Qualitative Analysis, Adaptive Control and Synchronization of a Seven-Term Novel 3-D Chaotic System with a Quartic Nonlinearity

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ABSTRACT

First, this paper introduces a seven-term novel 3-D polynomial chaotic system with two quadratic nonlinearities and a quartic nonlinearity. The phase portraits of the novel chaotic system are displayed and the qualitative properties are discussed. The Lyapunov exponents of the novel chaotic system are obtained as $L_1 = 6.11857$, $L_2 = 0$ and $L_3 = -18.13487$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is obtained as $L_1 = 6.11857$ and Lyapunov dimension as $D_L = 2.33739$. Next, we derive new results for the adaptive control design of the novel chaotic system with unknown parameters. The adaptive controller is designed to achieve global exponential stability for the novel chaotic system with unknown parameters. Next, we derive new results for the adaptive synchronization design of the identical novel chaotic systems with unknown parameters. The adaptive control and synchronization results have been established using Lyapunov stability theory. Numerical simulations with MATLAB have been shown to validate and illustrate all the new results derived in this paper.

Keywords: Chaos, chaotic systems, novel chaotic system, adaptive control, adaptive synchroization.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions. The sensitivity of a nonlinear chaotic system in response to small changes in the initial conditions is commonly called as *butterfly effect* [1] and this is one of the characterizing features of a chaotic system.

The Lyapunov exponent of a dynamical system is a quantitative measure that characterizes the rate of separation of infinitesimally close trajectories of the system. Thus, a chaotic system is also defined mathematically as a dynamical system having at least one positive Lyapunov exponent.

In the last four decades, many chaotic systems have been found in the literature using modelling and other techniques. In 1963, Lorenz modelled a 3-D chaotic system to study convection in the atmosphere and experimentally verified that a very small difference in the initial conditions resulted in very large changes in his deterministic weather model [2].

In the last four decades, many important chaotic systems were discovered such as Rössler system [3], Shimizu-Morioka system [4], Shaw system [5], Chen system [6], Lü system [7], Chen-Lee system [8], Cai system [9], Tigan system [10], Li system [11], Sundarapandian-Pehlivan system [12], Sundarapandian system [13], Vaidyanathan systems [14-18], Vaidyanathan-Madhavan system [19], Pehlivan-Moroz-Vaidyanathan system [20], etc.

Chaos control and chaos synchronization are important research problems in the chaos literature, which have been studied extensively in the last four decades. Chaos theory finds applications in a variety of fields such as lasers [21], oscillators [22-23], chemical reactors [24-25], biology [26-28],ecology [29-30], neural networks [31-33], robotics [34-35], etc.

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The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [36-37]. Some popular methods for chaos control are active control [38-39], adaptive control [40-41], sliding mode control [42-43], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. Because of the butterfly effect which results in the exponential divergence of two identical chaotic systems with nearly the same initial conditions, the synchronization of two identical chaotic systems or different chaotic systems is a challenging problem in the chaos literature. The synchronization of chaotic systems has applications in secure communications [44-46], cryptosystems [47-48], encryption [49-50] etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [51-52], many important methods have been derived for the chaos synchronization problem such as active control method [53-70], adaptive control method [71-87], sampled-data feedback control method [88-90], time-delay feedback approach [91], backstepping method [92-100], sliding mode control method [101-106], etc.

In this paper, we have proposed a seven-term novel 3-D chaotic system with two quadratic nonlinearities and a quartic nonlinearity. We have obtained the Lyapunov exponents of the novel chaotic system as $L_1 = 6.11857$, $L_2 = 0$ and $L_3 = -18.13487$. The maximal Lyapunov exponent (MLE) for the novel chaotic system is found as $L_1 = 6.11857$ and Lyapunov dimension as $D_L = 2.33739$. We have also derived new results for the adaptive control of the novel chaotic system and adaptive synchronization of identical novel chaotic systems with unknown parameters. The main adaptive results of this paper are proved using Lyapunov stability theory.

The rest of this paper is organized as follows. Section 2 describes the equations and phase portraits of the seven-term novel 3-D chaotic system with two quadratic nonlinearities and a quartic nonlinearity. Section 3 describes the qualitative properties of the novel 3-D chaotic system. Section 4 describes new results for the design of adaptive control to stabilize the novel chaotic system with unknown system parameters. Section 5 describes new results for the design of adaptive synchronization of the identical novel chaotic systems with unknown system parameters. Section 5 contains a summary of the main results derived in this paper.

2.   A SEVEN-TERMNOVEL CHAOTIC SYSTEM

In this section, we describe the equations and properties of a seven-term novel 3-D polynomial chaotic system with two quadratic nonlinearities and a quartic nonlinearity.

The proposed seven-term novel chaotic system is modelled by the 3-D dynamics

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\
\dot{x}_2 &= bx_1 - x_1 x_3 \\
\dot{x}_3 &= -cx_3 + dx_1^4
\end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the state variables and a, b, c, d are constant, positive parameters of the system.

