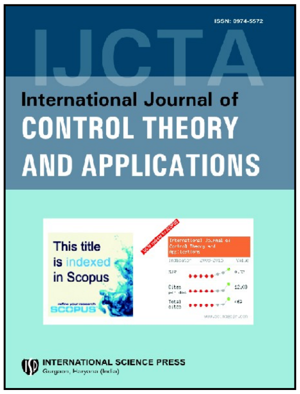

International Journal of Control Theory and Applications

ISSN : 0974-5572

Volume 10 • Number 11 • 2017

A Fuzzy based Two-warehouse Inventory Model for Non-instantaneous Deteriorating Items with Conditionally Permissible Delay in Payment

Ajay Singh Yadav[1], Sanjai Sharma[2] and Anupam Swami[3]

[1]*Department of Mathematics, SRM University, NCR Campus, Ghaziabad (U.P.), India, E-mail: ajay29011984@gmail.com*
[2]*Centre for Mathematical Sciences, Banasthali University, Rajasthan, India*
[3]*Department of Mathematics, Govt. P.G. College, Sambhal, (U.P.), India*

Abstract: In this paper, a fuzzy based two-ware house inventory model for non-instantaneous deteriorating items with conditionally permissible delay in payment is formulated and solved. Fuzziness is introduced by allowing cost components (holding cost, purchasing cost, selling price etc.), demand rate. To increase sales, supplier/wholesaler officers trade credit financing scheme to their retailers. Due to the uncertainty in the present business scenario, fuzzy concept may involve to deal with such problems. In fuzzy environment, all related parameters are assumed to be triangular fuzzy numbers. Shortages are not permissible. This paper mainly deals with non-instantaneous deteriorating items and trade credit financing with objective to derive the optimal replenishment policy which maximizes the total profit function of the retailer. First crisp model is developed and corresponding fuzzy model is formulated and solved. Numerical example is presented to illustrate and validate the model. Sensitivity analysis is performed to study the effect in the profit function with respect to change in parameter's value.

Keywords: Two warehouses, Non-instantaneous deterioration, Permissible delay in Payment, fuzzy triangular number, signed distance method, fuzzy demand.

1. INTRODUCTION

Due to the uncertainty factor involved in the business transaction and introduction of new products (fashionable items, garments, electronic items mobile etc.), a change in any cost components of an inventory model may occurs and cannot be determined exactly in advance until the exact situation is not known. There is long history of gradual progress in the development of inventory models related to decision making process to optimize the inventory cost. The most important role of the management is to decide when and how much to be ordered so that the cost of an inventory system could minimize and hence tomaximize the profit. Usually researcher the holding cost rate, demand rate and other inventory cost rate etc. as fixed, but all of them probably will have some little fluctuations for each cycle in real life situation. So in practical situations, if these quantities are treated as fuzzy variable then it will be more realistic. The concept of fuzziness was first introduced by the Lotfi A. Zadeh[1].

Many authors developed inventory models using fuzzy concept. L.A. Zadeh and R.E.Bellman[2] considered an inventory model on decision making in fuzzy environment. R. Jain[3] developed a fuzzy inventory model on decision making in the presence of fuzzy variables. D. Dubois and H. Prade[4] defined some operations on fuzzy numbers. In general, the demand is to be considered either constant or increasing with time. Sujit D. Kumar, P.K. Kund and A. Goswami[5] developed an economic production quantity model with fuzzy demand and deterioration rate. J.K. Syed and L.A.Aziz[6] consider a signed distance method for a fuzzy inventory model without shortages. P.K. De and A. Rawat[7] developed a fuzzy inventory model without shortages using triangular fuzzy number. C.K. Jaggi, S.Pareek, A. Shrma and Nidhi[8] developed a fuzzy inventory model for deteriorating items with time varying demand and shortages. D. Datta and Pawan Kumar[9]considers an optimal replenishment policy for an inventory model without shortages assuming fuzziness in demand. Halim *et al.*[10]. developed a fuzzy inventory model for perishable items with stochastic demand, partial backlogging and fuzzy deterioration rate. Goni and Maheshwari[11] discussed the retailer's ordered policy under the two level of delay in payments considering the demand and selling price as triangular fuzzy numbers. They used graded mean integration representation method for defuzzification. Halim et al.[12] addressed a lot sizing problem in an unreliable production system with stochastic machine breakdown and fuzzy repair time using the signed distance method. Singh, S.R. and Singh, C[13]. Developed a fuzzy inventory model for finite rate of replenishment using signed distance method.

In real life each product has self-life span and after the expiry of self- life period, these products become useless. Some product deteriorated with time span. Most of the inventory models consider that products start deteriorated as soon as it entered into inventory system. Many researchers have developed inventory models under consideration that the products are deteriorated instantly which is not always realistic in real life situations. There are many items which do not deteriorate at instant such as steel furniture; plastic items, electronic goods etc. and such items are known as "Non-instantaneous" deteriorating items. Many researchers have developed inventory models for non-instantaneous deteriorating items. K.S. Wu *et al.*[14] defines non-instantaneous deteriorating term and considered the problem of determining the optimal replenishment policy for such items with stock dependent demand. Soon, L.Y.Ouyang *et al.*[15] further developed an inventory model for non-instantaneous deteriorating items with permissible delay in payment. Partha G. *et. al.*[16] developed an inventory model under stock and selling price dependent demand rate. Maiti M.K., Maiti Manmohan[17] developed an inventory model under two warehouse facilities with advertisement price and displayed inventory level–dependent demand rate in fuzzy environment. Singh Ajay Yadav and Swami Anupam[18]developed a two warehouse inventory model for decaying items with exponential demand and variable holding cost.

