# SOLVING AN UNBALANCED FUZZY TRANSPORTATION PROBLEM USING A HEPTAGONAL FUZZY NUMBERS BY ROBUST RANKING METHOD 

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#### Abstract

In this paper, we consider an unbalanced fuzzy transportation problem where cost, requirement and availability are heptagonal fuzzy numbers. We use the proposed method to solve an unbalanced fuzzy transportation problem. Fuzzy transportation problem can be converted into a crisp valued transportation problem using Robust Ranking method.


Keywords: Fuzzy Transportation Problem, Heptagonal fuzzy number and ranking.

## 1. INTRODUCTION

In today's highly competitive market, many organizations trying to find better ways to create and deliver value to customers become stronger. How and when to send the products safely to the customers in the quantities with minimum cost become more challenging. To meet this challenging, transportation models provide a powerful framework. Transportation models have wide applications in logistics and supply chain for reducing the transportation cost. Various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter, and the penalty factors.

The Fuzzy transportation Problem (FTP) is one of the special kind of fuzzy linear programming problem. A fuzzy transportation problem is a transportation problem in which the transportation costs, available and requirement quantities are fuzzy quantities. Transportation problem was originally introduced and developed by Hithchcock in 1941, in which the parameters like transportation cost, available and requirement are crisp values. But in the present world the transportation parameters may be uncertain due to many uncontrolled factors. So to deal the problems with imprecise information Zadeh introduced the concept of fuzziness.

In the present paper a FTP with heptagonal fuzzy numbers is introduced using Vogel's Approximation Method, New Method, Best Candidate Method with a comparative study.

## 2. PRELIMINARIES

### 2.1 Types of Transportation Problem:

There are two types of transportation problem namely

* Unbalanced Transportation Problem


### 2.2 Fuzzy balanced and unbalanced Transportation problem

The balanced fuzzy transportation, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

$$
\operatorname{Minimize} \sum_{i=1}^{p} \sum_{j=1}^{q} c_{i j} * x_{i j}
$$

Subject to $\sum_{j=1}^{q} x_{i j}=\alpha_{i}, \mathrm{i}=1,2,3 \ldots \ldots, \mathrm{p}$

$$
\begin{aligned}
& \quad \sum_{i=1}^{p} x_{i j}=\beta_{j}, \mathrm{j}=1,2,3, \ldots \ldots ., \mathrm{q} \\
& \sum_{i=1}^{p} \alpha_{i}=\sum_{j=1}^{q} \beta_{j}
\end{aligned}
$$

$x_{i j}$ is a non-negative fuzzy number,
Where $\mathrm{p}=$ total number of sources
$\mathrm{q}=$ total number of destinations
$\alpha_{i}=$ the fuzzy availability of the product at $\mathrm{i}^{\text {th }}$ source
$\beta_{i}=$ the fuzzy demand of the product at $\mathrm{j}^{\text {th }}$ destination
$\mathrm{C}_{\mathrm{ij}}=$ the fuzzy transportation cost for unit quantity of the product from $\mathrm{i}^{\text {th }}$ source to $\mathrm{j}^{\text {th }}$

Destination
$x_{i j}=$ the fuzzy quantity of the product that should be transported from $\mathrm{i}^{\text {th }}$ source to $\mathrm{j}^{\text {th }}$ destination to minimize the total fuzzy transportation cost. $\sum_{i=1}^{p} \alpha_{i}=$ total fuzzy availability of the product,
$\sum_{j=1}^{q} \beta_{j}=$ total fuzzy demand of the product
$\sum_{i=1}^{p} \sum_{j=1}^{q} c_{i j} * x_{i j}=$ total fuzzy transportation cost.
If $\sum_{i=1}^{p} \alpha_{i}=\sum_{j=1}^{q} \beta_{j}$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

Consider transportation with $m$ fuzzy origins (rows) and $n$ fuzzy destinations (columns).

