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AN APPROACH TO CRISPIFY THE INTUITIONISTIC FUZZY NUMBERS

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Abstract

In many problems the observed values of the variables are not exact yet the variables must satisfy a set of rigid relationship and each value is modified until they satisfy the relationship. In this paper we have presented a method that can be used to adjust the values of the variables in many engineering and mathematical problem. The approached method is to find the most appropriate set of crisp numbers. Here we have considered the approximate observed values as P-norm Generalized Trapezoidal Intuitionistic Fuzzy Numbers. As an application, a transportation problem and circuit system has been numerically illustrated.

Keywords: *Intuitionistic Fuzzy Number (IFN); Genaralized Trapezoidal Intuitionistic Fuzzy Number (GTrIFN); p-norm Generalized Trapezoidal Intuitionistic Fuzzy Number*$(GTrIFN)_p$*; Transportation Problem; Circuit System.*

1. INTRODUCTION

In many practical engineering problems we come across in a situation in which the observed set of values are approximate and they are required to be adjusted so that they satisfy one or more relationship between them. Such an adjustment is normally done by trial and error method.

Kikuchi [9] proposed a method that adjusts the approximate observed values in the case of above problems by considering the observed values as Triangular Fuzzy Number and used fuzzy linear programming to find the true value. De, Yadav [8] considered the observed values as Trapezoidal Fuzzy Number in their paper. In the present paper we have described the generalized form of Kikuchi's

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method and have assumed all the observed values as P-norm Generalized Trapezoidal Intuitionistic Fuzzy Numbers and it employs the fuzzy optimization approach.

Here the objective is to find the set of values such that the smallest membership grade among them is maximized and largest non-membership grade among them is minimized. The paper presents the approach, optimization problem formulation with realistic application example [9] in transportation engineering and a circuit system.

2. PRELIMINARY CONCEPTS

Definition 2.1: Intuitionistic Fuzzy Set: Let $U = \{x_1, x_2, \dots, x_n\}$ be a finite universal set. An Intuitionistic Fuzzy Set $\tilde{A}^i$ in a given universal set U is an object having the form

$$\tilde{A}^i = \left\{\langle x_i, \mu_{\tilde{A}^i}(x_i), \upsilon_{\tilde{A}^i}(x_i)\rangle : x_i \in U\right\}$$

Where the functions

$$\mu_{\tilde{A}^i}: U \to [0,1]; \text{ i.e., } x_i \in U \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$$

and

$$\upsilon_{\tilde{A}^i}: U \to [0,1]; \text{ i.e., } x_i \in U \to \upsilon_{\tilde{A}^i}(x_i) \in [0,1]$$

define the degree of membership and the degree of non-membership of an element $x_i \in U$, such that they satisfy the following conditions:

$$0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1, \forall x_i \in U$$

which is known as Intuitionistic Condition. The degree of acceptance $\mu_{\tilde{A}^i}(x_i)$and of non-acceptance $\upsilon_{\tilde{A}^i}(x_i)$ can be arbitrary.

Definition 2.2: $(\boldsymbol{\alpha}, \boldsymbol{\beta})$-cuts: A set of (α, β)-cut, generated by IFS $\tilde{A}^i$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$$\tilde{A}^i{}_{\alpha,\beta} = \begin{cases} \left(x, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x)\right);\ x \in U \\ \mu_{\tilde{A}^i}(x) \ge \alpha, \upsilon_{\tilde{A}^i}(x) \le \beta;\ \alpha, \beta \in [0,1] \end{cases},$$

where, (α, β)-cut, denoted by $\tilde{A}^i{}_{\alpha,\beta}$, is defined as the crisp set of elements x which belong to $\tilde{A}^i$ at least to the degree α and which does belong to $\tilde{A}^i$ at most to the degree β.

Definition 2.3: Intuitionistic Fuzzy Number (IFN): An Intuitionistic Fuuzy Number $\tilde{A}^i$ is

An Intuitionistic Fuuzy Subset on the real line

Normal i.e. there exists at least one $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}^i}(x_0) = 1$ (so $\upsilon_{\tilde{A}^i}(x_0) = 0$)

Convex for the membership function $\mu_{\tilde{A}^i}$ i.e.

$$\mu_{\tilde{A}^i}\left(\lambda x_1 + (1 - \lambda x_2)\right) \geq \min\left\{\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)\right\}; \forall\, x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$$

Concave for the non-membership function $\upsilon_{\tilde{A}^i}$ i.e.

$$\upsilon_{\tilde{A}^i}\left(\lambda x_1 + (1 - \lambda x_2)\right) \leq \max\left\{\upsilon_{\tilde{A}^i}(x_1), \upsilon_{\tilde{A}^i}(x_2)\right\}; \forall\, x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$$

Definition 2.4: Genaralized Trapezoidal Intuitionistic Fuzzy Number: A Generalized Trapezoidal Intuitionistic Fuzzy Number (GTrIFN) is denoted by $\tilde{A}^i = \langle (a_1, a_2, a_3, a_4; w), (a_1', a_2, a_3, a_4'; u) \rangle$ is a special Intuitionistic Fuzzy Set on a real number set $\mathbb{R}$, whose membership function and non-membership function are defined as:

$$\mu_{\tilde{A}^i}(x) = \begin{cases} 0\ x \leq a_1 \\ w\frac{x-a_1}{a_2-a_1}, a_1 \leq x \leq a_2 \\ w, a_2 \leq x \leq a_3 \\ w\frac{a_4-x}{a_4-a_3}, a_3 \leq x \leq a_4 \\ 0, a_4 \leq x \end{cases},$$

$$\upsilon_{\tilde{A}^i}(x) = \begin{cases} 1\ x \leq a_1' \\ 1-(1-u)\frac{x-a_1'}{a_2-a_1'}, a_1' \leq x \leq a_2 \\ u, a_2 \leq x \leq a_3 \\ 1-(1-u)\frac{a_4'-x}{a_4'-a_3}, a_3 \leq x \leq a_4' \\ 1, a_4' \leq x \end{cases}$$

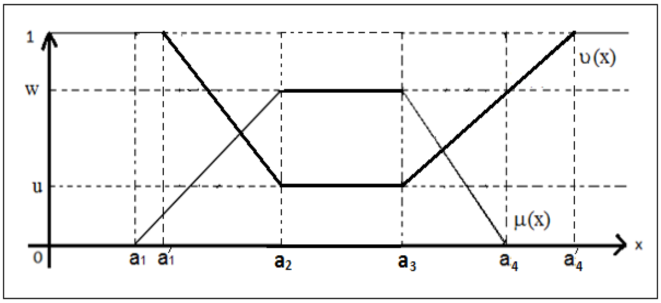


Figure 1: Rough sketch of membership function and non-membership function of GTrIFN

where, w and u represent the maximum degree of membership and minimum degree of non-membership respectively such that they satisfy the condition.