The system (1) exhibits a chaotic attractor for the values

$$a = 8, \quad b = 55, \quad c = 4, \quad d = 5 \tag{2}$$

Figure 1 shows the strange chaotic attractor of the system (1). Figures 2-4 show the 2-D view of the chaotic attractor of the system (1) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes respectively.
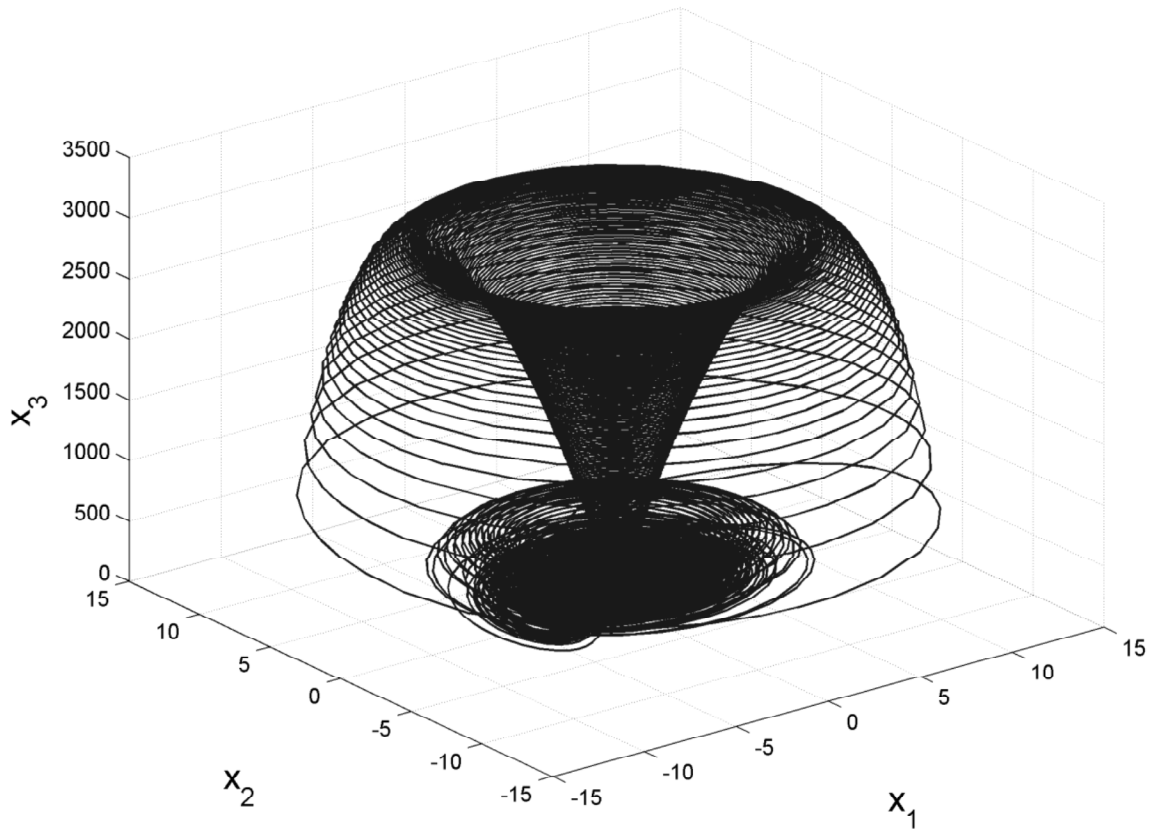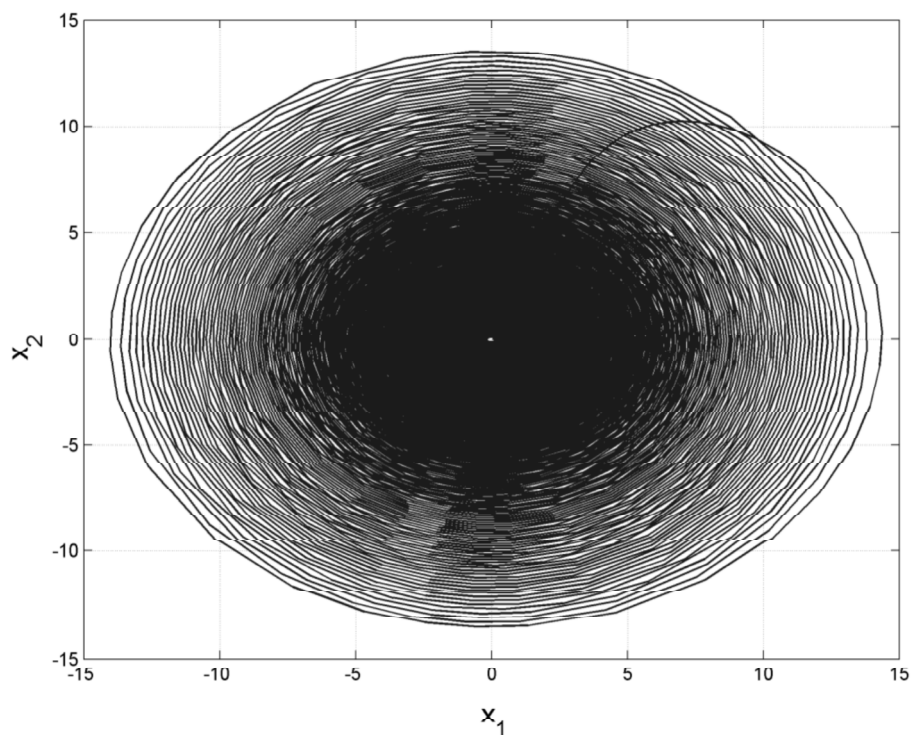

Figure 1: Strange Attractor of the Novel Chaotic System

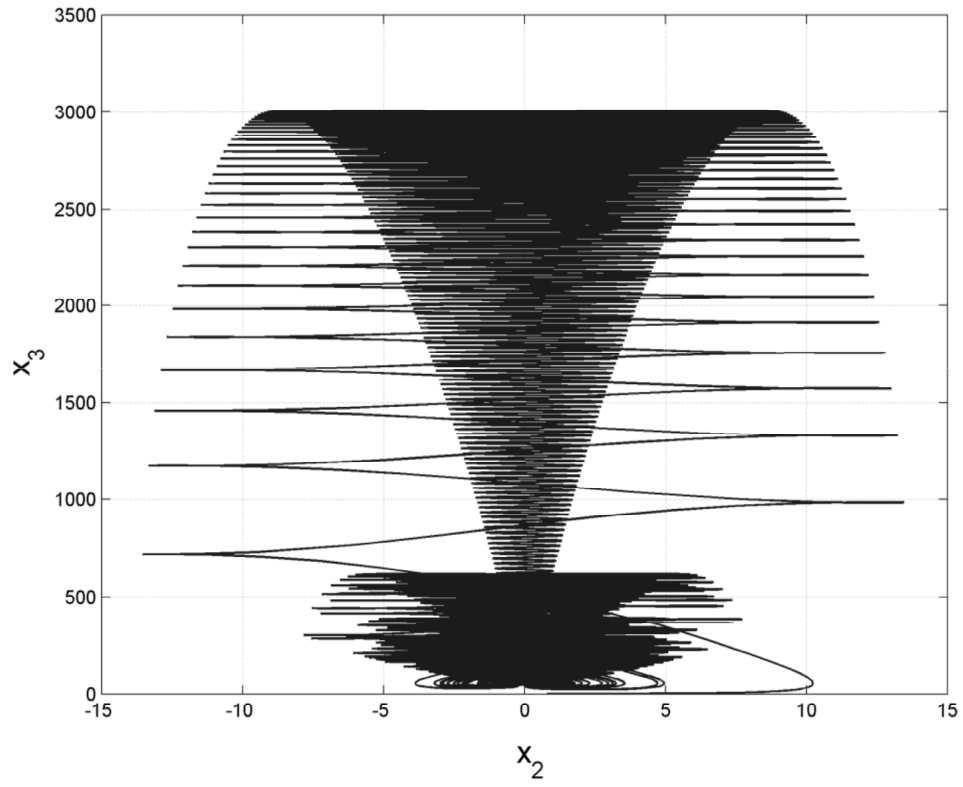**Figure 2: 2-D View of the Novel Chaotic System in (x_1, x_2) Plane**