Let $\mathrm{C}_{\mathrm{ij}}=\left[\mathrm{C}_{\mathrm{ij}}{ }^{(1)}, \mathrm{C}_{\mathrm{ij}}{ }^{(2)}\right.$, $\left.\mathrm{C}_{\mathrm{ij}}{ }^{(3)}\right]$ be the cost of transporting one unit of the product from $\mathrm{i}^{\text {th }}$ fuzzy origin to $\mathrm{j}^{\text {th }}$ fuzzy destination $\alpha_{i}=\left[\alpha_{i}{ }^{(1)}, \alpha_{i}{ }^{(2)}, \alpha_{i}{ }^{(3)}\right]$ be the quantity of commodity available at fuzzy origin i $\beta_{i}=\left[\beta_{i}^{(1)}, \beta_{i}^{(2)}, \beta_{i}^{(3)}\right]$ be the quantity of commodity requirement at fuzzy destination $\mathrm{j} \cdot \mathrm{x}_{\mathrm{ij}}=\left[\mathrm{x}_{\mathrm{ij}}{ }^{1}, \mathrm{x}_{\mathrm{ij}}{ }^{2}, \mathrm{x}_{\mathrm{ij}}{ }^{3}\right]$ is quantity transported from $\mathrm{i}^{\text {th }}$ fuzzy origin to $\mathrm{j}^{\text {th }}$ fuzzy destination.

An unbalanced transportation problem is converted into a balanced transportation problem by introducing a dummy origin or dummy destinations
which will provide for the excess availability or the requirements the cost of transporting a unit from this dummy origin(or dummy destination) to any place is taken to be zero. After converting the unbalanced problem into a balanced problem, we adopt the usual procedure for solving a balanced transportation problem.

### 2.3 The Initial Basic Feasible Solution

Let us consider a T.P involving $m$ origins and $n$ destinations. Since the sum of origin capacities equals the sum of destinations requirements, a feasible solution always exists. Any feasible solution satisfying $m+n-1$ of the $m+n$ constraints is a redundant one and hence can be deleted. This also means that a feasible solution to a T.P can have at the most only $\mathrm{m}+\mathrm{n}-1$ strictly positive component, otherwise the solution will degenerate.

### 2.4 Feasible Solution (F.S)

A set of non-negative allocations $\mathrm{x}_{\mathrm{ij}}>0$ which satisfies the row and column restrictions is known as feasible solution.

### 2.5 Basic Feasible Solution (B.F.S)

A feasible solution to a m-origin and n-destination problem is said to be basic feasible solution if the number of positive allocations are ( $\mathrm{m}+\mathrm{n}-1$ ).

If the number of allocations in a basic feasible solutions are less than ( $\mathrm{m}+\mathrm{n}-1$ ), it is called degenerate basic feasible solution (DBFS) (Otherwise non-degenerate).

### 2.6 Definition of Fuzzy set:

If y is a collection of objects denoted generally by Y , then a fuzzy set $\bar{B}$ in explained as a set of ordered pairs $\bar{B}=\left\{\left(\mathrm{y}, \mu_{\bar{B}}(\mathrm{y})\right) / \mathrm{y} \in \mathrm{Y}\right\}$ where $\mu_{\bar{B}}(\mathrm{y})$ is termed as the membership function for the fuzzy set A . The membership function maps each element of Y to a membership value between 0 and 1 .

### 2.7 Crisp set:

A crisp set is a special case of fuzzy set, in which the membership function takes only two values 0 and 1 .

## $2.8 \alpha$ - cut:

Given a fuzzy set B defined on Y and any number the $\alpha \in[0,1]$ the $\alpha-\mathrm{cut}, \alpha_{B}$ is the crisp set $\alpha_{B}$ is the crisp set $\alpha_{B}=\{\mathrm{y} \in \mathrm{Y} / \mathrm{B}(\mathrm{y}) \geq \alpha, \alpha \in[0,1]\}$.