$$0 \le w \le 1, 0 \le u \le 1, 0 \le w + u \le 1$$

And $$a_1{}' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4{}'$$

If $a_2 = a_3$ then Generalized Trapezoidal Intuitionistic Fuzzy Number (GTrIFN) is transformed into Generalized Triangular Intuitionistic Fuzzy Number (GTIFN) $(\langle(a_1, a_2, a_3,; w), (a_1{}', a_2, a_3{}'; u)\rangle)$

Definition 2.5: P-norm Generalized Trapezoidal Intuitionistic Fuzzy Number: A P-norm Generalized Trapezoidal Intuitionistic Fuzzy Number$(\text{GTrIFN})_p$ is denoted by $\tilde{A}_p^i = \langle(a_1, a_2, a_3, a_4; w), (a_1{}', a_2, a_3, a_4{}'; u)\rangle_p$ is a special Intuitionistic Fuzzy Set on a real number set ℝ, whose membership function and non-membership functions are defined as

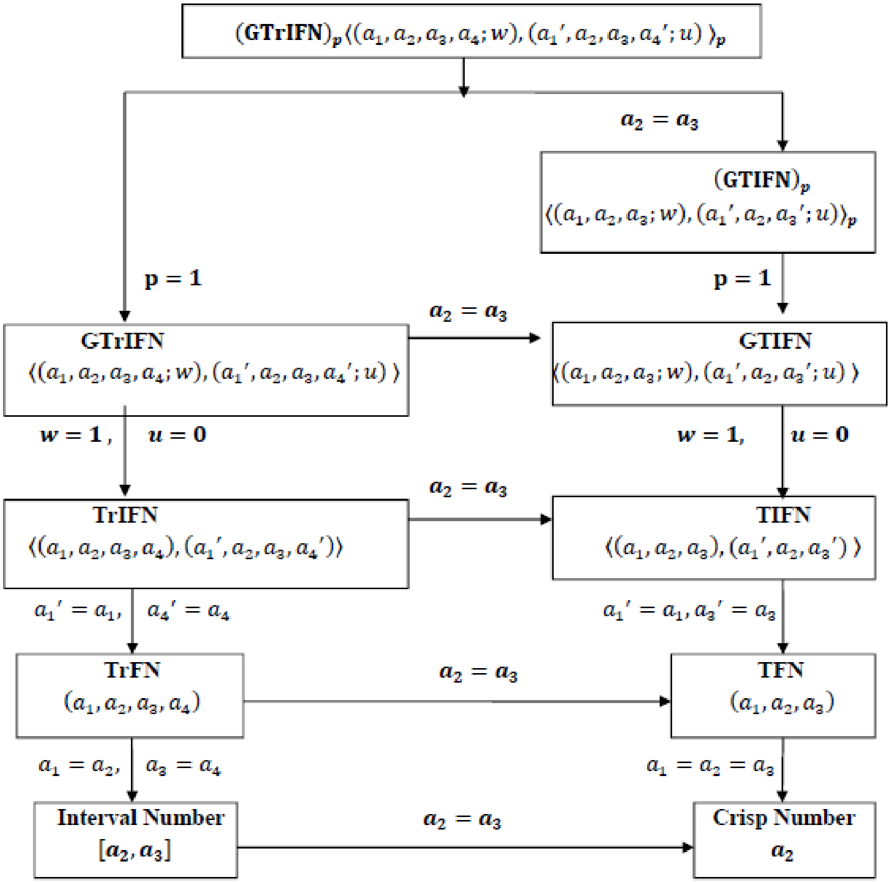
$$\mu_{\tilde{A}^i}(x) = \begin{cases} 0 \; x \le a_1 \\ w\left[1 - \left(\frac{a_2 - x}{a_2 - a_1}\right)^p\right]^{1/p}, a_1 \le x \le a_2 \\ w, a_2 \le x \le a_3 \\ w\left[1 - \left(\frac{x - a_3}{a_4 - a_3}\right)^p\right]^{1/p}, a_3 \le x \le a_4 \\ 0, a_4 \le x \end{cases},$$

$$\upsilon_{\tilde{A}^i}(x) = \begin{cases} 1 \; x \le a_1{}' \\ u\left[1 + \frac{1-u^p}{u^p}\left(\frac{a_2 - x}{a_2 - a_1{}'}\right)^p\right]^{1/p}, a_1{}' \le x \le a_2 \\ u, a_2 \le x \le a_3 \\ u\left[1 + \frac{1-u^p}{u^p}\left(\frac{x - a_3}{a_4{}' - a_3}\right)^p\right]^{1/p}, a_3 \le x \le a_4{}' \\ 1, a_4{}' \le x \end{cases}$$

here *w* and *u* represent the maximum degree of membership and minimum degree of non membership respectively, satisfying $0 \le w \le 1, 0 \le u \le 1, 0 \le w + u \le 1$. Also $a_1{}' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4{}'$ and p is a positive integer.

Definition 2.6: A $(\text{GTrIFN})_p$ $\tilde{A}_p^i = \langle(a_1, a_2, a_3, a_4; w), (a_1{}', a_2, a_3, a_4{}'; u)\rangle_p$ is said to be positive if and only if $a_1{}' \ge 0$ and is said to be negative if $a_1{}' \le 0$ and is equal to zero if all the values$a_1{}', a_1, a_2, a_3, a_4, a_4{}'$ are zero

Definition 2.7: A flowchart containing the relationship between different Intuitionistic Fuzzy Numbers is shown below:



Definition 2.8: Arithmetic Operations:

The arithmetic operations of $(\text{GTrIFN})_p$ are defined as follows:

Let $\tilde{A}_p^i = \langle(a_1,a_2,a_3,a_4;w_a),(a_1',a_2,a_3,a_4';u_a)\rangle_p$ and $\tilde{B}_p^i = \langle(b_1,b_2,b_3,b_4;w_b),\ (b_1',b_2,b_3,b_4';u_b)\rangle_p$ be two $(\text{GTrIFN})_p$. Then

1. $\tilde{A}_p^i + \tilde{B}_p^i = \langle(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4;\ \min(w_a,w_b)),(a_1'+b_1',a_2+b_2,a_3+b_3,a_4'+b_4';\ \max(u_a,u_b)\rangle_p$

2. $\tilde{A}_p^i - \tilde{B}_p^i = \langle(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1;\min(w_a,w_b)),(a_1'-b_4',a_2-b_3,a_3-b_2,a_4'-b_1';\ \max(u_a,u_b)\rangle_p$