Figure 3: 2-D View of the Novel Chaotic System in (x_2, x_3) Plane
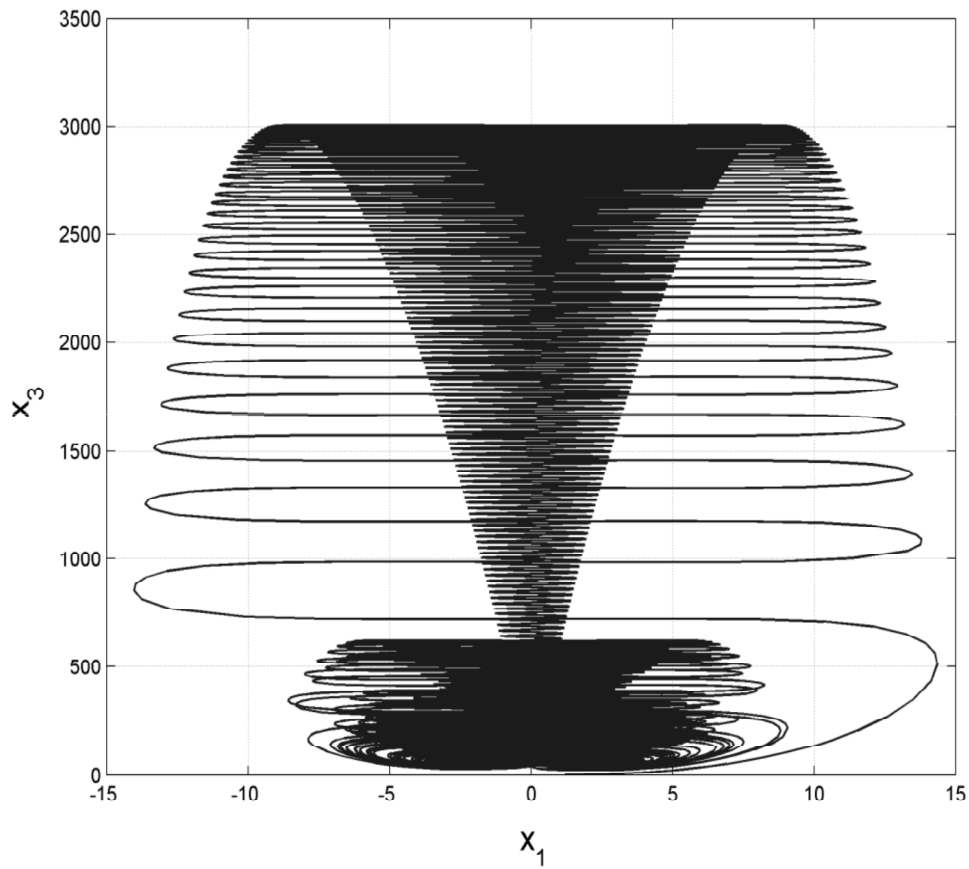

Figure 4: 2-D View of the Novel Chaotic System in (x_1, x_3) Plane

3. PROPERTIES OF THE NOVEL CHAOTIC SYSTEM

(A) Symmetry

The novel 3-D chaotic system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) \rightarrow (-x_1, -x_2, x_3) \tag{3}$$

Since the transformation (3) persists for all values of the system parameters, the novel chaotic system (1) has rotation symmetry about the x_3–axis and that any non-trivial trajectory must have a twin trajectory.

(B) Invariance

The x_3–axis ($x_1 = 0$, $x_2 = 0$) is invariant for the system (1). Hence, all orbits of the system (1) starting on the x_3– axis stay in the x_3– axis for all values of time.

(C) Dissipativity

We write the system (1) in vector notation as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} \tag{4}$$

where

$$\begin{aligned}
f_1(x) &= a(x_2 - x_1) + x_2 x_3 \\
f_2(x) &= bx_1 - x_1 x_3 \\
f_3(x) &= -cx_3 + dx_1^4
\end{aligned} \tag{5}$$

The divergence of the vector field f on R^3 is obtained as

$$\operatorname{div} f = \frac{\partial f_1(x)}{\partial x_1} + \frac{\partial f_2(x)}{\partial x_2} + \frac{\partial f_3(x)}{\partial x_3} = -(a+c) < 0 \tag{6}$$

since a and c are positive constants.

Let Ω be any region in R^3 having a smooth boundary.

Let $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, it follows that

$$\frac{dV(t)}{dt} = \iiint_{\Omega(t)} (\operatorname{div} f)\, dx_1\, dx_2\, dx_3 \tag{7}$$

By substituting the value of $\operatorname{div} f$ from (6) into (7), we obtain

$$\frac{dV(t)}{dt} = -(a+c) \iiint_{\Omega(t)} dx_1\, dx_2\, dx_3 = -(a+c)V(t) \tag{8}$$

Integrating the linear differential equation (8), we get the solution as

$$V(t) = V(0)\exp(-(a+c)t) = V(0)\exp(-\mu t) \tag{9}$$

where $\mu = a + c > 0$.

From Eq. (9), it follows that the volume $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Thus, the novel chaotic system (1) is dissipative. Hence, the asymptotic motion of the system (1) settles exponentially onto a set of measure zero, *i.e.* a strange attractor.