### 2.9 Definition of Fuzzy Number:

A fuzzy set $\bar{B}$ is defined on universal set of real numbers is said to be a generalized fuzzy number if its membership function has the following attributes:
a) $\mu_{\bar{B}}(\mathrm{y}): \mathrm{R} \rightarrow[0,1]$ is continuous;
b) $\mu_{\bar{B}}(\mathrm{y})=0$ for all $\mathrm{y} \in \mathrm{A}(-\infty, \mathrm{a}] \mathrm{U}[\mathrm{d}, \infty)$;
c) $\mu_{\bar{B}}(\mathrm{y})$ is strictly increasing on $[\mathrm{a}, \mathrm{b}]$ and strictly decreasing on $[\mathrm{c}, \mathrm{d}]$;
d) $\mu_{\bar{B}}(\mathrm{y})=\mathrm{w}$ for all $\mathrm{y} \in[\mathrm{b}, \mathrm{c}]$, where $0<\mathrm{w} \leq 1$

## 3. HEPTAGONAL FUZZY NUMBERS (HFN)

A fuzzy number $\hat{H}$ in R is said to be a heptagonal fuzzy number if its membership function $\mu_{\hat{\mathrm{H}}}: \mathrm{R} \rightarrow[0,1]$ has the following characteristics. We denote the heptagonal fuzzy number by $\hat{H}=\left(h_{1}, h_{2}, h_{3}, h_{4}, h_{5}, h_{6}, h_{7}\right)$

$$
\mu_{\hat{\mathrm{H}}}(y)=\left\{\begin{array}{lc}
0, & y<h_{1} \\
\frac{1}{2}\left(\frac{y-h_{1}}{h_{2}-h_{1}}\right), & h_{1} \leq y \leq h_{2} \\
\frac{1}{2}, & h_{2} \leq y \leq h_{3} \\
\frac{1}{2}+\frac{1}{2}\left(\frac{y-h_{3}}{h_{4}-h_{3}}\right), & h_{3} \leq y \leq h_{4} \\
\frac{1}{2}+\frac{1}{2}\left(\frac{h_{5}-y}{h_{5}-h_{4}}\right), & h_{4} \leq y \leq h_{5} \\
\frac{1}{2}, & h_{5} \leq y \leq h_{6} \\
\frac{1}{2}\left(\frac{h_{7}-y}{h_{7}-h_{6}}\right), & h_{6} \leq y \leq h_{7} \\
0, & x \geq h_{7}
\end{array}\right.
$$

### 3.1 Arithmetic operations on Heptagonal fuzzy numbers:

If $\widetilde{G_{h}}=\left(\mathrm{g}_{1}, \mathrm{~g}_{2}, \mathrm{~g}_{3}, \mathrm{~g}_{4}, \mathrm{~g}_{5}, \mathrm{~g}_{6}, \mathrm{~g}_{7}\right)$ and $\widetilde{H_{h}}=\left(\mathrm{h}_{1}, \mathrm{~h}_{2}, \mathrm{~h}_{3}, \mathrm{~h}_{4}, \mathrm{~h}_{5}, \mathrm{~h}_{6}, \mathrm{~h}_{7}\right)$ are two heptagonal fuzzy numbers then the following three Arithmetic Operations can be performed as follows:

## *Addition:

$$
\widetilde{G_{h}}+\widetilde{H_{h}}=\left(\mathrm{g}_{1}+\mathrm{h}_{1,} \mathrm{~g}_{2}+\mathrm{h}_{2,} \mathrm{~g}_{3}+\mathrm{h}_{3,} \mathrm{~g}_{4}+\mathrm{h}_{4,} \mathrm{~g}_{5}+\mathrm{h}_{5,} \mathrm{~g}_{6}+\mathrm{h}_{6,} \mathrm{~g}_{7}+\mathrm{h}_{7}\right)
$$