3. $$\lambda\tilde{A}_p^i = \begin{cases}\left\langle\begin{matrix}(\lambda a_1,\lambda a_2,\lambda a_3,\lambda a_4;w_a),\\(\lambda a_1',\lambda a_2,\lambda a_3,\lambda a_4';u_a)\end{matrix}\right\rangle_p & for\ \lambda>0\\ \left\langle\begin{matrix}(\lambda a_4,\lambda a_3,\lambda a_2,\lambda a_1;w_a),\\(\lambda a_4',\lambda a_3,\lambda a_2,\lambda a_1';u_a)\end{matrix}\right\rangle_p & for\ \lambda<0\end{cases}$$

4. $\tilde{A}_p^i . \tilde{B}_p^i$

$$\approx \langle \begin{matrix} (a_1b_1, a_2b_2, a_3b_3, a_4b_4; \min(w_a, w_b)), \\ (a_1'b_1', a_2b_2, a_3b_3, a_4'b_4'; \max(u_a, u_b)) \end{matrix} \rangle_p \text{ if } \tilde{A}_p^i > 0, \tilde{B}_p^i > 0.$$

$$\approx \langle \begin{matrix} (a_1b_4, a_2b_3, a_3b_2, a_4b_1; \min(w_a, w_b)), \\ (a_1'b_4', a_2b_3, a_3b_2, a_4'b_1'; \max(u_a, u_b)) \end{matrix} \rangle_p \text{ if } \tilde{A}_p^i < 0, \tilde{B}_p^i > 0.$$

$$\approx \langle \begin{matrix} (a_4b_1, a_3b_2, a_2b_3, a_1b_4; \min(w_a, w_b)), \\ (a_4'b_1', a_3b_2, a_2b_3, a_1'b_4'; \max(u_a, u_b)) \end{matrix} \rangle_p \text{ if } \tilde{A}_p^i > 0, \tilde{B}_p^i < 0.$$

$$\approx \langle \begin{matrix} (a_4b_4, a_3b_3, a_2b_2, a_1b_1; \min(w_a, w_b)), \\ (a_4'b_4', a_3b_3, a_2b_2, a_1'b_1'; \max(u_a, u_b)) \end{matrix} \rangle_p \text{ if } \tilde{A}_p^i < 0, \tilde{B}_p^i < 0.$$

5. $$\frac{\tilde{A}_p^i}{\tilde{B}_p^i} \approx \langle \begin{matrix} \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}; \min(w_a, w_b)\right), \\ \left(\frac{a_1'}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4'}{b_1'}; \max(u_a, u_b)\right) \end{matrix} \rangle_p \text{ if } \tilde{A}_p^i > 0, \tilde{B}_p^i > 0.$$

3. THE APPROACH

The approach is the following:

Here we discuss the approach is the case of $n + 1$ observed values.

Suppose the problem is to

Find $\tilde{X}_r^i, \tilde{Y}^i, r = 1,2, \dots, n$

Such that $\tilde{Y}^i = f\left(\tilde{X}_1^i, \tilde{X}_2^i, \dots, \tilde{X}_n^i\right)$

$$\tilde{X}_r^i, \tilde{Y}^i \geq 0, r = 1,2, \dots, n$$

Let us consider all observed values as Intuitionistic Fuzzy Sets.

i.e. $\tilde{X}_r^i = \langle x_r, \mu_{\tilde{X}_r^i}(x_r), \upsilon_{\tilde{X}_r^i}(x_r) \rangle, r = 1,2, \dots, n$ and $\tilde{Y}^i = \langle y, \mu_{\tilde{Y}^i}(y), \upsilon_{\tilde{Y}^i}(y) \rangle$

Here the membership function of each value is $\mu_{\tilde{X}_1^i}(x_1),\ \mu_{\tilde{X}_2^i}(x_2), \dots, \mu_{\tilde{X}_n^i}(x_n)$ and $\mu_{\tilde{Y}^i}(y)$ and non membership function of each value is $\upsilon_{\tilde{X}_1^i}(x_1), \upsilon_{\tilde{X}_2^i}(x_2), \dots,$ $\upsilon_{\tilde{X}_n^i}(x_n)$ and $\upsilon_{\tilde{Y}^i}(y)$.

Then the problem becomes

Find $x_r, y, r = 1,2, \dots, n$ where

$$\text{Max Min}\left(\mu_{\tilde{X}_1^i}(x_1),\ \mu_{\tilde{X}_2^i}(x_2), \dots, \mu_{\tilde{X}_n^i}(x_n), \mu_{\tilde{Y}^i}(y)\right)$$

$$\text{Min Max}\left(\upsilon_{\tilde{X}_1^i}(x_1), \upsilon_{\tilde{X}_2^i}(x_2), \dots, \upsilon_{\tilde{X}_n^i}(x_n), \upsilon_{\tilde{Y}^i}(y)\right)$$

Such that $y = f(x_1, x_2, \dots, x_n)$

$$x_1, x_2, \dots, x_n, y \geq 0$$

Now if we let $\alpha = \text{Min}\left(\mu_{\tilde{X}_1^i}(x_1), \mu_{\tilde{X}_2^i}(x_2), \dots, \mu_{\tilde{X}_n^i}(x_n), \mu_{\tilde{Y}^i}(y)\right)$

and $$\beta = \text{Max}\left(\upsilon_{\tilde{X}_1^i}(x_1), \upsilon_{\tilde{X}_2^i}(x_2), \dots, \upsilon_{\tilde{X}_n^i}(x_n), \upsilon_{\tilde{Y}^i}(y)\right)$$

Then the problem can be rewritten as

Find $x_1, x_2, \dots, x_n, y, \alpha, \beta$ where

Max α

Min β

Such that $y = f(x_1, x_2, \dots, x_n)$

$$\mu_{\tilde{X}_1^i}(x_1) \geq \alpha, \mu_{\tilde{X}_2^i}(x_2) \geq \alpha, \dots, \mu_{\tilde{X}_n^i}(x_n) \geq \alpha,$$

$$\mu_{\tilde{Y}^i}(y) \geq \alpha$$

$$\upsilon_{\tilde{X}_1^i}(x_1) \leq \beta, \upsilon_{\tilde{X}_2^i}(x_2) \leq \beta, \dots, \upsilon_{\tilde{X}_n^i}(x_n) \leq \beta,$$