(D) Equilibrium Points

The equilibrium points of the novel chaotic system (1) are obtained by solving the nonlinear equations

$$
\begin{aligned}
f_1(x) &= a(x_2 - x_1) + x_2 x_3 = 0 \\
f_2(x) &= bx_1 - x_1 x_3 = 0 \\
f_3(x) &= -cx_3 + dx_1^4 = 0
\end{aligned} \tag{10}
$$

We take the parameter values as in the chaotic case, *viz.*

$$a = 8, \quad b = 55, \quad c = 4, \quad d = 5 \tag{11}$$

Solving the equations (10) using the values (11), we obtain three equilibrium points of the novel chaotic system (1) as follows:

$$
E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad
E_1 = \begin{bmatrix} 2.5755 \\ 0.3270 \\ 55.0 \end{bmatrix}, \quad
E_2 = \begin{bmatrix} -2.5755 \\ -0.3270 \\ 55.0 \end{bmatrix} \tag{12}
$$

The Jacobian matrix of the novel chaotic system (1) is obtained as

$$
J(x) = \begin{bmatrix} -a & a + x_3 & x_2 \\ b - x_3 & 0 & -x_1 \\ 4dx_1^3 & 0 & -c \end{bmatrix} \tag{13}
$$

The Jacobian matrix at the equilibrium E_0 is obtained as

$$
J_0 = J(E_0) = \begin{bmatrix} -8 & 8 & 0 \\ 55 & 0 & 0 \\ 0 & 0 & -4 \end{bmatrix}, \tag{14}
$$

which has the eigenvalues

$$\lambda_1 = -4, \quad \lambda_2 = -25.3542, \quad \lambda_3 = 17.3542 \tag{15}$$

This shows that the equilibrium E_0 is a saddle-point, which is unstable.

The Jacobian matrix at the equilibrium E_1 is obtained as

$$
J_1 = J(E_1) = \begin{bmatrix} -8 & 63 & 0.3270 \\ 0 & 0 & -2.5755 \\ 341.6761 & 0 & -4 \end{bmatrix}, \tag{16}
$$

which has the eigenvalues

$$\lambda_1 = -43.3460, \ \lambda_2 = 15.6730 + 14.3283i, \ \lambda_3 = 15.6730 - 14.3283i \tag{17}$$

This shows that the equilibrium E_1 is a saddle-focus, which is unstable.

The Jacobian matrix at the equilibrium E_2 is obtained as

$$J_2 = J(E_2) = \begin{bmatrix} -8 & 63 & -0.3270 \\ 0 & 0 & 2.5755 \\ -341.6761 & 0 & -4 \end{bmatrix}, \tag{18}$$

which has the eigenvalues

$$\lambda_1 = -43.3460, \ \lambda_2 = 15.6730 + 14.3283i, \ \lambda_3 = 15.6730 - 14.3283i \tag{19}$$

This shows that the equilibrium E_2 is a saddle-focus, which is unstable.

Hence, E_0, E_1, E_2 are all unstable equilibrium points, where E_0 is a saddle point, and E_1, E_2 are saddle-focus points.

(E) Lyapunov Exponents

We take the parameter values of the system (1) as

$$a = 8, \ b = 55, \ c = 4, \ d = 5 \tag{20}$$

We take the initial state as

$$x_1(0) = 1.6, \ x_2(0) = 0.8, \ x_3(0) = 1.2 \tag{21}$$

The Lyapunov exponents of the system (1) are numerically obtained with MATLAB as

$$L_1 = 6.11857, \ L_2 = 0, \ L_3 = -18.13487 \tag{22}$$

Eq. (22) shows that the system (1) is chaotic, since it has a positive Lyapunov exponent.

Also, the maximal Lyapunov exponent (MLE) of the system (1) is obtained as $L_1 = 6.11857$.

Since $L_1 + L_2 + L_3 = -12.0163 < 0,$ it is immediate that (1) is a dissipative chaotic system.

The dynamics of the Lyapunov exponents is depicted in Figure 5.

(F) Lyapunov Dimension

The Lyapunov dimension of the chaotic system (1) is determined as

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.33739, \tag{23}$$

which is fractional. Thus, the seven-term 3-D system (1) is a dissipative novel chaotic system with fractional Lyapunov dimension.

4.   ADAPTIVE CONTROL OF THE NOVEL CHAOTIC SYSTEM

In this section, we derive new results for the adaptive controller to stabilize the unstable novel chaotic system with unknown parameters for all initial conditions.
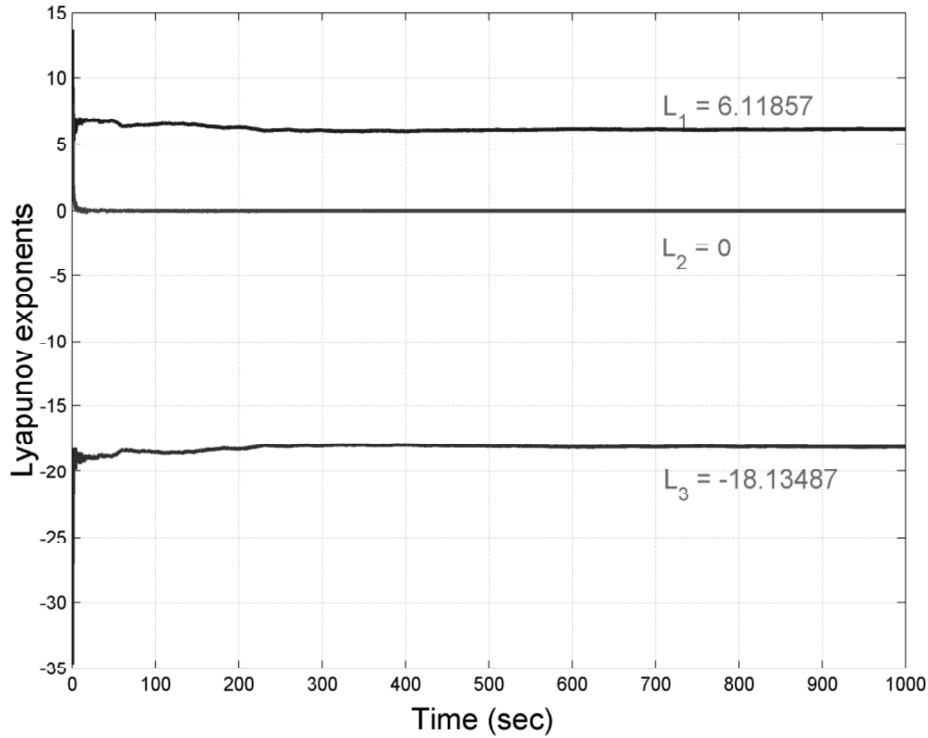
Figure 5: Dynamics of the Lyapunov Exponents

Thus, we consider the controlled novel 3-D chaotic system

$$\dot{x}_1 = a(x_2 - x_1) + x_2 x_3 + u_1$$
$$\dot{x}_2 = bx_1 - x_1 x_3 + u_2$$
$$\dot{x}_3 = -cx_3 + dx_1^4 + u_3$$

(24)

where x_1, x_2, x_3 are state variables, a, b, c, d are constant, unknown, parameters of the system and u_1, u_2, u_3 are adaptive controls to be designed.