## *Subtraction:

$\widetilde{G_{h}}-\widetilde{H_{h}}=\left(\mathrm{g}_{1}-\mathrm{h}_{1,}, \mathrm{~g}_{2}-\mathrm{h}_{2}, \mathrm{~g}_{3}-\mathrm{h}_{3}, \mathrm{~g}_{4}-\mathrm{h}_{4}, \mathrm{~g}_{5}-\mathrm{h}_{5}, \mathrm{~g}_{6}-\mathrm{h}_{6}, \mathrm{~g}_{7}-\mathrm{h}_{7}\right)$
*Multiplication:
$\widetilde{G_{h}} * \widetilde{H_{h}}=\left(\mathrm{g}_{1} * \mathrm{~h}_{1, \mathrm{~g}_{2}} * \mathrm{~h}_{2,} \mathrm{~g}_{3} * \mathrm{~h}_{3,} \mathrm{~g}_{4} * \mathrm{~h}_{4,} \mathrm{~g}_{5} * \mathrm{~h}_{5,} \mathrm{~g}_{6} * \mathrm{~h}_{6,} \mathrm{~g}_{7} * \mathrm{~h}_{7}\right)$

## 4.ROBUST'S RANKING METHOD

Robust's ranking which satisfy compensation, linearity and additively properties and provides results which are consistent with human intuition. If $\widetilde{A_{H}}$ is a fuzzy number then the ranking is defined by
$\mathrm{R}\left(\widetilde{A_{H}}\right)=\int_{0}^{1} 0.5\left(b_{h \alpha}^{L}, b_{h \alpha}^{U}\right) \mathrm{d} \alpha$
Where ( $b_{h \alpha}^{L}, b_{h \alpha}^{U}$ ) is the $\alpha$ level cut of fuzzy number $\widetilde{A_{G}}$.

### 4.1 Proposed Ranking:

If $\left(h_{1}, h_{2}, h_{3}, h_{4}, h_{5}, h_{6}, h_{7}\right)$ are a heptagonal fuzzy numbers then
$\left(b_{h \alpha}^{L}, b_{h \alpha}^{U}\right)=\left\{\left(\mathrm{h}_{2}-\mathrm{h}_{1}\right) \alpha+\mathrm{h}_{\left.\left.1, \mathrm{~h}_{4}-\left(\mathrm{h}_{4}-\mathrm{h}_{3}\right) \alpha,\left(\mathrm{h}_{6}-\mathrm{h}_{5}\right) \alpha+\mathrm{h}_{5}, \mathrm{~h}_{7^{-}}\left(\mathrm{h}_{7}-\mathrm{h}_{5}\right) \alpha\right\},{ }^{2}\right)}\right.$
Hence, the fuzzy version of heptagonal ranking is
$\mathrm{R}\left(\widetilde{A_{H}}\right)=\int_{0}^{1} 0.5\left\{\left(\mathrm{~h}_{2^{-}} \mathrm{h}_{1}\right) \alpha+\mathrm{h}_{1}, \mathrm{~h}_{4-}\left(\mathrm{h}_{4-} \mathrm{h}_{3}\right) \alpha,\left(\mathrm{h}_{6}-\mathrm{h}_{5}\right) \alpha+\mathrm{h}_{5}, \mathrm{~h}_{7-}\left(\mathrm{h}_{7-} \mathrm{h}_{5}\right)\right.$ $\alpha\} \mathrm{d} \alpha$ $\qquad$ (A)

## 5. FUZZY VERSION OF PROPOSED METHOD

Even though there are many methods to find the basic feasible solution for a transportation problem we have analyzed a fuzzy version of Vogel's Approximation Method (VAM), New Method and Best Candidate Method for solving transportation problem.

## Proposed Algorithm:

## Step 1:

Using the above ranking as per equation (A), the fuzzy transportation problem is converted into a crisp value problem and solved using Vogel's Approximation Method (VAM), New Method and Best Candidate Method.

## Step 2:

Construct the fuzzy transportation table for the given fuzzy transportation problem and then, convert it into a balanced one, if it is not.