Maiti M. K., MaitiManmohan[19]developed a multi-item inventory models with stock dependent demand and two storage facilities are developed in fuzzy environment. Rong M, *et.al.*[20] introduced an optimization inventory policy for a deteriorating item with imprecise lead time has been developed under two warehouse system in their paper. Datta P. Chaudhary P.D.[21] developed a fuzzy based two warehouse inventory model has been developed for decaying items with cubic demand under fuzzy environment. Mandal A. W. and Islam S.[22]developed afuzzy inventory model for non–deteriorating item with power demand pattern under partially backlogged shortages with two warehouse system. A fuzzy inventory model has been introduced under two warehouse management with price dependent demand rate in fuzzy environment by Maragatham M. and Lakshmi Devi P.K.[23] Kumar Neeraj *et. al.*[24] developed a finite horizon inventory problem for deteriorating item with two warehouses in fuzzy environment. Nita H. Shah, *et. al.*[25] introduces an inventory system with non-instantaneous deteriorating item and advertisement and selling price dependent demand rate. Panda S. *et.al.*[26] has proposed a dynamic pre and post deterioration cumulative discount policy to enhance inventory depletion rate resulting low volume of deterioration cost, holding cost to increase the profit. Chin T.Y.[27] developed an inventory model under a stock-dependent demand rate and stock dependent holding cost rate with relaxed terminal conditions. Maiti, M.K.[28] used genetic algorithm to solve the developed inventory model with credit linked promotional demand rate in an imprecise planning horizon. The research paper of Sharma V. and Chuadhary R.R.[29] deals with in developing an inventory model for deteriorating items with the weibull deterioration rate following two parameters. Sarkar B., Sarkar S. introduces an improved inventory model with partial backlogging, time varying deterioration and stock dependent demand with time varying backlogging rate as well as time varying deterioration rate.

In literature, it is assumed that the deterioration rate in the both warehouse are of same type i.e. either constant or time dependent but it is realistic in practical situations. It may vary in both warehouses depending upon the facilities provided there at. During the permissible delay period retailer accumulates money by earning interest on sales revenue to reduce his total inventory cost and maximize profit and therefore, order for more items may be placed. Due toan attractive price discount or when the cost of procuring goods is higher than the other inventory related costs or, when demand for items is very high or, when the item under consideration is a seasonal product such as the yield of a harvest or, when there are some problems in frequent procurement, retailer may purchase goods in bulk which require more space for storage which force to rent another house known as RW.

In this paper, first a deterministic crisp inventory model for non-instantaneous deteriorating items with two levels of storage and constant demand is developed under consideration that the delay in payment is permitted. It is assumed that items are deteriorated after a fixed time span and deterioration rate in the both warehouses are different and deterioration cost rate is taken equal in both warehouses. Assuming fuzziness in demand rate, holding cost rates in both warehouses, and deterioration rate a corresponding fuzzy model is developed and solved using signed distance method. Different cases depending upon the permissible delay period offered by supplier are discussed and results are obtained. Numerical example is presented to illustrate the developed mode.

2. ASSUMPTION AND NOTATIONS

The model is developed under the following assumptions and notations:

2.1. Assumptions

- Replenishment rate is infinite and lead time is zero.
- Shortages are not permitted.
- The time horizon of the inventory system is infinite.
- Goods of OW are consumed only after the consumption of goods stored into RWto reduce the holding cost in RW.
- The OW has the limited capacity of storage and RW has unlimited capacity.
- Demand rate is constant i.e. $f(t) = d$ and is fuzzy triangular numberin case of fuzzy model.
- Goods are not deteriorated till a fixed time or self-life period of product. To ensure the solution of model the deteriorated amount of goods in OW must be less than the total demand i.e. $\alpha W < \int_{t_\mu}^{t_v} d\ dt$
- The unit inventory cost (Holding cost + deterioration cost) in RW>OW i.e. $(h_r - h_w) > c(\theta(t) - \alpha)$ where $c > 0$.
- Retailer pays purchase cost to supplier either at the end of permissible delay period or at the time of ordering for next cycle and earns revenue in terms of interest after sales of goods till the payment is made.
- Goods are instantly transported from RW to retails shop on the basis of continuous release pattern and transportation cost is incurred.
- The items are deteriorated at different rate in both warehouses. In OW the deterioration rate is linear time dependent.

2.2. Notations

A_c : Cost of Ordering per Order.

W : Capacity of OW.

T : The length of replenishment cycle ant time point up to which inventory vanishes in OW.

M : Permissible delay period.

t_μ : The point of time up to which inventory does not deteriorated.

t_v : The point of time at which inventory level vanishes in RW.

h_r : The holding cost per unit time in OW.

h_w : The holding cost per unit time in RW.

α : Deterioration rate in RW which is constant and $0 < \alpha < 1$.

$\theta(t)$: Deterioration rate in OW which is time dependent and given by βt where $\beta > 0$

S_p : Selling price per unit of item

I_p : Interest charges per unit of time.

I_e : Interest earned per unit of time.

$I_{r,i}(t)$: The level of inventory in RW at time point t for $i = 1, 2$

$I_{w,i}(t)$: The level of inventory in OW at time epoch t for, 3.

Q_{max} : Number of inventory ordered at.

R^i : Revenue earned for cases

$Л^{i,j}(t_w, t_v, T)$: The total profit function per unit time for cases $i = 1, 2, 3, 4$ and $j = 0, 1, 2$

$Л^{max}(t_w, t_v, T)$: The optimal profit function per unit time for cases.

{~ Sign represent the fuzziness of the parameters}

Triangular fuzzy number: A triangular fuzzy number is specified by the triplet (a, b, c) where $a < b < c$ and defined by its continuous membership function $\lambda_{A^\sim} : X \to [0,1]$ as follows:

$$\lambda_{A^\sim}(x) = \begin{cases} \dfrac{x - a_1}{c - a_1} & if\ a_1 \le x \le b_1 \\ \dfrac{b_1 - x}{c_1 - b_1} & if\ c_1 \le x \le c_1 \\ 0 & otherwise \end{cases}$$

3. MATHEMATICAL FORMULATION OF MODEL

3.1. Crisp model

Initially, Q_{max} unitsof itemsare arrived and W units is kept intoOW and remaining $Q_{max} - W$ are stocked in RW. (See Figure 1).