We aim to solve the adaptive control problem by considering the adaptive feedback control law

$$u_1 = -A(t)(x_2 - x_1) - x_2 x_3 - k_1 x_1$$
$$u_2 = -B(t)x_1 + x_1 x_3 - k_2 x_2$$
$$u_3 = C(t)x_3 - D(t)x_1^4 - k_3 x_3$$

(25)

where $A(t), B(t), C(t), D(t)$ are estimates for the unknown parameters a, b, c, d, respectively, and k_1, k_2, k_3 are positive gain constants.

The closed-loop system is obtained by substituting (25) into (24) as

$$\dot{x}_1 = (a - A(t))(x_2 - x_1) - k_1 x_1$$
$$\dot{x}_2 = (b - B(t))x_1 - k_2 x_2$$
$$\dot{x}_3 = -(c - C(t))x_3 + (d - D(t))x_1^4 - k_3 x_3$$

(26)

To simplify (26), we define the parameter estimation error as

$$e_a(t) = a - A(t)$$
$$e_b(t) = b - B(t)$$
$$e_c(t) = c - C(t) \tag{27}$$
$$e_d(t) = d - D(t)$$

Substituting (27) into (26), we obtain

$$\dot{x}_1 = e_a(x_2 - x_1) - k_1 x_1$$
$$\dot{x}_2 = e_b x_1 - k_2 x_2 \tag{28}$$
$$\dot{x}_3 = -e_c x_3 + e_d x_1^4 - k_3 x_3$$

Differentiating the parameter estimation error (27) with respect to t, we get

$$\dot{e}_a(t) = -\dot{A}(t)$$
$$\dot{e}_b(t) = -\dot{B}(t)$$
$$\dot{e}_c(t) = -\dot{C}(t) \tag{29}$$
$$\dot{e}_d(t) = -\dot{D}(t)$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$V(x_1, x_2, x_3, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right), \tag{30}$$

which is positive definite on R^7.

Differentiating V along the trajectories of (28) and (29), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{A}\right] + e_b\left[x_1 x_2 - \dot{B}\right]$$
$$+ e_c\left[-x_3^2 - \dot{C}\right] + e_d\left[x_1^4 x_3 - \dot{D}\right] \tag{31}$$

In view of (31), we define an update law for the parameter estimates as

$$\dot{A} = x_1(x_2 - x_1)$$
$$\dot{B} = x_1 x_2$$
$$\dot{C} = -x_3^2 \tag{32}$$
$$\dot{D} = x_1^4 x_3$$

Theorem 1. The novel chaotic system (24) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (25) and the parameter update law (32), where k_i, $(i = 1, 2, 3)$ are positive constants.

Proof. The result is proved using Lyapunov stability theory [107]. We consider the quadratic Lyapunov function V defined by (30), which is a positive definite function on R^7.

Substituting the parameter update law (32) into (31), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{33}$$

which is a negative semi-definite function on R^7.

Therefore, it can be concluded that the state vector $x(t)$ and the parameter estimation error are globally bounded, i.e.

$$\begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & e_a(t) & e_b(t) & e_c(t) \end{bmatrix}^T \in L_\infty. \tag{34}$$

We define

$$k = \min\{k_1, k_2, k_3\}. \tag{35}$$

Then it follows from (33) that

$$\dot{V} \le -k\|x\|^2 \text{ or } k\|x\|^2 \le -\dot{V}. \tag{36}$$

Integrating the inequality (36) from 0 to t, we get

$$k\int_o^t \|x(\tau)\|^2 \, d\tau \le -\int_0^t \dot{V}(\tau)d\tau = V(0) - V(t) \tag{37}$$

From (37), it follows that $x(t) \in L_2$.

Using (28), we can conclude that $\dot{x}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof.

Numerical Results

For the novel chaotic system (24), the parameter values are taken as in the chaotic case, *viz.*

$$a = 8, \quad b = 55, \quad c = 4, \quad d = 5 \tag{38}$$

We take the feedback gains as $k_i = 5$ for $i = 1, 2, 3$.

The initial values of the chaotic system (24) are taken as

$$x_1(0) = 8.5, \ x_2(0) = -9.7, \ x_3(0) = 5.4 \tag{39}$$

The initial values of the parameter estimates are taken as

$$A(0) = 12, \ B(0) = 7, \ C(0) = 16, \ D(0) = 20 \tag{40}$$

Figure 6 depicts the time-history of the controlled novel chaotic system.

5. ADAPTIVE SYNCHRONIZATION OF THE IDENTICAL NOVEL CHAOTIC SYSTEMS

In this section, we derive new results for the adaptive synchronization design of the identical novel chaotic systems with unknown parameters.

