ is also true, i.e., $t'_{(m+1)j,2} = t_{(m+1)j,2}$.

Since,

$$t'_{(m+1)j,2} = \max\left(t_{(m+1)j,1}, t'_{m,2}\right) + A_{m+1,2} + S_{mj,2}$$

$$= \max\left(t_{(m+1)j,1}, L_2 + \sum_{i=1}^{m} A_{i,2} + \sum_{i=1}^{m-1} S_{ij,2}\right) + A_{m+1,2} + S_{mj,2}$$

$$= \max\left(t_{(m+1)j,1}, \left(\sum_{i=1}^{m} I_{i,2} + \sum_{i=1}^{m} A_{i,2} + \sum_{i=1}^{m-1} S_{ij,2}\right) + I_{m+1}\right) + A_{m+1,2} + S_{mj,2}$$

$$= \max\left(t_{(m+1)j,1}, t_{mj,2} + I_{m+1}\right) + A_{m+1,2} + S_{mj,2}$$

$$= \max\left(t_{(m+1)j,1}, t'_{mj,2} + \max\left(\left(t_{(m+1)j,1} - t_{mj,2}\right), 0\right)\right) + A_{m+1,2} + S_{mj,2} \quad (\because t'_{mj,2} = t_{mj,2})$$

$$= \max(t_{(m+1)j,1}, t_{mj,2}) + A_{m+1,2} + S_{mj,2}$$

$$= t_{(m+1)j,2}.$$

Therefore, $P(m+1)$ is true whenever $P(m)$ is true.

Hence, by the principle of mathematical induction $P(n)$ is true for all n i.e., $t'_{nj,2} = t_{nj,2}$ for all n.

Remark: If M_2 starts processing the job at $L_2 = t_{nj,2} - \sum_{i=1}^{n} A_{i,2} - \sum_{i=1}^{n-1} S_{ij,2}$, then total time elapsed $t_{nj,2}$ is not altered and M_2 is engaged for minimum time. If M_2 starts processing the jobs at time L_2 then $t_{nj,2} = L_2 + \sum_{i=1}^{n} A_{i,2} + \sum_{i=1}^{n-1} S_{ij,2}$.

8. ALGORITHM

The following algorithm is proposed to optimize the bicriteria in two stage flowshop scheduling in which the processing times are associated with probabilities under sequence dependent setup time. The bicriteria problem addressed in this research can be referred to as $F_2 / S_{sd} / R(S), C_{\max}$.

Step 1: Calculate the expected processing times of the given attributes on two machines M_1 and M_2 as follows $A_{i,j} = a_{i,j} \times p_{i,j} \ \forall\, i, j$.

Step 2: Using Johnson's technique (1954), obtain the sequences S_k having minimum total elapsed time. Let these be sequences be $S_1, S_2, \ldots$.

Step 3: Compute total elapsed time $t_{n,2}(S_l)$, $l = 1, 2, 3, \ldots$, for second machine by preparing in-out tables for sequence S_l.

Step 4: Compute $L_2(S_l)$ for each sequence S_l as $L_2 = t_{n,2} - \sum_{i=1}^{n} A_{i,2} - \sum_{i=1}^{n-1} S_{ij,2}$.

Step 5: Find utilization time of 2nd machine for each sequence S_l as $U_2(S_l) = t_{n,2}(S_l) - L_2(S_l)$.

Step 6: Find minimum of $\{U_2(S_l)\}$; $l = 1, 2, 3, \ldots$.

Let it for sequence S_p. Then S_p is the optimal sequence and minimum rental cost for the sequence S_p is $R(S_p) = t_{n,1}(S_p) \times C_1 + U_2(S_p) \times C_2$.

9. NUMERICAL ILLUSTRATION

Consider a two stage furniture production system where each stage represents a machine. At stage one, sheets of raw materials (MDF, DDF, Plywood, Plyboard etc.) are cut and subsequently painted in the second stage according to the market demand. The painted pieces are then assembled on an assembly line and delivered to the customers. A setup change over is needed in cutting department when the thickness of two successive jobs differs substantially. In the painting department, a setup is required when the colour of two successive jobs changes. The setup times are sequence dependent. Further the machines M_1 and M_2 are taken on rent under rental policy P.

Consider an instance consisting of seven jobs which are processed on two machines. On the first machine, there are four different attributes while the second machine is capable of handing six attributes. The attributes, processing times as well as setup times on the first and second machine are shown in Tables 5, 6, 7 and 8 respectively.