## Step 3:

## Vogel's Approximation Method (VAM)

The Vogel's Approximation Method takes into account not only the least cost $\mathrm{C}_{\mathrm{ij}}$ but also the costs that just exceed $\mathrm{C}_{\mathrm{ij}}$. The steps of the method are given below:
i) For each row of the transportation table identify the smallest and the next-tosmallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly, compute the differences for each column.
ii) Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to $i^{\text {th }}$ row and let $\mathrm{C}_{\mathrm{ij}}$ be the smallest cost in the $\mathrm{i}^{\text {th }}$ row. Allocate the maximum feasible amount $\mathrm{x}_{\mathrm{ij}}=\min .\left(\mathrm{a}_{\mathrm{i}}, \mathrm{b}_{\mathrm{j}}\right)$ in the $(\mathrm{i}, \mathrm{j})^{\text {th }}$ cell and cross off the $\mathrm{i}^{\text {th }}$ row or the $\mathrm{j}^{\text {th }}$ column in the usual manner.
iii) Recompute the column and row differences for the reduced transportation table and go to step 2 . Repeat the procedure until all the rim requirements are satisfied.

## Step 4:

## New Method

i) Construct the fuzzy transportation table for the given fuzzy transportation problem.
ii) Using the Modified Ranking Method and the fuzzy transportation problem is converted into a crisp value problem.
iii) Find the difference between the greatest and next greatest costs in each column and write them in bracket also find the difference between the greatest and next greatest costs in each row and write them in bracket.
iv) Identify the largest distribution choose the smallest entry along the largest distribution, if there are two or more smallest element choose any one of them arbitrary.
v) Allocate $X_{i j}=\min \left(\mathrm{a}_{\mathrm{i}}, \mathrm{b}_{\mathrm{j}}\right)$ on the left top of the smallest entry in the cell $(\mathrm{i}, \mathrm{j})$ of the transportation table.
vi) Recomputed the column and row difference for the reduce transportation table and go to step (iii).Repeat the procedure until the rim requirements are satisfied.

## Step 5:

## The Best Candidates Method (BCM)

## Algorithm

i) Prepare the BCM matrix. If the matrix unbalanced, we balance it and don't use the added row or column candidates in our solution procedure.
ii) Select the best candidates that are for minimizing problems to the minimum cost, and maximizing profit to the maximum cost. Therefore, this step can be done by electing the best two candidates in each row. If the candidate repeated more than two times, then the candidate should be elected again. As well as, the columns must be checked such that if it is not have candidates so that the candidates will be elected for them. However, if the candidate is repeated more than one time, the elect it again.
iii) Find the combinations by determining one candidate for each row and column, this should be done by starting from the row that have the least candidates, and then delete that row and column. If there are situations that have no candidate for some rows or columns, then directly elect the best available candidate. Repeat Step (iii) by determining the next candidate in the row that started from.

Compute and compare the summation of candidates for each combination. This is to determine the best combination that gives the optimal solution.

## 6. NUMERICAL EXAMPLE

We consider the fuzzy transportation problem (FTP) where Costs, Requirements and Availables are Heptagonal Fuzzy Numbers. It can be solved using Vogel's Approximation Method, New Method and Best Candidates Method and FTP can be converted into a crisp valued TP using Robust's Ranking method.

Table-6.1 Fuzzy Transportation Problem

|  | I | II | III | Fuzzy <br> Available |
| :---: | :---: | :---: | :---: | :---: |
| I | $(6,0,5,4,1,2,0)$ | $(10,12,13,2,4,6,8)$ | $(6,5,2,1,3,4,2)$ | $(6,5,2,4,3,2,1)$ |
| II | $(3,6,5,2,3,4,1)$ | $(1,1,0,4,3,2,3)$ | $(11,13,3,5,7,9,1)$ | $(5,0,1,2,1,4,3)$ |
| III | $(6,3,2,1,2,7,7)$ | $(2,1,8,7,3,6,3)$ | $(5,0,4,3,6,2,1)$ | $(1,1,0,2,1,2,2)$ |
| Fuzzy <br> Requirement | $(6,3,2,1,2,7,7)$ | $(6,0,5,0,1,2,4)$ | $(6,2,0,4,0,6,4)$ |  |