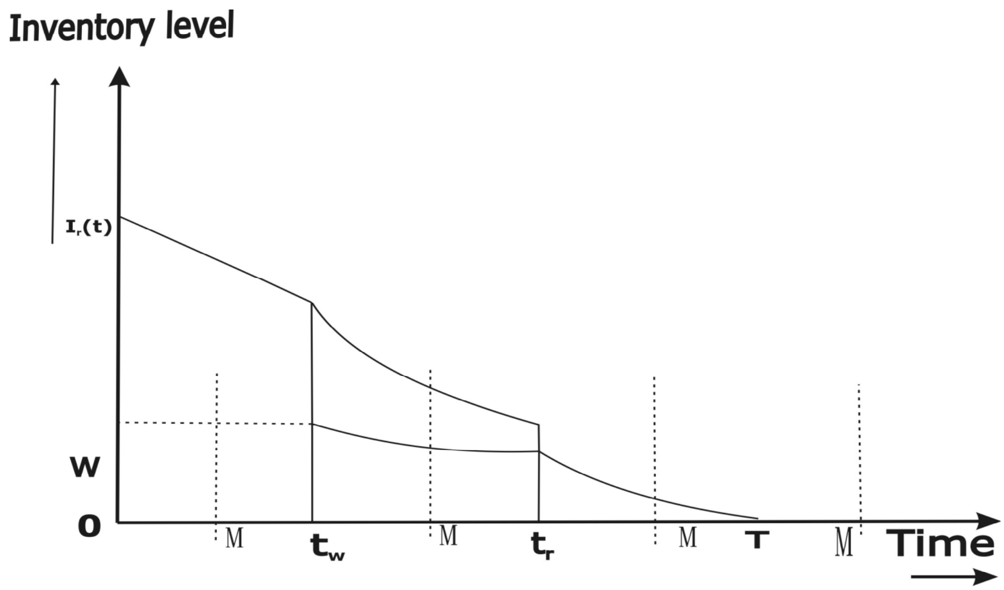

Figure 1: Graphical representation for the inventory model

During time interval [0 t_μ], inventory level in RW depletes due to demand only and in this period inventory level in OW remains W unit. The situation describing inventory level is governed by the following differential equations:

$$\frac{dI_{r,1}(t)}{dt} = -f(t); \qquad 0 \le t \le t_\mu \tag{1}$$

$$\frac{dI_{w,1}(t)}{dt} = 0; \qquad 0 \le t \le t_\mu \tag{2}$$

In time interval [0 t_v], inventory level in RW depletes due to the combined effect of demand and deterioration and reaches to zero at time $t = t_1$. In OW it depletes due to deterioration only. The situation is governed by following differential equations:

$$\frac{dI_{r,2}(t)}{dt} = -\alpha I_r(t) - f(t); \qquad t_w \le t \le t_v \tag{3}$$

$$\frac{dI_{w,2}(t)}{dt} = -\theta(t) I_{w,2}(t); \qquad t_w \le t \le t_v \tag{4}$$

Now, attime $t = t_v$, when stock vanishes in RW and supply for demand is made from OW stocks. Stocks in OW in the time interval [t_v T] depletesdue to combined effects of demand and deterioration. Differential equation describing the situation is given as

with boundary conditions $I_{r,1}(t) = I_{r,2}(t)\ at\ t = t_\mu, I_{r,2}(t) = 0\ at\ t = t_v, I_{w,1}(t) = W\ at\ t = 0,$ solutions of equations. (1) to (5) are respectively given by

$I_{w,2}(t) = W\ at\ t = t_\mu\ and\ I_{w,3}(t) = 0\ at\ t = T$ are as follows:

$$I_{r,1}(t) = D(t_\mu - t) + \frac{D}{\alpha}(e^{\alpha(t_1 - t_\mu)} - 1); \quad 0 \le t \le t_\mu \tag{6}$$

$$I_{r,2}(t) = \frac{D}{\alpha}(e^{\alpha(t_1 - t)} - 1); \quad t_\mu \le t \le t_v \tag{7}$$

$$I_{w,1}(t) = W; \qquad 0 \le t \le t_\mu \tag{8}$$

$$I_{w,2}(t) = W\ e^{\frac{\beta}{2}(t_w^2 - t^2)}; \qquad t_\mu \le t \le t_v \tag{9}$$

$$I_{w,3}(t) = D\left(\left(T - t\right) + \frac{\beta}{6}(T^3 - t^3)\right)e^{-\frac{\beta}{2}t^2}; \ t_\mu \le t \le T \tag{10}$$

Since $t = 0$, $Q_{max} - W = I_{r,1}(0)$ which gives

$$Q_{\max} = W + D\ t_\mu + \frac{D}{\alpha}(e^{\alpha(t_1 - t_\mu)} - 1) \tag{11}$$

The maximum ordered quantity is given by

$$Q_{\max} = W + Dt_\mu + \frac{D}{\alpha}(e^{\alpha(t_1 - t_\mu)} - 1) \tag{12}$$

Now, total inventory cost consist of the following components:

Ordering cost: C_o (13)

Holding cost: C_o

$$h_r\left(\int_0^{t_\mu} I_{r,1}(t)dt + \int_{t_\mu}^{t_1} I_{r,2}(t)dt\right) \text{ in RW and} \tag{14}$$

$$h_w\left(\int_0^{t_\mu} I_{w,1}(t)dt + \int_{t_\mu}^{t_v} I_{w,2}(t)dt + \int_{t_v}^{T} I_{w,3}(t)dt\right) \text{ in OW}$$

Purchase cost: $t_c\ Q_{max}$ (15)

Transportation cost: $t_c \int_0^{t_1} f(t)dt$ (16)

SinceM is permissible delay periodbeyond which an interest will be charged by the supplier. Asper pictorial representation of inventory level at Figure 1, there may arise the following cases(depending upon M) which are discussed separately in the section 4.0.