As the master system, we take the novel 3-D chaotic system

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= bx_1 - x_1 x_3 \\ \dot{x}_3 &= -cx_3 + dx_1^4 \end{aligned} \tag{41}$$

where x_1, x_2, x_3 are state variables and a, b, c, d are constant, unknown, parameters of the system.
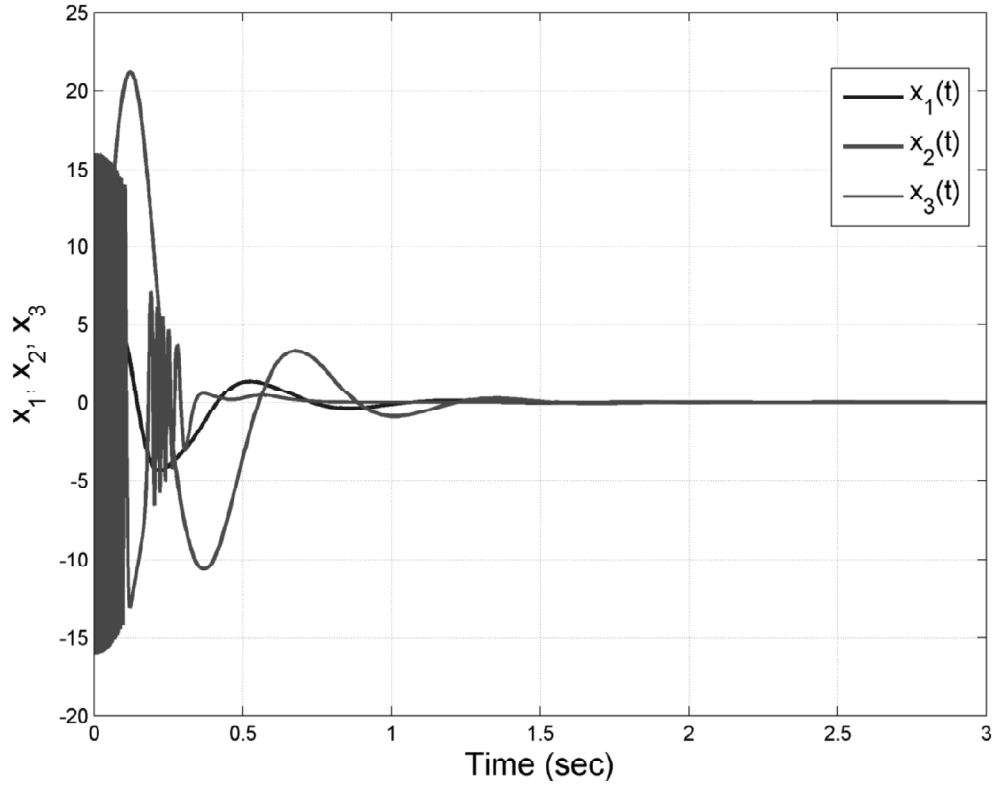
Figure 6: Time-History of the Controlled Novel Chaotic System

As the slave system, we take the controlled novel 3-D chaotic system

$$\dot{y}_1 = a(y_2 - y_1) + y_2 y_3 + u_1$$
$$\dot{y}_2 = by_1 - y_1 y_3 + u_2$$
$$\dot{y}_3 = -cy_3 + dy_1^4 + u_3$$

(42)

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are adaptive controllers to be designed.

The synchronization error is defined by

$$e_1 = y_1 - x_1$$
$$e_2 = y_2 - x_2$$
$$e_3 = y_3 - x_3$$

(43)

The error dynamics is easily obtained as

$$\dot{e}_1 = a(e_2 - e_1) + y_2 y_3 - x_2 x_3 + u_1$$
$$\dot{e}_2 = be_1 - y_1 y_3 + x_1 x_3 + u_2$$
$$\dot{e}_3 = -ce_3 + d(y_1^4 - x_1^4) + u_3$$

(44)

We consider the adaptive control law defined by

$$u_1 = -A(t)(e_2 - e_1) - y_2 y_3 + x_2 x_3 - k_1 e_1$$
$$u_2 = -B(t)e_1 + y_1 y_3 - x_1 x_3 - k_2 e_2$$
$$u_3 = C(t)e_3 - D(t)(y_1^4 - x_1^4) - k_3 e_3$$

(45)

where k_1, k_2, k_3 are positive gain constants.

Substituting (45) into (44), we get the closed-loop error dynamics as

$$
\begin{aligned}
\dot{e}_1 &= (a - A(t))(e_2 - e_1) - k_1 e_1 \\
\dot{e}_2 &= (b - B(t))e_1 - k_2 e_2 \\
\dot{e}_3 &= -(c - C(t))e_3 + (d - D(t))(y_1^4 - x_1^4) - k_3 e_3
\end{aligned}
\tag{46}
$$

To simplify the error dynamics (46), we define the parameter estimation error as

$$
\begin{aligned}
e_a(t) &= a - A(t) \\
e_b(t) &= b - B(t) \\
e_c(t) &= c - C(t) \\
e_d(t) &= d - D(t)
\end{aligned}
\tag{47}
$$

Using (43), we can simplify the error dynamics (46) as

$$
\begin{aligned}
\dot{e}_1 &= e_a(e_2 - e_1) - k_1 e_1 \\
\dot{e}_2 &= e_b e_1 - k_2 e_2 \\
\dot{e}_3 &= -e_c e_3 + e_d(y_1^4 - x_1^4) - k_3 e_3
\end{aligned}
\tag{48}
$$

Differentiating the parameter estimation error (47) with respect to t, we get

$$
\begin{aligned}
\dot{e}_a(t) &= -\dot{A}(t) \\
\dot{e}_b(t) &= -\dot{B}(t) \\
\dot{e}_c(t) &= -\dot{C}(t) \\
\dot{e}_d(t) &= -\dot{D}(t)
\end{aligned}
\tag{49}
$$

Next, we find an update law for parameter estimates using Lyapunov stability theory.

Consider the quadratic Lyapunov function defined by

$$
V(e_1, e_2, e_3, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right),
\tag{50}
$$

which is positive definite on R^7.

Differentiating V along the trajectories of (48) and (49), we obtain

$$
\begin{aligned}
\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 &+ e_a\left[e_1(e_2 - e_1) - \dot{A}\right] + e_b\left[e_1 e_2 - \dot{B}\right] \\
&+ e_c\left[-e_3^2 - \dot{C}\right] + e_d\left[e_3(y_1^4 - x_1^4) - \dot{D}\right]
\end{aligned}
\tag{51}
$$

In view of (51), we define an update law for the parameter estimates as

$$
\begin{aligned}
\dot{A} &= e_1(e_2 - e_1) \\
\dot{B} &= e_1 e_2 \\
\dot{C} &= -e_3^2 \\
\dot{D} &= e_3(y_1^4 - x_1^4)
\end{aligned}
\tag{52}
$$

where k_4, k_5, k_6, k_7 are positive gain constants.

Theorem 2. The identical novel chaotic systems(41) and (42) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (45) and the parameter update law (52), where k_i, $(i = 1, 2, 3)$ are positive constants.