## Solution:

## Step 1:

Using the proposed ranking method, the fuzzy transportation problem is converted into a crisp value as
The ranking indices for the cost $\mathrm{C}_{\mathrm{ij}}$ are calculated as:

| $\mathrm{C}_{11}=(6,0,5,4,1,2,0)$ | $\begin{aligned} \mathrm{R}\left(\mathrm{C}_{11}\right) & =\int_{0}^{1} 0.5\{(0-6) \alpha+6,4-(4-5) \alpha,(2-1) \alpha+1,0-(0-1) \\ \alpha\} \mathrm{d} \alpha & \\ & =\int_{0}^{1} 0.5\{(0-6) \alpha+6,4-(4-5) \alpha,(2-1) \alpha+1,0-(0-1) \\ \alpha\} \mathrm{d} \alpha & =4.75 \end{aligned}$ |
| :---: | :---: |
| $\mathrm{C}_{12}=(10,12,13,2,4,6,8)$ | $\begin{aligned} & \mathrm{R}\left(\mathrm{C}_{12}\right)=\int_{0}^{1} 0.5\{(12-10) \alpha+10,2-(2-13) \alpha,(6-4) \alpha+4,8-(8- \\ & \text { 4) } \alpha\} \mathrm{d} \alpha \\ & =14.75 \end{aligned}$ |
| $\mathrm{C}_{13}=(6,5,2,1,3,4,2)$ | $\begin{aligned} & \mathrm{R}\left(\mathrm{C}_{13}\right)=\int_{0}^{1} 0.5\{(5-6) \alpha+6,1-(1-2) \alpha,(4-3) \alpha+3,2-(2-3) \\ & \alpha\} \mathrm{d} \alpha \\ & \quad=6.5 \end{aligned}$ |
| $\mathrm{C}_{21}=(3,6,5,2,3,4,1)$ | $\begin{aligned} & \mathrm{R}\left(\mathrm{C}_{21}\right)=\int_{0}^{1} 0.5\{(6-3) \alpha+3,2-(2-5) \alpha,(4-3) \alpha+3,1-(1-3) \\ &\alpha\} \mathrm{d} \alpha \\ &=6.75 \end{aligned}$ |


| $\left.\mathrm{C}_{22}=1,1,0,4,3,2,3\right)$ | $\begin{aligned} \mathrm{R}\left(\mathrm{C}_{22}\right) & =\int_{0}^{1} 0.5\{(1-1) \alpha+1,4-(4-0) \alpha,(2-3) \alpha+3,3-(3-3) \\ \alpha\} \mathrm{d} \alpha & =4.25 \end{aligned}$ |
| :---: | :---: |
| $\mathrm{C}_{23}=(11,13,3,5,7,9,1)$ | $\begin{aligned} & \mathrm{R}\left(\mathrm{C}_{23}\right)=\int_{0}^{1} 0.5\{(13-11) \alpha+11,5-(5-3) \alpha,(9-7) \alpha+7,1-(1- \\ & \text { 7) } \alpha\} \mathrm{d} \alpha \\ & =14 \end{aligned}$ |
| $\mathrm{C}_{31}=(6,3,2,1,2,7,7)$ | $\begin{aligned} & \mathrm{R}\left(\mathrm{C}_{31}\right)=\int_{0}^{1} 0.5\{(3-6) \alpha+6,1-(1-2) \alpha,(7-2) \alpha+2,7-(7-2) \\ & \alpha\} \mathrm{d} \alpha \\ & =7.5 \end{aligned}$ |
| $\mathrm{C}_{32}=(2,1,8,7,3,6,3)$ | $\begin{aligned} \mathrm{R}\left(\mathrm{C}_{32}\right) & =\int_{0}^{1} 0.5\{(1-2) \alpha+2,7-(7-8) \alpha,(6-3) \alpha+3,3-(3-3) \\ \alpha\} \mathrm{d} \alpha & =8.25 \end{aligned}$ |
| $\mathrm{C}_{33}=(5,0,4,3,6,2,1)$ | $\begin{aligned} & \mathrm{R}\left(\mathrm{C}_{33}\right)=\int_{0}^{1} 0.5\{(0-5) \alpha+5,3-(3-4) \alpha,(2-6) \alpha+6,1-(1-6) \\ & \alpha\} \mathrm{d} \alpha \\ & =6.75 \end{aligned}$ |