4. CASE ANALYSIS

Case-1: $0 < M \le t_\mu$

Case-2: $t_\mu < M \le t_v$

Case-3: $t_v < M \le T$

Case-4: $M < T$

Case-1.0: 0 $< M \le t_\mu$

If retailer wishes to pay full amount to the supplier at $t = M$ then he does not pay any interest and earn interest form his sales revenue till end of cycle length. Therefore interest earned by the retailer is

Subcase-1 .1

$$\begin{aligned} IE_1 \quad = \quad & S_P I_e \int_0^M tf(t)dt + I_e \int_M^T \left(S_P \int_0^M f(t)dt + \left(S_P I_e \int_0^M tf(t)dt \right) \right) dt \\ & + S_P I_e \int_{t_\mu}^M tf(t)dt + I_e \int_{t_\mu}^T \left(S_P \int_{t_\mu}^M f(t)dt + \left(S_P I_e \int_{t_\mu}^M tf(t)dt \right) \right) dt \\ & + S_P I_e \int_{t_\mu}^{t_\nu} tf(t)dt + I_e \int_{t_\nu}^T \left(S_P \int_{t_\mu}^{t_1} f(t)dt + \left(S_P I_e \int_{t_\mu}^M tf(t)dt \right) \right) dt \\ & + \left(S_P I_e \int_{t_\nu}^T tf(t)dt \right) \end{aligned} \tag{1.1}$$

$$IP_{1.1} = 0 \tag{1.2}$$

Therefore totalrevenue earned is

$$R^{1.1} = \quad S_P \int_0^M f(t)dt + S_P \int_M^T f(t)dt + IE_{1.1} \tag{1.3}$$

Subcase-2: If retailer wishes topay total purchase cost at the end of cycle length that is at the time of next replenishment i.e. at t =T.In this case retailer has to pay interestbeyond M and simultaneously he earns interest on his sales revenue till the payment is made.

Interest earned is

$$\begin{aligned} IE_{1.1} = S_P I_e \int_0^M tf(t)dt + I_e \int_M^T \left(S_P \int_0^M f(t)dt + \left(S_P I_e \int_0^M tf(t)dt \right) \right) dt \\ + \quad S_P I_e \int_{t_\mu}^M tf(t)dt + I_e \int_{t_\mu}^T \left(S_P \int_{t_\mu}^M f(t)dt + \left(S_P I_e \int_{t_\mu}^M tf(t)dt \right) \right) dt \\ + \quad S_P I_e \int_{t_\mu}^{t_\nu} tf(t)dt + I_e \int_{t_\nu}^T \left(S_P \int_{t_\mu}^{t_1} f(t)dt + \left(S_P I_e \int_{t_\mu}^M tf(t)dt \right) \right) dt \\ + \left(S_P I_e \int_{t_\nu}^T tf(t)dt \right) \end{aligned} \tag{1.4}$$

$$IP_{1.2} = I_P \int_M^T P_c Q_{max} dt \ ; \tag{1.5}$$

Therefore totalrevenue earned is

$$R^{1.2} = \quad S_P \int_0^M f(t)dt + S_P \int_M^T f(t)dt + IE_1 \tag{1.6}$$

Case-2.0: $t_\mu < M \le t_\nu$

Subcase-2 .1 If retailer wishes to pay full amount to the supplier at then he does not pay any interest and earn interest form his sales revenue till end of cycle length. Therefore interest earned by the retailer is

$$\begin{aligned} IE_{2.1} = {} & S_P I_e \int_0^M t f(t)dt + I_e \int_M^T \left(S_P \int_0^M f(t)dt + \left(S_P I_e \int_0^M t f(t)dt \right) \right) dt \\ & + S_P I_e \int_M^{t_\nu} t f(t)dt + I_e \int_{t_\nu}^T \left(S_P \int_M^{t_1} f(t)dt + \left(S_P I_e \int_M^{t_\mu} t f(t)dt \right) \right) dt \\ & + \left(S_P I_e \int_{t_\nu}^T t f(t)dt \right) \end{aligned} \tag{2.1}$$

$$IP_{2.1} = I_P \int_M^T P_c Q_{max} dt \tag{2.2}$$

Therefore total revenue earned is

$$R^{2.1} = S_P \int_0^T f(t)dt + IE_1; \tag{2.3}$$

Subcase-2.2: If retailer pays total purchase cost at the end of cycle length that is at the time of next replenishment i.e. at t =T. In this case retailer has to pay interest beyond M and simultaneously he earns interest on his sales revenue till the payment is made.

$$\begin{aligned} IE_{2.2} = {} & S_P I_e \int_0^M t f(t)dt + I_e \int_M^T \left(S_P \int_0^M f(t)dt + \left(S_P I_e \int_0^{t_\mu} t f(t)dt \right) \right) dt \\ & + S_P I_e \int_M^{t_\nu} t f(t)dt \\ & + I_e \int_{t_\nu}^T \left(S_P \int_M^{t_\nu} f(t)dt + \left(S_P I_e \int_M^{t_\nu} t f(t)dt \right) \right) dt \\ & + \left(S_P I_e \int_{t_\nu}^T t f(t)dt \right) \end{aligned} \tag{2.4}$$

$$IP_{2.2} = I_P \int_M^T P_c Q_{max} dt \tag{2.5}$$

Therefore total revenue earned is

$$R^{2.2} = S_P \int_0^T f(t)dt + IE_{2.2} \tag{2.6}$$

Case-3.0: $t_\nu < M \le T$

Subcase-3 .1 If retailer wishes to pay full amount to the supplier at $t = M$ then he does not pay any interest and earn interest form his sales revenue till end of cycle length. Therefore interest earned by the retailer is

$$\begin{aligned} IE_{3.1} = {} & S_P I_e \int_0^M t f(t)dt + I_e \int_M^T \left(S_P \int_0^M f(t)dt + \left(S_P I_e \int_0^M t f(t)dt \right) \right) dt \\ & + \left(S_P I_e \int_M^T t f(t)dt \right) \end{aligned} \tag{3.1}$$

$$IP_{3.1} = I_P \int_{\mathrm{M}}^{T} P_c Q_{max} dt \tag{3.2}$$

Therefore total revenue earned is

$$\mathrm{R}^{3.1} = \quad S_P \int_0^T f(t)dt \ + IE_{3.1} \tag{3.3}$$

Subcase-3.2 If retailer pays total purchase cost at the end of cycle length that is at the time of next replenishment i.e. at t =T. In this case retailer has to pay interest beyond M and simultaneously he earns interest on his sales revenue till the payment is made.