$$\upsilon_{\tilde{Y}^i}(y) \leq \beta$$

$$x_1, x_2, \dots, x_n, y \geq 0$$

$$0 \leq \alpha, \beta \leq 1 \text{ and } 0 \leq \alpha + \beta \leq 1$$

Let the left and right hand sides of each membership function be $\mu_{\tilde{X}_r^i{}^L}(x_r), r = 1,2, \dots, n, \mu_{\tilde{Y}^i{}^L}(y)$ and $\mu_{\tilde{X}_r^i{}^R}(x_r), r = 1,2, \dots, n,$ $\mu_{\tilde{Y}^i{}^R}(y)$

and the left and right hand sides of each non-membership function be $\upsilon_{\tilde{X}_r^i{}^L}(x_r), r = 1,2, \dots, n, \upsilon_{\tilde{Y}^i{}^L}(y)$ and $\upsilon_{\tilde{X}_r^i{}^R}(x_r), r = 1,2, \dots, n,$ $\upsilon_{\tilde{Y}^i{}^R}(y)$

then we can write the above problem as

Find $x_1, x_2, \dots, x_n, y, \alpha, \beta$ where

Max α

Min β

Such that $y = f(x_1, x_2, \dots, x_n)$

$$\mu_{\tilde{X}_1^i{}^L}(x_1) \geq \alpha, \mu_{\tilde{X}_2^i{}^L}(x_2) \geq \alpha, \dots, \mu_{\tilde{X}_n^i{}^L}(x_n) \geq \alpha, \mu_{\tilde{Y}^i{}^L}(y) \geq \alpha$$

$$\mu_{\tilde{X}_1^i{}^R}(x_1) \geq \alpha, \mu_{\tilde{X}_2^i{}^R}(x_2) \geq \alpha, \dots, \mu_{\tilde{X}_n^i{}^R}(x_n) \geq \alpha, \mu_{\tilde{Y}^i{}^R}(y) \geq \alpha$$

$$\upsilon_{\tilde{X}_1^i{}^L}(x_1) \leq \beta, \upsilon_{\tilde{X}_2^i{}^L}(x_2) \leq \beta, \dots, \upsilon_{\tilde{X}_n^i{}^L}(x_n) \leq \beta, \upsilon_{\tilde{Y}^i{}^L}(y) \leq \beta$$

$$\upsilon_{\tilde{X}_1^i{}^R}(x_1) \leq \beta, \upsilon_{\tilde{X}_2^i{}^R}(x_2) \leq \beta, \dots, \upsilon_{\tilde{X}_n^i{}^R}(x_n) \leq \beta, \upsilon_{\tilde{Y}^i{}^R}(y) \leq \beta$$

$x_1, x_2, \dots, x_n, y \geq 0$

$$0 \leq \alpha, \beta \leq 1 \text{ and } 0 \leq \alpha + \beta \leq 1$$

3.1. Formulation of the Problem as Optimization Model:

The proposed formulation is formalized in the form of multi objective optimization model as

Max α

Min β

Subject to $y = f(x_1, x_2, \dots, x_n)$

$$\mu_{\tilde{X}_1^i{}^L}(x_1) \geq \alpha, \mu_{\tilde{X}_2^i{}^L}(x_2) \geq \alpha, \dots, \mu_{\tilde{X}_n^i{}^L}(x_n) \geq \alpha, \mu_{\tilde{Y}^i{}^L}(y) \geq \alpha$$

$$\mu_{\tilde{X}_1^i{}^R}(x_1) \geq \alpha, \mu_{\tilde{X}_2^i{}^R}(x_2) \geq \alpha, \dots, \mu_{\tilde{X}_n^i{}^R}(x_n) \geq \alpha, \mu_{\tilde{Y}^i{}^R}(y) \geq \alpha$$

$$\upsilon_{\tilde{X}_1^i{}^L}(x_1) \leq \beta, \upsilon_{\tilde{X}_2^i{}^L}(x_2) \leq \beta, \dots, \upsilon_{\tilde{X}_n^i{}^L}(x_n) \leq \beta, \upsilon_{\tilde{Y}^i{}^L}(y) \leq \beta$$

$$\upsilon_{\tilde{X}_1^i{}^R}(x_1) \leq \beta, \upsilon_{\tilde{X}_2^i{}^R}(x_2) \leq \beta, \dots, \upsilon_{\tilde{X}_n^i{}^R}(x_n) \leq \beta, \upsilon_{\tilde{Y}^i{}^R}(y) \leq \beta$$

$x_1, x_2, \dots, x_n, y \geq 0$

$$0 \leq \alpha, \beta \leq 1 \text{ and } 0 \leq \alpha + \beta \leq 1$$

Now we can covert the above multi objective optimization model to a single objective optimization model by using average operator as

$$\text{Max } \frac{\alpha + (1 - \beta)}{2}$$

Subject to $y = f(x_1, x_2, \dots, x_n)$

$$\mu_{\tilde{X}_1^i{}^L}(x_1) \geq \alpha, \mu_{\tilde{X}_2^i{}^L}(x_2) \geq \alpha, \dots, \mu_{\tilde{X}_n^i{}^L}(x_n) \geq \alpha, \mu_{\tilde{Y}^i{}^L}(y) \geq \alpha$$

$$\mu_{\tilde{X}_1^i{}^R}(x_1) \geq \alpha, \mu_{\tilde{X}_2^i{}^R}(x_2) \geq \alpha, \dots, \mu_{\tilde{X}_n^i{}^R}(x_n) \geq \alpha, \mu_{\tilde{Y}^i{}^R}(y) \geq \alpha$$

$$\upsilon_{\tilde{X}_1^i{}^L}(x_1) \le \beta, \upsilon_{\tilde{X}_2^i{}^L}(x_2) \le \beta, \dots, \upsilon_{\tilde{X}_n^i{}^L}(x_n) \le \beta, \upsilon_{\tilde{Y}^i{}^L}(y) \le \beta$$

$$\upsilon_{\tilde{X}_1^i{}^R}(x_1) \le \beta, \upsilon_{\tilde{X}_2^i{}^R}(x_2) \le \beta, \dots, \upsilon_{\tilde{X}_n^i{}^R}(x_n) \le \beta, \upsilon_{\tilde{Y}^i{}^R}(y) \le \beta$$

$$x_1, x_2, \dots, x_n, y \ge 0$$

$$0 \le \alpha, \beta \le 1 \text{ and } 0 \le \alpha + \beta \le 1$$

Now solving the above optimization model we can easily find the crispified values of $x_1, x_2, \dots, x_n$ and y.