Rank of all Fuzzy Available

| $\alpha_{1}=(6,5,2,4,3,2,1)$ | $\begin{aligned} & \mathrm{R}\left(\alpha_{1}\right)=\int_{0}^{1} 0.5\{(5-6) \alpha+6,4-(4-2) \alpha,(2-3) \alpha+3,1-(1-3) \\ & \alpha\} \mathrm{d} \alpha \\ & \quad=6.5 \end{aligned}$ |
| :---: | :---: |
| $\alpha_{2}=(5,0,1,2,1,4,3)$ | $\begin{aligned} & \mathrm{R}\left(\alpha_{2}\right)=\int_{0}^{1} 0.5\{(0-5) \alpha+5,2-(2-1) \alpha,(4-1) \alpha+1,3-(3-1) \\ &\alpha\} \mathrm{d} \alpha \\ &=4.25 \end{aligned}$ |
| $\alpha_{3}=(1,1,0,2,1,2,2)$ | $\begin{aligned} & \mathrm{R}\left(\alpha_{3}\right)=\int_{0}^{1} 0.5\{(1-1) \alpha+1,2-(2-0) \alpha,(2-1) \alpha+1,2-(2-1) \\ &\alpha\} \mathrm{d} \alpha \\ &=2.5 \end{aligned}$ |

## Rank of all Fuzzy Requirement

| $\beta_{1}=(6,3,2,1,2,7,7)$ | $\mathrm{R}\left(\beta_{1}\right)=\int_{0}^{1} 0.5\{(3-6) \alpha+6,1-(1-2) \alpha,(7-2) \alpha+2,7-(7-2) \alpha\} \mathrm{d} \alpha$ <br> $=7.5$ |
| :--- | :--- |
| $\beta_{2}=(6,0,5,0,1,2,4)$ | $\mathrm{R}\left(\beta_{2}\right)=\int_{0}^{1} 0.5\{(0-6) \alpha+6,0-(0-5) \alpha,(2-1) \alpha+1,4-(4-1)$ <br> $\alpha\} \mathrm{d} \alpha$ |
| $\beta_{3}=(6,2,5,4,0,6,4)$ | $\mathrm{R}\left(\beta_{3}\right)=\int_{0}^{1} 0.5\{(2-6) \alpha+6,4-(4-0) \alpha,(6-0) \alpha+0,4-(4-0)$ <br> $\alpha\} \mathrm{d} \alpha=5.5$ |

Table - 6.2 Crisp Transportation Problem

|  | I | II | III | Fuzzy <br> Available |
| :---: | :---: | :---: | :---: | :---: |
| I | 4.75 | 14.75 | 6.5 | 6.5 |
| II | 6.75 | 4.25 | 14 | 4.25 |
| III | 7.5 | 8.25 | 6.75 | 2.5 |
| Fuzzy <br> Requirement | 7.5 | 9.5 | 5.5 |  |

Step 2
Using the Dummy Variable we get the following table
Table-6.3 Balanced Fuzzy Transportation Problem

| I | II | III | Fuzzy <br> Available |  |
| :---: | :---: | :---: | :---: | :---: |
| I | 4.75 | 14.75 | 6.5 | 6.5 |
| II | 6.75 | 4.25 | 14 | 4.25 |
| III | 7.5 | 8.25 | 6.75 | 2.5 |
| Dummy Variable | 0 | 0 | 0 | 9.25 |
| Fuzzy <br> Require ment | 7.5 | 9.5 | 5.5 |  |

Since $\sum \alpha_{i}=\sum \beta_{j}$ there exist a basic feasible solution to this problem and is displayed in the following table by using VAM, New Method \& BCM.