$$IE_{3.2} = \ S_P I_e \int_0^M tf(t)dt + I_e \int_M^T \left(S_P \int_0^M f(t)dt \ + \left(S_P I_e \int_0^M tf(t)dt \right) \right) dt + \left(S_P I_e \int_M^T tf(t)dt \right) \tag{3.4}$$

$$IP_{3.2} = I_P \int_{\mathrm{M}}^{T} P_c Q_{max} dt \tag{3.5}$$

Therefore total revenue earned is

$$\mathrm{R}^{3.2} = \quad S_P \int_0^T f(t)dt \ + IE_{1.1} \tag{3.6}$$

Case-4.0: *M < T*

In this case also retailer has not to pay any interest since the permissible delay period is more than the ordering cycle length and accumulate interest on revenue collected from the sales, therefore

$$IE_{4.0} = \quad S_P I_e \int_0^T tf(t)dt + I_e \int_T^M \left(S_P \int_0^T f(t)dt \ + \left(S_P I_e \int_0^T tf(t)dt \right) \right) dt \tag{4.1}$$

and interest paid is

$$IP_{4.0} = 0;$$

Therefore total revenue earned is

$$\mathrm{R}^{4.0} = \quad S_P \int_0^T f(t)dt \ + IE_{4.0} \tag{4.2}$$

Now the total average profit function for the model in case and subcase is given by

$$Л^{i.j}(t_\mu, t_\nu \ , \mathrm{T}) = \frac{1}{T}\left[\mathrm{R}^{i.j} - (ordering\ cost + holding\ cost + purchase\ cost + transportation\ cost + interest\ paid)\right]$$

$$Л^{i.j}(t_\mu, t_\nu \ , \mathrm{T}) = \frac{1}{T}\Big[S_P \int_0^M d\ dt \ + S_P \int_{\mathrm{M}}^T d\ dt \ + IE_{1.1} - \left(A_c + h_r \left(\int_0^{t_\mu} I_{r,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{r,2}(t)dt\right) + h_w \left(\int_0^{t_\mu} I_{w,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{w,2}(t)dt + \int_{t_\nu}^{T} I_{w,3}(t)dt\right) + P_c \left(W + d\ t_\mu + \frac{d}{\alpha}\left(e^{\alpha(t_1 - t_\mu)} - 1\right)\right) + \ t_c \int_0^{t_\nu} d\ dt\right)\Big] \tag{4.3}$$

Optimality condition for crisp inventory model

The optimal problem can be formulated as

$$\text{Maximize: } Л^{i.j}(t_\mu, t_\nu\ , \mathrm{T})$$

$$\text{Subject to: } (t_\mu > 0, t_\nu > 0, \mathrm{T} > 0)$$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial Л^{i.j}(t_\mu, t_\nu\ , \mathrm{T})}{\partial t_\mu} = 0; \quad \frac{\partial Л^{i.j}(t_\mu, t_\nu\ , \mathrm{T})}{\partial t_\nu} = 0; \quad \frac{\partial Л^{i.j}(t_\mu, t_\nu\ , \mathrm{T})}{\partial \mathrm{T}} = 0\ ; \tag{4.4}$$

Solving equation (4.4) for t_μ, t_ν, T, we can obtain values as t_μ^*, t_ν^*, T^* and with these optimal values we can find the total profit from equation (4.3) for all cases discussed separately and from these cases optimal one can bechoosen depending upon the permissible delay period

3.2. Fuzzy Model

The above developed model is a crisp model in which all assumed parameters are fixed but in fuzzy environment, the parameter's values are not fixed and fluctuated around a fixed point in some interval.

Using equation (4.3) and fuzzy parameters $H_r = (\tilde{h}_{r1}, \tilde{h}_{r2}, \tilde{h}_{r3}), H_w = (\tilde{h}_{w1}, \tilde{h}_{w2}, \tilde{h}_{w3})$, $P_c = (\tilde{P}_{c\,1}, \tilde{P}_{c\,2}, \tilde{P}_{c\,3})$ and $d = (\tilde{d}_1, \tilde{d}_2, \tilde{d}_3)$, $S_p = (\tilde{S}_{p1}, \tilde{S}_{p2}, \tilde{S}_{p3})$; we have,

$$\tilde{ЛГ}^{i.j}(t_\mu, t_1\ , \mathrm{T}) = (\tilde{ЛГ}_1^{i.j}(t_\mu, t_1\ , \mathrm{T}), \tilde{ЛГ}_2^{i.j}(t_\mu, t_1\ , \mathrm{T}), \tilde{ЛГ}_3^{i.j}(t_\mu, t_1\ , \mathrm{T})) \tag{4.5}$$

where

$$\tilde{ЛГ}_1^{i.j}(t_\mu, t_1\ , \mathrm{T}) = \frac{1}{T}\Big[S_P \int_0^M \tilde{d}_1\, dt + S_P \int_M^T \tilde{d}_1\, dt + IE_{1.1} - \Big(\tilde{A}_1 + \tilde{h}_{r1}\Big(\int_0^{t_\mu} I_{r,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{r,2}(t)dt\Big) + \tilde{h}_{w1}\Big(\int_0^{t_\mu} I_{w,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{w,2}(t)dt + \int_{t_1}^{T} I_{w,3}(t)dt\Big) + P_c\ \Big(W + \tilde{d}_1\, t_\mu + \frac{\tilde{d}_1}{\alpha}(e^{\alpha(t_1 - t_\mu)} - 1)\Big) + t_c \int_0^{t_\nu} \tilde{d}_1\, dt\Big)\Big];$$