Table 5
Attributes of Jobs

		Machine M_2					
		1	2	3	4	5	6
Machine M_1	1	–	–	–	J_1	J_2	–
	2	–	J_3	–	J_4	–	–
	3	–	–	J_5	–	–	–
	4	J_7	–	–	J_6	–	–

Table 6
Processing Times of Attributes with Probabilities

		Machine M_1		Machine M_2	
Attributes	1	12	0.2	15	0.2
	2	10	0.4	8	0.2
	3	11	0.3	20	0.1
	4	24	0.1	6	0.2
	5	–	–	13	0.2
	6	–	–	30	0.1

Table 7
Setup Times on Machine M_1

		Attributes			
		1	2	3	4
Attributes	1	–	1	2	1
	2	2	–	1	2
	3	1	2	–	3
	4	3	4	2	–

(If the attribute in row i is processed immediately after the attribute in column j)

Table 8
Setup Times on Machine M_2

		Attributes					
		1	2	3	4	5	6
Attributes	1	–	3	1	1	1	2
	2	2	–	4	3	3	6
	3	3	4	–	2	1	2
	4	2	2	3	–	2	6
	5	3	1	2	1	–	4
	6	7	2	8	6	5	–

(If the attribute in row i is processed immediately after the attribute in column *j*)

Let the rental cost per unit for the Machines M_1 and M_2 be 8 units and 10 units respectively. Our objective is to find the sequence of jobs processing with minimum possible rental cost, when the machines are taken on rent under rental policy *P*.

Solution: *As per Step 1*: The expected processing times of the two machines for the possible attributes are

Table 9
Expected Processing Times of Attributes

		Machine M_1	*Machine M_2*
Attributes	1	2.4	3.0
	2	4.0	1.6
	3	3.3	2.0
	4	2.4	1.2
	5	–	2.6
	6	–	3.0

As per Step 2: Using Johnson's technique [1], the sequence S_p having minimum total elapsed time is $S_p = J_7 - J_6 - J_2 - J_5 - J_3 - J_4 - J_1$.

The In-Out flow table of jobs for the sequence $S_p = J_7 - J_6 - J_2 - J_5 - J_3 - J_4 - J_1$ is

Table 10
In-Out Flow Table of Jobs for Ssequence S_p

	Machine M_1	*Machine M_2*
Jobs	In – Out	In – Out
J_7	0.0 – 2.4	2.4 – 5.4
J_6	2.4 – 4.8	7.4 – 8.6
J_2	5.8 – 8.2	9.6 – 12.2
J_5	9.2 – 12.5	13.2 – 15.2
J_3	13.5 – 17.5	19.2 – 20.8
J_4	17.5 – 21.5	22.8 – 24.0
J_1	22.5 – 24.9	24.9 – 26.1

Therefore, Total elapsed time $t_{n,2}(S_p) =$ 26.1 units

The latest time at which Machine M_2 should be taken on rent is

$$L_2(S_p) = t_{n,2}(S_p) - \sum_{q=1}^{n} A_{q,2}(S_p) - \sum_{j=1}^{n-1} S_{ij,2}(S_p)$$

$$= 26.1 - 12.8 - 10 = 3.3 \text{ units.}$$

Therefore, the utilization time of Machine M_2 is

$$L_2(S) = t_{n,2}(S_p) - L_2(S_p)$$

$$= 26.1 - 3.3 = 22.8 \text{ units.}$$

The bi-objective In-Out flow table for the sequence S_p of jobs is

Table 11
Bi-Objective In-Out Flow Table of Jobs for Sequence S_p

	Machine M_1	*Machine M_2*
Jobs	In – Out	In – Out
J_7	0.0 – 2.4	3.3 – 6.3
J_6	2.4 – 4.8	8.3 – 9.5
J_2	5.8 – 8.2	10.5 – 13.1
J_5	9.2 – 12.5	14.1 – 16.1
J_3	13.5 – 17.5	20.1 – 21.7
J_4	17.5 – 21.5	23.7 – 24.9
J_1	22.5 – 24.9	24.9 – 26.1

Total Minimum Rental Cost = $R(S_p) = t_{n,1}(S_p) \times C_1 + U_2(S_p) \times C_2 = 427.2$ units.

10. CONCLUSION

If the machine M_2 is taken on rent when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at time $L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} A_{i,2}(S) - \sum_{i=1}^{n-1} S_{ij,2}(S)$ on M_2 will, reduce its utilization time. Therefore total rental cost of M_2 will be minimum. Also rental cost of M_1 will always be minimum as idle time of M_1 is minimum always due to our rental policy. The study may further be extending by introducing the concept of transportation time, Weightage of jobs, Breakdown Interval etc.

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