Proof. The result is proved using Lyapunov stability theory [107]. We consider the quadratic Lyapunov function V defined by (50), which is a positive definite function on R^7.

Substituting the parameter update law (52) into (51), we obtain $\dot{V}$ as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{53}$$

which is a negative semi-definite function on R^7.

Thus, it can be concluded that the synchronization vector $e(t)$ and the parameter estimation error are globally bounded, i.e.

$$\begin{bmatrix} e_1(t) & e_2(t) & e_3(t) & e_a(t) & e_b(t) & e_c(t) \end{bmatrix}^T \in L_\infty. \tag{54}$$

We define

$$k = \min\{k_1, k_2, k_3\}. \tag{55}$$

Then it follows from (53) that

$$\dot{V} \le -k\|e\|^2 \text{ or } k\|e\|^2 \le -\dot{V}. \tag{56}$$

Integrating the inequality (56) from 0 to t, we get

$$k \int_o^t \|e(\tau)\|^2 \, d\tau \le -\int_0^t \dot{V}(\tau) d\tau = V(0) - V(t) \tag{57}$$

Therefore, we can conclude that $e(t) \in L_2$.

Using (48), we can conclude that $\dot{e}(t) \in L_\infty$.

Hence, using Barbalat's lemma, we can conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof.

Numerical Results

For the novel chaotic systems, the parameter values are taken as in the chaotic case, *viz.*

$$a = 8, \ b = 55, \ c = 4, \ d = 5 \tag{58}$$

We take the feedback gains as $k_i = 5$ for $i = 1, 2, 3$.

The initial values of the master system (41) are taken as

$$x_1(0) = 5.2, \ x_2(0) = -2.8, \ x_3(0) = 3.9 \tag{59}$$

The initial values of the slave system (42) are taken as

$$y_1(0) = 3.1, \ y_2(0) = 9.4, \ y_3(0) = -6.7 \tag{60}$$

The initial values of the parameter estimates are taken as

$$A(0) = 7, \ B(0) = 21, \ C(0) = 9, \ D(0) = 18 \tag{61}$$

Figures7-9 depicts the complete synchronization of the identical novel chaotic systems.

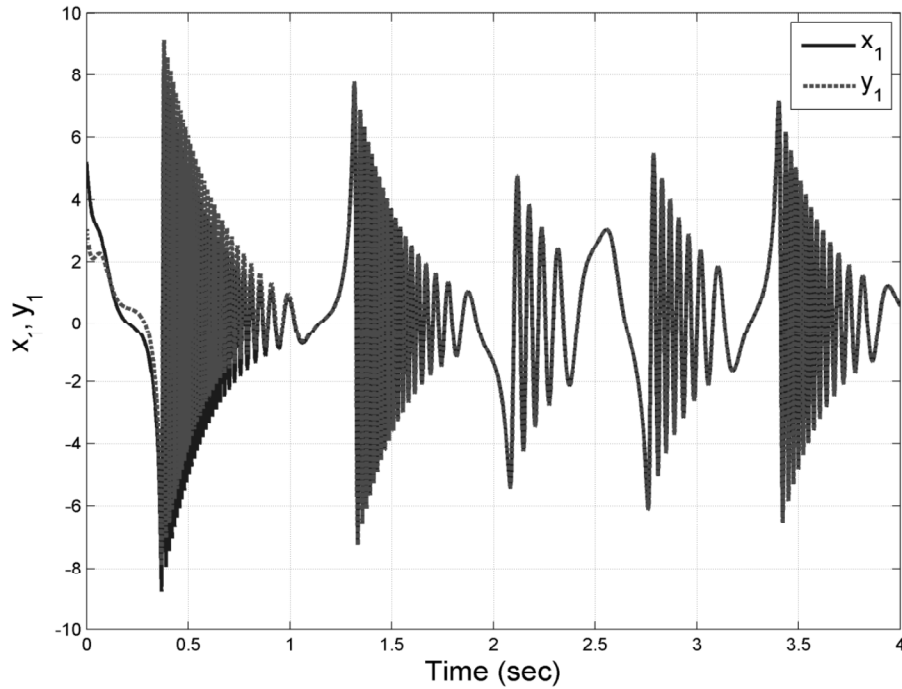Figure 10 depicts the time-history of the synchronization errors.

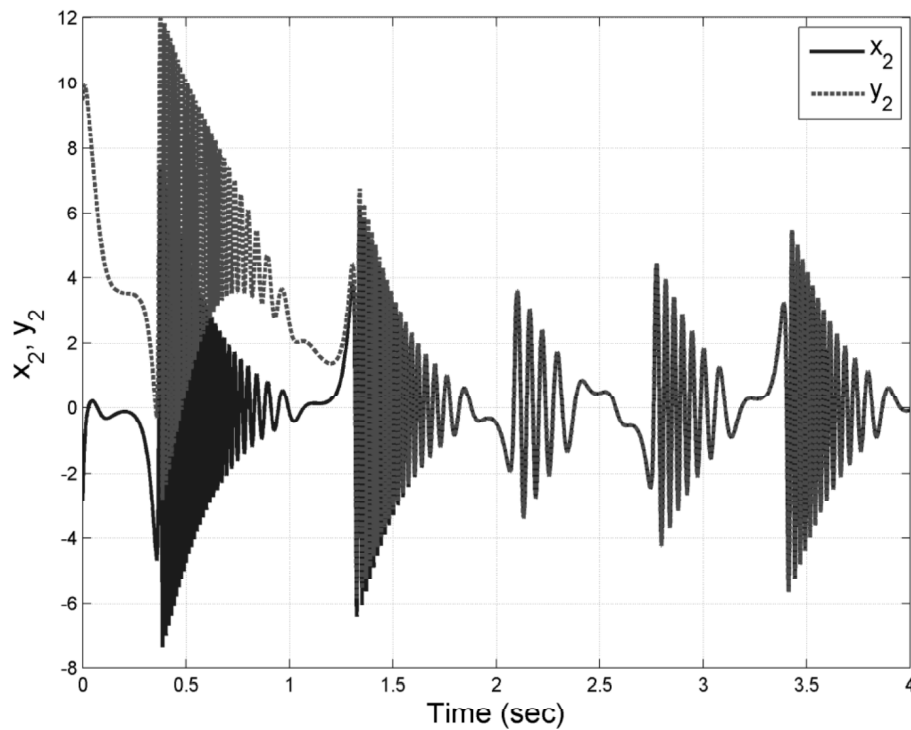**Figure 7: Complete Synchronization of the States x_1 and y_1**