$$\tilde{ЛГ}_2^{i.j}(t_\mu, t_\nu\ , \mathrm{T}) = \frac{1}{T}\Big[S_P \int_0^M \tilde{d}_2\, dt + S_P \int_M^T \tilde{d}_2\, dt + IE_{1.1} - \Big(\tilde{A}_2 + \tilde{h}_{r2}\Big(\int_0^{t_\mu} I_{r,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{r,2}(t)dt\Big) + \tilde{h}_{w2}\Big(\int_0^{t_\mu} I_{w,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{w,2}(t)dt + \int_{t_\nu}^{T} I_{w,3}(t)dt\Big) + P_c\ \Big(W + \tilde{d}_2\, t_\mu + \frac{\tilde{d}_2}{\alpha}(e^{\alpha\ (t_\nu - t_\mu)} - 1)\Big) + t_c \int_0^{t_\nu} \tilde{d}_2\, dt\Big)\Big];$$

$$\tilde{ЛГ}_2^{i.j}(t_\mu, t_\nu\ , \mathrm{T}) = \frac{1}{T}\Big[S_P \int_0^M \tilde{d}_2\, dt + S_P \int_M^T \tilde{d}_2\, dt + IE_{1.1} - \Big(\tilde{A}_2 + \tilde{h}_{r2}\Big(\int_0^{t_\mu} I_{r,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{r,2}(t)dt\Big) + \tilde{h}_{w2}\Big(\int_0^{t_\mu} I_{w,1}(t)dt + \int_{t_\mu}^{t_\nu} I_{w,2}(t)dt + \int_{t_\nu}^{T} I_{w,3}(t)dt\Big) + P_c\ \Big(W + \tilde{d}_2\, t_\mu + \frac{\tilde{d}_2}{\alpha}(e^{\alpha(t_\nu - t_\mu)} - 1)\Big) + t_c \int_0^{t_\nu} \tilde{d}_2\, dt\Big)\Big];$$

Optimality condition for fuzzy inventorysystem

The optimal problem can be formulated as

$$\text{Maximize: } Л^{\sim i.j}(t_\mu, t_\nu \ , \mathrm{T})$$

$$\text{Subject to : } \left(t_\mu > 0, t_\nu > 0, \mathrm{T} > 0\right)$$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial Л^{\sim i.j}(t_\mu t_\nu \ ,\mathrm{T})}{\partial t_\mu} = 0; \frac{\partial Л^{\sim i.j}(t_\mu, t_\nu \ ,\mathrm{T})}{\partial t_\nu} = 0; \frac{\partial Л^{\sim i.j}(t_\mu, \ t_\nu \ ,\mathrm{T})}{\partial \mathrm{T}} = 0 \ ; \tag{4.6}$$

Note: In numerical examples, the model for subcase-1.1 of case-1.0 is solved for crisp as well as for fuzzy model and similarly other cases may be solved. The calculation is performed for each subcase of three cases and for case 4.0 separately,for both models. Results of both models have been compared in the result analysis given in the section -4.1.

4. NUMERICAL EXAMPLES

In order to illustrate the above model with the help of above solution procedure, we consider the following examples:

Parameter	C_0	d	W	P_c	S_p	I_p	β	α	h_r	h_w	I_e	M
Example-1	1500	2000	100	10	15	0.15	0.10	0.06	3	1	0.10	1/12
Example-2	1200	2500	300	12	18	0.15	0.10	0.06	5	3	0.20	½

Table 1.1
Crisp Model

M=1/12						
Case	*Subcase*	t_μ	t_ν	*T*	*Average Profit*	*Remarks*
1	1.1	4.4726	13.8577	20.8225	24474.40	
	1.2	4.5961	14.5983	21.6988	22452.80	
2	2.1	4.7720	7.6209	14.2371	27626.70	
	2.2	5.6823	8.5557	15.4251	25976.80	
3	3.1	17.2034	23.5803	30.0138	9834.18	
	3.2	17.3076	23.7201	30.2147	7473.15	
4	-	-	-	-	-	Infeasible

M=1/4						
Case	*Subcase*	t_μ	t_ν	*T*	*Average Profit*	*Remarks*
1	1.1	4.6265	14.0443	21.0135	24239.30	
	1.2	4.7558	14.8608	21.9682	22223.70	
2	2.1	5.8665	8.7959	15.6457	25718.20	
	2.2	5.3607	8.2289	14.8885	27051.00	
3	3.1	17.2034	23.5803	30.0138	9846.18	
	3.2	17.3075	23.7201	30.2147	7485.04	
4	-	-	-	-	-	Infeasible

M=1/2						
Case	*Subcase*	t_μ	t_v	T	*Average Profit*	*Remarks*
1	1.1	4.8548	14.3154	21.2903	23912.00	
	1.2	4.9664	15.0087	22.1134	21878.90	
2	2.1	6.1331	9.1062	15.9702	25355.40	
	2.2	5.0187	7.8672	14.5016	27387.00	
3	3.1	17.2033	23.5801	30.0136	9885.78	
	3.2	17.3074	23.7200	30.2145	7524.76	
4	-	-	-	-	-	Infeasible

Table 1.2
Fuzzy model

M=1/12						
Case	*Subcase*	t_μ	t_v	T	*Average Profit*	*Remarks*
1	1.1	14.5853	18.0150	24.2463	14381.10	
	1.2	14.7140	18.1864	24.4768	10862.40	
2	2.1	14.6040	20.0965	26.9053	17238.40	
	2.2	14.8569	20.4347	27.3685	9752.86	
3	3.1	21.8985	29.9010	34.9463	-71458.80	
	3.2	20.8290	27.7919	33.3481	-81618.70	
4	-	-	-	-	-	Infeasible