4. APPLICATIONS

4.1. Example: Traffic Volume Consistency

Let us consider the case shown in Figure 2. The number of observed values (traffic volumes) is 7. For each value, a intuitionistic fuzzified range is assumed. Let $X_1, X_2, \dots, X_7$ be the obtained values in $(\text{GTrIFN})_p$, and let $x_1, x_2, \dots, x_7$, the unknown values which satisfy the following relationship must be satisfied at points P and Q; therefore, the relationships are:

at P : $x_1 + x_2 + x_3 = x_6$

at Q: $x_6 + x_4 - x_5 = x_7$

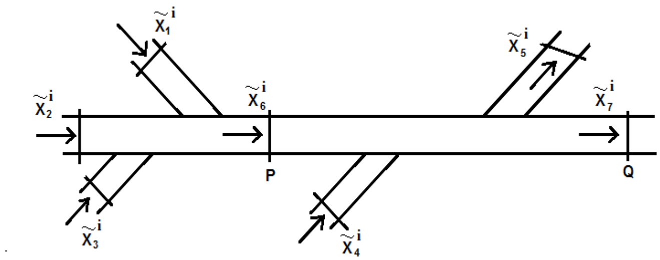


Figure 2: The described traffic volumes

Let,

$$\widetilde{X_1}^i = \langle {(2000,2500,2600,3150; 0.80), \atop (1950,2500,2600,3200; 0.15)} \rangle_p ,$$

$$\widetilde{X_2}^i = \langle {(6240,7800,8500,10100; 0.85), \atop (6200,7800,8500,10200; 0.12)} \rangle_p ,$$

$$\widetilde{X_3}^i = \langle {(960,1200,1300,1520; 0.89), \atop (950,1200,1300,1560; 0.07)} \rangle_p ,$$

$$\widetilde{X_4}^i = \langle {(4000,5000,5200,6100; 0.70), \atop (3900,5000,5200,6240; 0.26)} \rangle_p ,$$

$$\widetilde{X_5}^i = \langle {(1440,1800,1800,2000; 0.75), \atop (1400,1800,2000,2400; 0.20)} \rangle_p ,$$

$$\widetilde{X_6}^i = \langle {(10800,13500,15000,17800; 0.83), \atop (10600,13500,15000,18000; 0.14)} \rangle_p ,$$

$$\widetilde{X_7}^i = \langle {(13760,17200,19000,22500; 0.9), \atop (13700,17200,19000,22800; 0.09)} \rangle_p ,$$

where,

$$\mu_{\widetilde{X_1}^i}(x) = \begin{cases} 0, x \le 2000 \\ 0.80\left[1-\left(\frac{2500-x}{500}\right)^p\right]^{1/p}, 2000 \le x \le 2500 \\ 0.80, 2500 \le x \le 2600 \\ 0.80\left[1-\left(\frac{x-2600}{550}\right)^p\right]^{1/p}, 2600 \le x \le 3150 \\ 0, 3150 \le x \end{cases}$$

$$\upsilon_{\widetilde{X_1}^i}(x) = \begin{cases} 1, x \le 1950 \\ 0.15\left[1+\frac{1-(0.15)^p}{(0.15)^p}\left(\frac{2500-x}{550}\right)^p\right]^{1/p}, 1950 \le x \le 2500 \\ 0.15, 2500 \le x \le 2600 \\ 0.15\left[1+\frac{1-(0.15)^p}{(0.15)^p}\left(\frac{x-2600}{600}\right)^p\right]^{1/p}, 2600 \le x \le 3200 \\ 1, 3200 \le x \end{cases}$$

$$\mu_{\widetilde{X_2}^i}(x) = \begin{cases} 0, x \le 6240 \\ 0.85\left[1-\left(\frac{7800-x}{1560}\right)^p\right]^{1/p}, 6240 \le x \le 7800 \\ 0.85, 7800 \le x \le 8500 \\ 0.85\left[1-\left(\frac{x-8500}{1600}\right)^p\right]^{1/p}, 8500 \le x \le 10100 \\ 0, 10100 \le x \end{cases}$$

$$\upsilon_{\widetilde{X_2}^i}(x) = \begin{cases} 1, x \le 6200 \\ 0.12\left[1+\frac{1-(0.12)^p}{(0.12)^p}\left(\frac{7800-x}{1600}\right)^p\right]^{1/p}, 6200 \le x \le 7800 \\ 0.12, 7800 \le x \le 8500 \\ 0.12\left[1+\frac{1-(0.12)^p}{(0.12)^p}\left(\frac{x-8500}{1700}\right)^p\right]^{1/p}, 8500 \le x \le 10200 \\ 1, 10200 \le x \end{cases}$$

$$\mu_{\widetilde{X_3}^i}(x) = \begin{cases} 0, x \le 960 \\ 0.89\left[1-\left(\frac{1200-x}{240}\right)^p\right]^{1/p}, 960 \le x \le 1200 \\ 0.89, 1200 \le x \le 1300 \\ 0.89\left[1-\left(\frac{x-1300}{220}\right)^p\right]^{1/p}, 1300 \le x \le 1520 \\ 0, 1520 \le x \end{cases}$$

$$\upsilon_{\widetilde{X_3}^i}(x) = \begin{cases} 1, x \le 950 \\ 0.07\left[1+\frac{1-(0.07)^p}{(0.07)^p}\left(\frac{1200-x}{250}\right)^p\right]^{1/p}, 950 \le x \le 1200 \\ 0.07, 1200 \le x \le 1300 \\ 0.07\left[1+\frac{1-(0.07)^p}{(0.07)^p}\left(\frac{x-1300}{260}\right)^p\right]^{1/p}, 1300 \le x \le 1560 \\ 1, 1560 \le x \end{cases}$$

$$\mu_{\widetilde{X_4}^i}(x) = \begin{cases} 0, x \le 4000 \\ 0.70\left[1-\left(\frac{5000-x}{1000}\right)^p\right]^{1/p}, 4000 \le x \le 5000 \\ 0.91, 5000 \le x \le 5200 \\ 0.70\left[1-\left(\frac{x-5200}{900}\right)^p\right]^{1/p}, 5200 \le x \le 6100 \\ 0, 6100 \le x \end{cases}$$

$$\upsilon_{\widetilde{X_4}^i}(x) = \begin{cases} 1, x \le 3900 \\ 0.26\left[1+\frac{1-(0.26)^p}{(0.26)^p}\left(\frac{5000-x}{1100}\right)^p\right]^{1/p}, 3900 \le x \le 5000 \\ 0.26, 5000 \le x \le 5200 \\ 0.26\left[1+\frac{1-(0.26)^p}{(0.26)^p}\left(\frac{x-5200}{1040}\right)^p\right]^{1/p}, 5200 \le x \le 6240 \\ 1, 6240 \le x \end{cases}$$