Figure 8: Complete Synchronization of the States x_2 and y_2

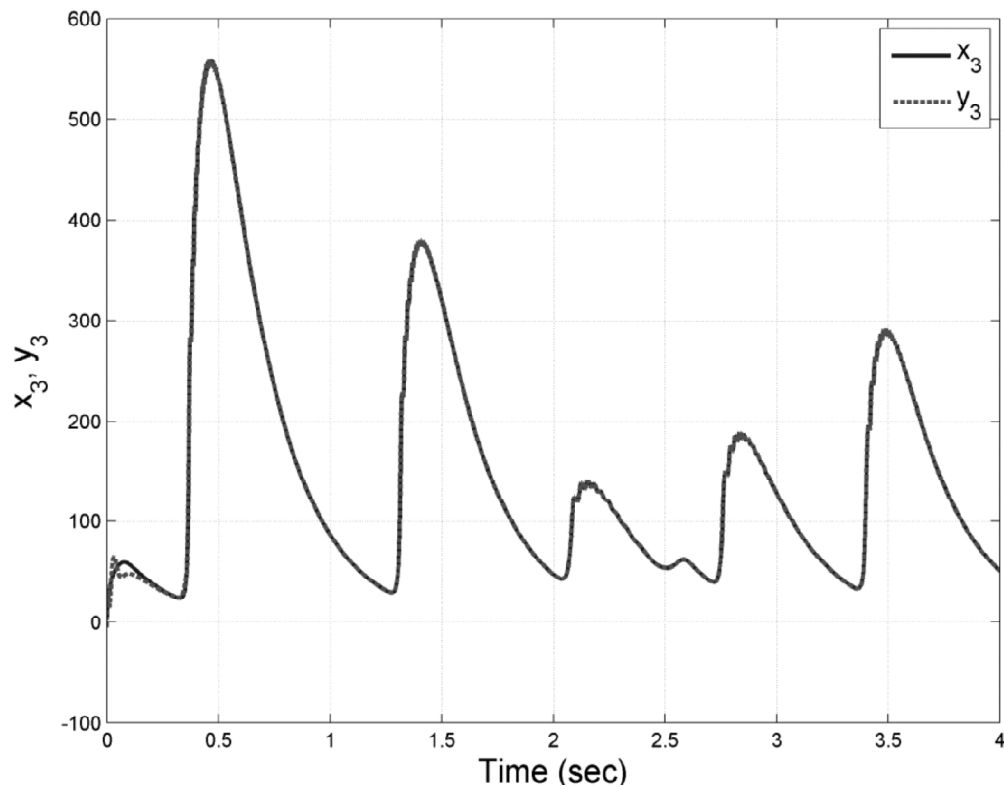**Figure 9: Complete Synchronization of the States x_3 and y_3**
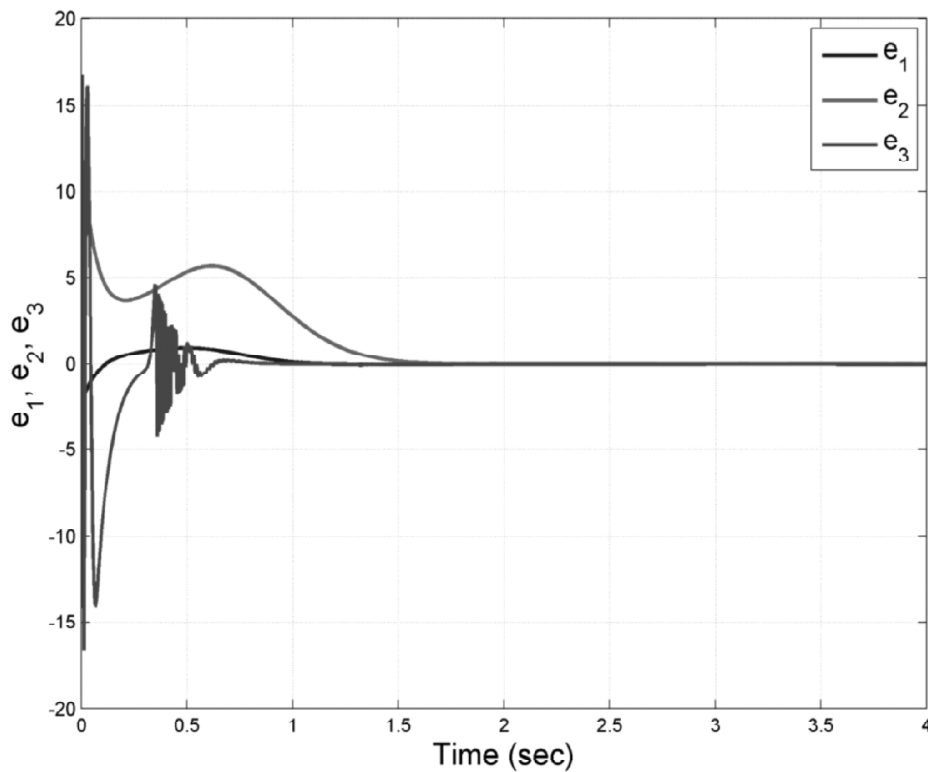

Figure 10: Time History of the Chaos Synchronization Errors e_1, e_2, e_3

6. CONCLUSIONS

In this paper, we have derived a seven-term novel 3-D polynomial chaotic system with two quadratic nonlinearities and a quartic nonlinearity. We gave a qualitative analysis of the main properties of the novel

3-D chaotic system. We determined the Lyapunov exponents of the novel chaotic system as $L_1 = 6.11857$, $L_2 = 0$ and $L_3 = -18.34837$. The maximal Lyapunov exponent (MLE) for the novel chaotic system was found as $L_1 = 6.11857$, and Lyapunov dimension as $D_L = 2.33739$. Next, we have derived adaptive control and synchronization results for the novel chaotic system with unknown parameters, which have been established using Lyapunov stability theory. Numerical simulations with MATLAB wereexhibited to validate and illustrate the phase portraits of the novel chaotic system and the adaptive results derived in this paper.

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