Table 2.1
Crisp Model

Case	*Subcase*	t_μ	t_v	T	*Average Profit.*	*Remarks*
1	1.1	7.1814	23.8175	35.4715	36451.70	
	1.2	10.0684	12.9634	17.5822	9425.12	
2	2.1	9.6269	13.5051	18.3863	12820.00	
	2.2	9.7086	13.6116	18.5477	9486.85	
3	3.1	11.6060	16.1065	20.2349	-3055.79	
	3.2	11.6602	16.1782	20.4486	-6632.25	
4	-	-	-	-	-	Infeasible

Table 2.2
Fuzzy Model

Case	*Subcase*	t_μ	t_v	T	*Average I.C.*	*Remarks*
1	1.1	11.3130	14.5523	19.2457	17317.60	
	1.2	11.0489	14.2093	18.7421	32580.30	
2	2.1	11.2399	15.6227	20.7152	16217.10	
	2.2	13.8797	19.1290	23.8411	-62721.00	
3	3.1	13.6285	18.7938	23.0779	-15086.30	
	3.2	13.6760	18.8573	23.8474	-20022.20	
4	-	-	-	-	-	Infeasible

4.1. Numerical analysis

- From Table-1.1, forcrisp model, it is observed that in subcase-2.1, profit and optimal ordering cycle length both are optimal as compared to other subcases. When the permissible delay period is beyond the ordering cycle length the solution is infeasible.
- In subcases-2.1 & 2.2, and subcases-1.1&1.2, as permissible delay period increases the total profit decreases while in subcases-3.1 & 3.2 the profit is directly proportional to permissible delay period.
- From Table-1.2, for fuzzy model, it is observed that, in subcase-2.1 the profit is optimal.
- From Table-2.1, for crisp model, the profit is optimal in subcase1.1 and the replenishment is less frequent as compared to remaining cases.
- From Table-2.2, for fuzzy model, the profit is optimal in subcase1.2 and the replenishment cycle is more frequent as compared to remaining cases.
- Fixing the value of, the concavity of the optimal profit function with respect to optimal and in each subcase is depicted in Figure-2a for crisp model and in Figure-2b for fuzzy model. Because of high non-linearity of the function; concavity of the model cannot be tested analytically. The concavity of the graph shows that the solution under constrained is unique and global one.

Table 3
Table presenting sensitivity performance

Variation in parameter d^{-}			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_{μ}	4.5726	4.7019	
t_{v}	14.2267	14.9814	
T	21.2229	22.1128	
Average Profit	23211.30	21285.10	

Variation in parameter S_p			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_{μ}	0.1133	1.7789	
t_{v}	1.3227	3.7581	
T	7.3910	10.3768	
Average Profit	33030.70	32052.40	

Variation in parameter P_c			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_{μ}	4.7814	4.9003	
t_{v}	15.7871	16.6329	
T	23.1125	24.1126	
Average Profit	14000.00	11832.90	

Variation in parameter h_r			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_μ	19.4920	19.1096	
t_v	23.7210	23.8232	
T	32.2220	32.3711	
Average Profit	-779.88	-2454.07	

Variation in parameter h_w			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_μ	9.1185	9.3526	
t_v	10.3699	10.7252	
T	14.9988	15.4391	
Average Profit	14580.50	13013.70	

Variation in parameter S_p and d			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_μ	0.3901	0.9056	
t_v	0.7045	1.7850	
T	6.9363	8.3753	
Average Profit	48627.50	48149.00	

Variation in parameter S_p and P_c			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_μ	0.1473	5.0892	
t_v	3.3132	6.7004	
T	10.1037	14.2067	
Average Profit	3127.50	26396.80	

Variation in parameter h_r and h_w			
Case-1.0	Subcase:1.1	Subcase-1.2	Remarks
t_μ	15.4381	15.4979	
t_v	20.2564	20.3311	
T	25.8632	25.9771	
Average Profit	-6430.17	-8247.00	

4.2. Sensitivity Analysis

Considering example-1 mentioned in section 4.0, sensitivity analysis is performed to study the effect of fuzziness on the parameters for the optimal policy and the results are given in Table 3.

OBSERVATIONS

From Table 3, the following observations can be made:

1) The profit function is sensitive to the demand, holding cost in RW,holding cost in OW, selling price and purchase cost.
2) If the selling price is a fuzzy parameter then the profit increases as compared to the crisp model and if purchase cost is a fuzzy parameter then the profit decreases as compared to the crisp model.
3) If the demand is a fuzzy parameter then the profit decreases as compared to the crisp model and if holding cost in RW is a fuzzy parameter then the profit function is highly sensitive to it and profit decreases rapidly.
4) When holding cost in OW is a fuzzy parameter, the profit function decreases as compared to the crisp model.
5) When demand and selling price both are fluctuated together, the profit function is highly sensitive to these parameters and increases rapidly.
6) When purchase cost and selling price both fluctuated together, the profit function is highly sensitive to these parameters in subcase1.1 and decrease rapidly while in subcase1.2 increases at slow rate.
7) When holding cost in RW and holding cost in RW, both fluctuated together, the profit function is highly sensitive to these parameters in each subcase of case-1.0 and decrease rapidly.

Note: Sensitivity analysis is performed only for case-1.0 of example-1.

5. CONCLUDING REMARKS

In this paper, a deterministic two-warehouse inventory model for non-instantaneousdeteriorating items with constant demand and permissible delay period in payment is proposed under assumption that the items are transported from RW to retails shop under continuous release pattern with the objective of maximizing the total profitfunction of the inventory system. First, a crisp model is developed withfixed value of parameters and corresponding fuzzy model is developed in a fuzzy environment where the parameter's values are fluctuating around a crisp value. It isobserved thatprofit function is influenced by selling price, purchase cost,demand and holding cost rates in RW. In the different cases and subcases discussed in the model, it isobserved that in some cases, the crisp model provide the optimal solution while in some cases fuzzy model. Now it depends upon the decision maker to take decision according to the situation to optimize the profit function.Furthermore the proposed model can be used in inventory control of certain non- instantaneous deteriorating items and may be further extended by incorporating time dependent demand, probabilisticdemand pattern and variable holding cost etc.