$$\mu_{\widetilde{X_5}^i}(x) = \begin{cases} 0, x \le 1440 \\ 0.75\left[1-\left(\frac{1800-x}{360}\right)^p\right]^{1/p}, 1440 \le x \le 1800 \\ 0.75, 1800 \le x \le 2000 \\ 0.75\left[1-\left(\frac{x-2000}{300}\right)^p\right]^{1/p}, 2000 \le x \le 2300 \\ 0, 2300 \le x \end{cases}$$

$$\upsilon_{\widetilde{X_5}^i}(x) = \begin{cases} 1, x \leq 1400 \\ 0.20\left[1 + \frac{1-(0.20)^p}{(0.20)^p}\left(\frac{1800-x}{300}\right)^p\right]^{1/p}, 1400 \leq x \leq 1800 \\ 0.08, 1800 \leq x \leq 2000 \\ 0.20\left[1 + \frac{1-(0.20)^p}{(0.20)^p}\left(\frac{x-2000}{400}\right)^p\right]^{1/p}, 2000 \leq x \leq 2400 \\ 1, 2400 \leq x \end{cases}$$

$$\mu_{\widetilde{X_6}^i}(x) = \begin{cases} 0, x \leq 10800 \\ 0.83\left[1 - \left(\frac{13500-x}{2700}\right)^p\right]^{1/p}, 10800 \leq x \leq 13500 \\ 0.83, 13500 \leq x \leq 15000 \\ 0.83\left[1 - \left(\frac{x-15000}{2800}\right)^p\right]^{1/p}, 15000 \leq x \leq 17800 \\ 0, 17800 \leq x \end{cases}$$

$$\upsilon_{\widetilde{X_6}^i}(x) = \begin{cases} 1, x \leq 10600 \\ 0.14\left[1 + \frac{1-(0.14)^p}{(0.14)^p}\left(\frac{13500-x}{2900}\right)^p\right]^{1/p}, 10600 \leq x \leq 13500 \\ 0.14, 13500 \leq x \leq 15000 \\ 0.14\left[1 + \frac{1-(0.14)^p}{(0.14)^p}\left(\frac{x-15000}{3000}\right)^p\right]^{1/p}, 15000 \leq x \leq 18000 \\ 1, 18000 \leq x \end{cases}$$

$$\mu_{\widetilde{X_7}^i}(x) = \begin{cases} 0, x \leq 13760 \\ 0.90\left[1 - \left(\frac{17200-x}{3440}\right)^p\right]^{1/p}, 13760 \leq x \leq 17200 \\ 0.90, 17200 \leq x \leq 19000 \\ 0.90\left[1 - \left(\frac{x-19000}{3500}\right)^p\right]^{1/p}, 19000 \leq x \leq 22500 \\ 0, 22500 \leq x \end{cases}$$

$$\upsilon_{\widetilde{X_7}^i}(x) = \begin{cases} 1, x \leq 13700 \\ 0.09\left[1 + \frac{1-(0.09)^p}{(0.09)^p}\left(\frac{17200-x}{3500}\right)^p\right]^{1/p}, 13700 \leq x \leq 17200 \\ 0.09, 17200 \leq x \leq 19000 \\ 0.09\left[1 + \frac{1-(0.09)^p}{(0.09)^p}\left(\frac{x-19000}{3800}\right)^p\right]^{1/p}, 19000 \leq x \leq 22800 \\ 1, 22800 \leq x \end{cases}$$

The optimization model formulation is as follows: (using average operator)

$$\max \frac{\alpha + (1-\beta)}{2}$$

Subject to

$$x_1 + x_2 + x_3 = x_6$$

$$x_6 + x_4 - x_5 = x_7$$

$$0.80\left[1-\left(\frac{2500-x_1}{500}\right)^p\right]^{1/p} \geq \alpha, 0.80\left[1-\left(\frac{x_1-2600}{550}\right)^p\right]^{1/p} \geq \alpha,$$

$$0.15\left[1+\frac{1-(0.15)^p}{(0.15)^p}\left(\frac{2500-x_1}{550}\right)^p\right]^{1/p}$$

$$\leq \beta, 0.15\left[1+\frac{1-(0.15)^p}{(0.15)^p}\left(\frac{x_1-2600}{600}\right)^p\right]^{1/p} \leq \beta$$

$$0.85\left[1-\left(\frac{7800-x_2}{1560}\right)^p\right]^{1/p} \geq \alpha, 0.85\left[1-\left(\frac{x_2-8500}{1600}\right)^p\right]^{1/p} \geq \alpha,$$

$$0.12\left[1+\frac{1-(0.12)^p}{(0.12)^p}\left(\frac{7800-x_2}{1600}\right)^p\right]^{1/p}$$

$$\leq \beta, 0.12\left[1+\frac{1-(0.12)^p}{(0.12)^p}\left(\frac{x_2-8500}{1700}\right)^p\right]^{1/p} \leq \beta$$

$$0.89\left[1-\left(\frac{1200-x_3}{240}\right)^p\right]^{1/p} \geq \alpha, 0.89\left[1-\left(\frac{x_3-1300}{220}\right)^p\right]^{1/p} \geq \alpha,$$

$$0.07\left[1+\frac{1-(0.07)^p}{(0.07)^p}\left(\frac{1200-x_3}{250}\right)^p\right]^{1/p}$$

$$\leq \beta, 0.07\left[1+\frac{1-(0.07)^p}{(0.07)^p}\left(\frac{x_3-1300}{260}\right)^p\right]^{1/p} \leq \beta$$

$$0.70\left[1-\left(\frac{5000-x_4}{1000}\right)^p\right]^{1/p} \geq \alpha, 0.70\left[1-\left(\frac{x_4-5200}{900}\right)^p\right]^{1/p} \geq \alpha,$$

$$0.26\left[1+\frac{1-(0.26)^p}{(0.26)^p}\left(\frac{5000-x_4}{1100}\right)^p\right]^{1/p}$$

$$\leq \beta, 0.26\left[1+\frac{1-(0.26)^p}{(0.26)^p}\left(\frac{x_4-5200}{1040}\right)^p\right]^{1/p} \leq \beta$$

$$0.75\left[1-\left(\frac{1800-x_5}{360}\right)^p\right]^{1/p} \geq \alpha, 0.75\left[1-\left(\frac{x_5-2000}{300}\right)^p\right]^{1/p} \geq \alpha,$$

$$0.20\ \left[1+\frac{1-(0.20)^p}{(0.20)^p}\left(\frac{1800-x_5}{300}\right)^p\right]^{1/p} \leq \beta, 0.20\ \left[1+\frac{1-(0.20)^p}{(0.20)^p}\left(\frac{x_5-2000}{400}\right)^p\right]^{1/p} \leq \beta$$