## Step 3:

Table-6.4 IBFS by using fuzzy version of Vogel's Approximation Method


The above table satisfies the rim conditions with ( $\mathrm{m}+\mathrm{n}-1$ ) non-negative allocations at independent position.

The generalized fuzzy optimal solutions are
$\mathrm{x}_{11}=6.5, \mathrm{x}_{22}=4.25, \mathrm{x}_{32}=2.5, \mathrm{x}_{41}=1, \mathrm{x}_{42}=2.75, \mathrm{x}_{43}=5.5$
Therefore, the total transportation cost using Vogel's Approximation Method is

Minimize $\mathrm{z}=(6.5 * 4.75)+(4.25 * 4.25)+(2.5 * 8.25)+(1 * 0)+(2.75 * 0)+(5.5 * 0)$

$$
\begin{aligned}
& =30.875+18.0625+20.625+0+0+0 \\
& =69.5625
\end{aligned}
$$

## Step 4:

Table 6.5 IBFS by using fuzzy version of New Method


The above table satisfies the rim conditions with ( $\mathrm{g}+\mathrm{h}-1$ ) non-negative allocations at independent position.

The generalized fuzzy optimal solutions are
$\mathrm{x}_{11}=6.5, \mathrm{x}_{22}=4.25, \mathrm{x}_{31}=1, \mathrm{x}_{32}=1.5, \mathrm{x}_{42}=3.75, \mathrm{x}_{43}=5.5$
Therefore, The total transportation cost using New Method is
Minimize $\mathrm{z}=(6.5 * 4.75)+(5.5 * 0)+(3.75 * 0)+(4.25 * 4.25)+(1.5 * 8.25)+(1 * 7.5)$

$$
\begin{aligned}
& =30.875+0+0+18.0625+12.375+7.5 \\
& =68.8125
\end{aligned}
$$

## Step 5:

Table 6.6 IBFS by using fuzzy version of Best Candidate Method

| I | $\begin{array}{ll}6.5 \\ & \\ & \underline{4.75} \\ \end{array}$ |  | 14.75 |  | 6.5 |  | 6.5/0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| II | 6.75 |  | $\begin{array}{ll}4.25 \\ & \\ & \underline{4.25} \\ & \end{array}$ |  | 14 |  | 4.25/0 |
|  |  |  |  |  |  |  |  |
| III | 7.5 |  | 8.25 |  | 2.5 |  | 2.5/0 |
|  |  |  | 6.75 |  |  |  |
|  | 1 |  |  |  | 5.25 |  | 3 | 0 | 9.25/8.25/3/0 |
|  |  | 0 | 0 |  |  |  |  |  |
| Fuzzy <br> Require ment | 7.5/1/0 |  | 9.5/5.25/0 |  | 5.5/3/0 |  |  |  |  |

The above table satisfies the rim conditions with ( $\mathrm{m}+\mathrm{n}-1$ ) non-negative allocations at independent position.

The generalized fuzzy optimal solutions are
$x_{11}=6.5, x_{22}=4.25, x_{33}=2.5, x_{41}=1, x_{42}=5.25, x_{43}=3$
Therefore, The total transportation cost using Best Candidate Method is
Minimize $\mathrm{z}=(6.5 * 4.75)+(4.25 * 4.25)+(2.5 * 6.75)+(1 * 0)+(5.25 * 0)+(3 * 0)$

$$
\begin{aligned}
& =30.875+18.0625+16.875+0+0+0 \\
& =65.8125
\end{aligned}
$$

COMPARATIVE STUDY
COMPARISION TABLE

| METHOD | OPTIMUM SOLUTION |
| :---: | :---: |
| Vogel's Approximation <br> Method | 69.5625 |
| New method | 68.8125 |
| Best Candidate Method | 65.8125 |

## CONCLUSION

In the above problem we have obtained an optimal solution for a FTP using heptagonal fuzzy number. A new approach called Fuzzy version of Vogel's Approximation method, New Method and Best Candidate Method is analyzed. By comparing these methods, Best Candidate Method gives a better result than the other methods. This method is an easy approach compared to other methods to solve a fuzzy transportation problem.

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