REFERENCES

[1] Zadeh, L. A., (1965), Fuzzy Set, Information Control, (8), 338-353.

[2] Zadeh, L.A. and Bellman, R.E. (1970), Decision making in a fuzzy environment, *Management Science*, (17), 140-164.

[3] Jain, R., (1976), Decision making in the presence of fuzzy variables, *IIIE Transaction on Systems, Man and Cybernetics* (17), 698-703.

[4] Dubois,Prade, H. (1978), Operations on fuzzy numbers, InternationalJournals of System Science (9) 613-626.

[5] Kumar,S., Kundu, D. and Goswami A. (2003), An Economic Production Inventory Model Involving Fuzzy Demand Rate and Fuzzy Deterioration Rate. *Journal of Applied Mathematics and Computing,* (12)251-260.

[6] Syed, J. K., Aziz, L. A. (2007), Fuzzy Inventory Model without shortages by Using Signed Distance method.*Applied Mathematics and Information Science*, (1), 203-209.

[7] De,P.K., Rawat, A. (2001), Fuzzy Inventory Model without Shortage Using Triangular Fuzzy Number, *Fuzzy Information and Engineering*(3) 59-61.

[8] Jaggi, C. K., Pareek, S., Sharma, A., Nidhi. (2012), Fuzzy Inventory Model for Deteriorating Items wiyh Time Varying Demand and Shortages. *American Journal of Operational Research*, (2)81-92.

[9] Datta,D., Kumar P. (2012), Fuzzy Inventory Model without shortages by Using Trapezoidal Fuzzy Number with Sensitivity Analysis. *IOSR Journal of Mathematics*, (4)32-37.

[10] Halem,K.A., Giri,B.C.,Chaudhari,K.S. (2008), Fuzzy economic order quantity model for perishable items with stochastic demand, partial backlogging and fuzzy deteriorating rate. *International Journal of Operational Research,* (3)77-96.

[11] Goni,Maheshwari S. (2010), Supply chain model for retailer's ordering policy under two level of delay payments in fuzzy A environment. *Applied Mathematical Sciences*, (4), 1155-1164.

[12] Halem, K.A., Giri, B.C., Chaudhari, K.S. (2010), Lot sizing in an unreliable manufacturing system with fuzzy demand and repair time. *International Journal of industrial and System Engineering,* (5), 485-500.

[13] Singh,S.R.,Singh,C. (2008), Fuzzy Inventory Model for finite rate of Replenishment using Signed Distance Method. *International Transactions in Mathematical Sciences and Computer,* 1(1), 27-34.

[14] Wu, K.S., Ouyang, L.Y., Yang, C.T. (2006), An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging, *International Journal of Production Economics*. (101), 369-384.

[15] Ouyang,L.Y., Wu,K.S., Yang,C.T. (2006), A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments.*Computers and Industrial Engineering* (51), 637-651.

[16] Partha, G.,Maiti, M.K.,Maiti, M. (2013), Two storage inventory model of a deteriorating item with variable demand under parial credit period. *Applied Soft Computing.* (13), 428-448.

[17] Maiti, M.K., Maiti, Manmohan. (2006), Fuzzy inventory model with two ware house under possibility constraints, Fuzzy Sets and Systems (157), 52-73.

[18] Singh, A Y, Swami, A., (2013), A two warehouse inventory model for decaying items with exponential demand and variable holding cost. *International Journal of Inventive Engineering and Sciences* Vol. (1) Issue 5.

[19] Maiti, M.K.,Maiti, M. (2007), Two storage inventory model with lot size dependent fuzzy lead time under possibility constraints via genetic algorithm. *European Journal of Operation Research*(179), 352-371.

[20] Rong, M., Mahapatra, N.K., Maiti M., (2008), A two warehouse inventory model for a deteriorating item wit partially /fully backlogged shortages and fuzzy lead time.*European Journal of Operation Research*(189), 59-75.

[21] Datta, P., Chaudhary, P.D. (2015), A fuzzy based two warehouse inventory model for deteriorating items with cubic demand and differerent fuzzy cost parameters.*International Journal of Engineering Research and General Science* Volume(3), Issue 5.

[22] Mandal, A. W., Islam, S. (2015), A fuzzy two warehouse inventory model for power demand pattern, shortages with partially backlogged, *International Journal of Advance Engineering and Nano Technology (IJAENT)* Volume (2) Issue 8.

[23] Maragatham, M., Lakshmidevi, P.K.(2014), A fuzzy inventory model for deteriorating items with price dependent demand, International *Journal of Fuzzy Mathematical Archive* Vol. 5, No. (1), 39-47.

[24] Kumar, N.,Singh, S.R., Kumari R. (2013), Two warehouse inventory model of deteriorating items with three component demand rate and time-proportional backlogging rate in fuzzy environment, *International Journal of Industrial Engineering Computations* (4), 587-598.

[25] Nita, H.Shah, Hardic N.Soni, Kamlesh, A.P. (2013), Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates, Omega(41), 421-430.

[26] Panda, S., Saha, S., Basu, M. (2013), Optimal pricing and lot sizing for perishable inventory with price and time dependent ramp –type demand, *International Journal of System Science,* 44 (1), 127-138.

[27] Chin, T.Y. (2014), An inventory model with both stock-dependent demand rate and stock dependent holding cost rate, *International Journal of Production Economics*(155) 214-221.

[28] Maiti, M.K. (2011), A fuzzy genetic algorithm with varying population size to solve an inventory model with credit–linked promotional demand in an imprecise planning horizon, *European Journal of Operational Research*(213), 96-106.

[29] Sharma, V., Chuadhary, R.R. (2013), An inventory model for deteriorating items with weibull deterioration with time dependent demand and shortages, *Research Journal of Management Sciences,* Vol. 2(3), 28-30.

[30] Sarkar, B., Sarkar, S. (2013), An improved inventory model with partial backlogging, time varying deterioration and stock dependent demand, *Economic Modelling* (30), 924-932.