$$0.83\left[1-\left(\frac{13500-x_6}{2700}\right)^p\right]^{1/p} \geq \alpha, 0.83\left[1-\left(\frac{x_6-15000}{2800}\right)^p\right]^{1/p} \geq \alpha,$$

$$0.14\ \left[1+\frac{1-(0.14)^p}{(0.14)^p}\left(\frac{13500-x_6}{2900}\right)^p\right]^{1/p} \leq \beta, 0.14\ \left[1+\frac{1-(0.14)^p}{(0.14)^p}\left(\frac{x_6-15000}{3000}\right)^p\right]^{1/p} \leq \beta$$

$$0.90\left[1-\left(\frac{17200-x_7}{3440}\right)^p\right]^{1/p} \geq \alpha, 0.90\left[1-\left(\frac{x_7-19000}{3500}\right)^p\right]^{1/p} \geq \alpha,$$

$$0.09\left[1+\frac{1-(0.09)^p}{(0.09)^p}\left(\frac{17200-x_7}{3500}\right)^p\right]^{1/p} \leq \beta, 0.09\ \left[1+\frac{1-(0.09)^p}{(0.09)^p}\left(\frac{x_7-19000}{3800}\right)^p\right]^{1/p} \leq \beta$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

$0 \leq \alpha, \beta \leq 1$ and $0 \leq \alpha+\beta \leq 1$

The crispified results (traffic volumes) are shown for different values of p:

	$p=1$	$p=2$	$p=3$	$p=4$
$\alpha \approx$	0.65	0.34	0.70	0.35
$\beta \approx$	0.31	0.66	0.26	0.65
$x_1 \approx$	2701	2855	2743	2856
$x_2 \approx$	8867	8839	8920	8837
$x_3 \approx$	1358	1363	1290	1362
$x_4 \approx$	5186	5686	5103	5256
$x_5 \approx$	1759	1835	1738	1776
$x_6 \approx$	12926	13058	12952	13056
$x_7 \approx$	16353	16909	16317	16536

4.2. Example: Equivalent Resistance of Circuit:

Let us consider the case shown in Figure 4.2. The number of observed values (resistance) is 5. For each value, a intuitionistic fuzzified range is assumed. Let $X_1, X_2, \dots, X_5$ be the obtained values in $(\text{GTrIFN})_p$, and let $x_1, x_2, \dots, x_5$, the unknown values which satisfy the following relationship must be satisfied; therefore, the relationships are

$$x_4 = \frac{x_2 . x_3}{x_2 + x_3}$$

$$x_1 + x_4 = x_5$$

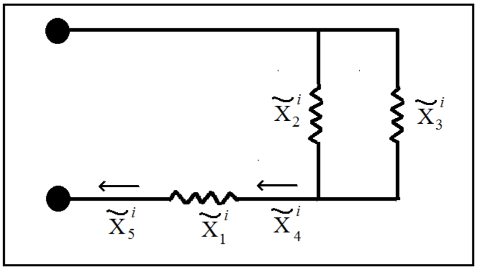


Figure 3: The described circuit system

Let,

$$\widetilde{X_1}^i = \langle {(48,60,70,80; 0.80), \atop (45,60,70,86; 0.15)} \rangle_p ,$$

$$\widetilde{X_2}^i = \langle {(320,400,450,535; 0.85), \atop (315,400,450,540; 0.12)} \rangle_p ,$$

$$\widetilde{X_3}^i = \langle {(84,120,150,175; 0.89), \atop (80,120,150,180; 0.07)} \rangle_p ,$$

$$\widetilde{X_4}^i = \langle {(80,100,120,141; 0.70), \atop (75,100,120,144; 0.26)} \rangle_p ,$$

$$\widetilde{X_5}^i = \langle {(135,170,200,236; 0.75), \atop (131,170,200,240; 0.20)} \rangle_p .$$

where,

$$\mu_{\widetilde{X_1}^i}(x) = \begin{cases} 0 \; x \le 48 \\ 0.80\left[1 - \left(\frac{60 - x}{12}\right)^p\right]^{1/p}, 48 \le x \le 60 \\ 0.80, 60 \le x \le 70 \\ 0.80\left[1 - \left(\frac{x - 70}{10}\right)^p\right]^{1/p}, 70 \le x \le 80 \\ 0, 80 \le x \end{cases}$$

$$\upsilon_{\widetilde{X_1}^i}(x)=\begin{cases} 1 \; x\le 45 \\ 0.15\left[1+\frac{1-(0.15)^p}{(0.15)^p}\left(\frac{60-x}{15}\right)^p\right]^{1/p}, 45\le x\le 60 \\ 0.15, 60\le x\le 70 \\ 0.15\left[1+\frac{1-(0.15)^p}{(0.15)^p}\left(\frac{x-70}{16}\right)^p\right]^{1/p}, 70\le x\le 86 \\ 1, 86\le x \end{cases}$$

$$\mu_{\widetilde{X_2}^i}(x)=\begin{cases} 0 \; x\le 320 \\ 0.85\left[1-\left(\frac{400-x}{80}\right)^p\right]^{1/p}, 320\le x\le 400 \\ 0.85, 400\le x\le 450 \\ 0.85\left[1-\left(\frac{x-450}{85}\right)^p\right]^{1/p}, 450\le x\le 535 \\ 0, 535\le x \end{cases}$$

$$\upsilon_{\widetilde{X_2}^i}(x)=\begin{cases} 1 \; x\le 315 \\ 0.12\left[1+\frac{1-(0.12)^p}{(0.12)^p}\left(\frac{400-x}{85}\right)^p\right]^{1/p}, 315\le x\le 400 \\ 0.12, 400\le x\le 450 \\ 0.12\left[1+\frac{1-(0.12)^p}{(0.12)^p}\left(\frac{x-450}{90}\right)^p\right]^{1/p}, 450\le x\le 540 \\ 1, 540\le x \end{cases}$$

$$\mu_{\widetilde{X_3}^i}(x)=\begin{cases} 0 \; x\le 84 \\ 0.89\left[1-\left(\frac{120-x}{36}\right)^p\right]^{1/p}, 84\le x\le 120 \\ 0.89, 120\le x\le 150 \\ 0.89\left[1-\left(\frac{x-150}{25}\right)^p\right]^{1/p}, 150\le x\le 175 \\ 0, 175\le x \end{cases}$$

$$\upsilon_{\widetilde{X_3}^i}(x)=\begin{cases} 1 \; x\le 80 \\ 0.07\left[1+\frac{1-(0.07)^p}{(0.07)^p}\left(\frac{120-x}{40}\right)^p\right]^{1/p}, 80\le x\le 120 \\ 0.07, 120\le x\le 150 \\ 0.07\left[1+\frac{1-(0.07)^p}{(0.07)^p}\left(\frac{x-150}{30}\right)^p\right]^{1/p}, 150\le x\le 180 \\ 1, 180\le x \end{cases}$$

$$\mu_{\widetilde{X_4}^i}(x) = \begin{cases} 0 \; x \leq 80 \\ 0.70\left[1-\left(\frac{100-x}{20}\right)^p\right]^{1/p}, 80 \leq x \leq 100 \\ 0.91, 100 \leq x \leq 120 \\ 0.70\left[1-\left(\frac{x-120}{21}\right)^p\right]^{1/p}, 120 \leq x \leq 141 \\ 0, 141 \leq x \end{cases}$$

$$\upsilon_{\widetilde{X_4}^i}(x) = \begin{cases} 1 \; x \leq 75 \\ 0.26\left[1+\frac{1-(0.26)^p}{(0.26)^p}\left(\frac{100-x}{25}\right)^p\right]^{1/p}, 75 \leq x \leq 100 \\ 0.26, 100 \leq x \leq 120 \\ 0.26\left[1+\frac{1-(0.26)^p}{(0.26)^p}\left(\frac{x-120}{24}\right)^p\right]^{1/p}, 120 \leq x \leq 144 \\ 1, 144 \leq x \end{cases}$$

$$\mu_{\widetilde{X_5}^i}(x) = \begin{cases} 0 \; x \leq 135 \\ 0.75\left[1-\left(\frac{170-x}{35}\right)^p\right]^{1/p}, 135 \leq x \leq 170 \\ 0.75, 170 \leq x \leq 200 \\ 0.75\left[1-\left(\frac{x-200}{36}\right)^p\right]^{1/p}, 200 \leq x \leq 236 \\ 0, 236 \leq x \end{cases}$$

$$\upsilon_{\widetilde{X_5}^i}(x) = \begin{cases} 1 \; x \leq 131 \\ 0.20\left[1+\frac{1-(0.20)^p}{(0.20)^p}\left(\frac{170-x}{39}\right)^p\right]^{1/p}, 131 \leq x \leq 170 \\ 0.08, 170 \leq x \leq 200 \\ 0.20\left[1+\frac{1-(0.20)^p}{(0.20)^p}\left(\frac{x-200}{40}\right)^p\right]^{1/p}, 200 \leq x \leq 240 \\ 1, 240 \leq x \end{cases}$$

The optimization model formulation is as follows: (using average operator)

$$\max \frac{\alpha + (1-\beta)}{2}$$

Subject to

$$x_4 = \frac{x_2 . x_3}{x_2 + x_3}$$

$$x_1 + x_4 = x_5$$

$$0.80\left[1-\left(\frac{60-x}{12}\right)^{p}\right]^{1/p} \geq \alpha, 0.80\left[1-\left(\frac{x-70}{10}\right)^{p}\right]^{1/p} \geq \alpha,$$

$$0.15\left[1+\frac{1-(0.15)^{p}}{(0.15)^{p}}\left(\frac{60-x}{15}\right)^{p}\right]^{1/p} \leq \beta, 0.15\left[1+\frac{1-(0.15)^{p}}{(0.15)^{p}}\left(\frac{x-70}{16}\right)^{p}\right]^{1/p} \leq \beta$$

$$0.85\left[1-\left(\frac{400-x}{80}\right)^{p}\right]^{1/p} \geq \alpha, 0.85\left[1-\left(\frac{x-450}{85}\right)^{p}\right]^{1/p} \geq \alpha,$$

$$0.12\left[1+\frac{1-(0.12)^{p}}{(0.12)^{p}}\left(\frac{400-x}{85}\right)^{p}\right]^{1/p} \leq \beta, 0.12\left[1+\frac{1-(0.12)^{p}}{(0.12)^{p}}\left(\frac{x-450}{90}\right)^{p}\right]^{1/p} \leq \beta$$

$$0.89\left[1-\left(\frac{120-x}{36}\right)^{p}\right]^{1/p} \geq \alpha, 0.89\left[1-\left(\frac{x-150}{25}\right)^{p}\right]^{1/p} \geq \alpha,$$

$$0.07\left[1+\frac{1-(0.07)^{p}}{(0.07)^{p}}\left(\frac{120-x}{40}\right)^{p}\right]^{1/p} \leq \beta, 0.07\left[1+\frac{1-(0.07)^{p}}{(0.07)^{p}}\left(\frac{x-150}{30}\right)^{p}\right]^{1/p} \leq \beta$$

$$0.70\left[1-\left(\frac{100-x}{20}\right)^{p}\right]^{1/p} \geq \alpha, 0.70\left[1-\left(\frac{x-120}{21}\right)^{p}\right]^{1/p} \geq \alpha,$$

$$0.26\left[1+\frac{1-(0.26)^{p}}{(0.26)^{p}}\left(\frac{100-x}{25}\right)^{p}\right]^{1/p} \leq \beta, 0.26\left[1+\frac{1-(0.26)^{p}}{(0.26)^{p}}\left(\frac{x-120}{24}\right)^{p}\right]^{1/p} \leq \beta$$

$$0.75\left[1-\left(\frac{170-x}{35}\right)^{p}\right]^{1/p} \geq \alpha, 0.75\left[1-\left(\frac{x-200}{36}\right)^{p}\right]^{1/p} \geq \alpha,$$

$$0.20\left[1+\frac{1-(0.20)^{p}}{(0.20)^{p}}\left(\frac{170-x}{39}\right)^{p}\right]^{1/p} \leq \beta, 0.20\left[1+\frac{1-(0.20)^{p}}{(0.20)^{p}}\left(\frac{x-200}{40}\right)^{p}\right]^{1/p} \leq \beta$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

$$0 \leq \alpha, \beta \leq 1 \text{ and } 0 \leq \alpha + \beta \leq 1$$

The crispified results (resistances) are shown for different values of p:

	$p = 1$	$p = 2$
$\alpha \approx$	0.93	0.41
$\beta \approx$	0.02	0.58
$x_1 \approx$	68	69
$x_2 \approx$	440	449
$x_3 \approx$	148	143
$x_4 \approx$	111	109
$x_5 \approx$	179	177

5. CONCLUSION

The discussed method provides a mechanism of adjustment, given small information about the nature of the observed values. The method uses the concept of fuzzy optimization in such a way so as to find a set of values that respects the original observed values as much as possible and at the same time they satisfy the rigid relationship that underlies among the variables. The method can be expanded so that it handles a fuzzy or intuitionistic fuzzy relationship among the variables as well as the rigid relationship shown in the given models. This method will be very useful for various data handling problems in engineering mathematics and physics.

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