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DISCUSSION ON FUZZY OPTIMIZATION MODEL FOR WATER QUALITY MANAGEMENT OF A RIVER SYSTEM

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Abstract: In this discussion, we show that the results of Sasikumar and Mujumdar [2] are not correct. Sasikumar and Mujumdar method [2] is unable to provide the best optimal solution to decision maker, if the alternative optimal solutions exists in fuzzy waste load allocation model.

Keywords: Fuzzy sets, Fuzzy optimizations

1. DISCUSION

Bellman and Zadeh [1] proposed concept of fuzzy decision. In space of alterna- tives the goals and constraints are defined roughly by fuzzy set. Fuzzy decision is defined by the union of fuzzy goals and fuzzy constraints. Intersection of fuzzy goal F and fuzzy constraints C gives us fuzzy decision Z which is defined as fuzzy set. The membership function of the fuzzy decision Z is given by

$$\mu_z(x) = \min[\mu_F(\mathrm{x}), \mu_C(\mathrm{x})]$$

The solution x^* corresponding to the maximum value of the membership function of the resulting decision Z is the optimum solution. That is,

$$\mu_Z(x^*) = \max_{\mathrm{x}\in Z}[\mu_Z(x)]$$

Crisp constraints are defined in the space of alternatives X (i.e., the decision space) which is restricted by exactly defined constraints (e.g., mass balance of flows at a junction in a river network for a water allocation problem; minimum waste treatment level imposed on the dischargers by the pollution control agency for a waste load allocation problem). By including these crisp constraints, $g_h(X)$ 0, h = 1, 2, ..., n_G, the crisp analogue of the fuzzy multiple objective optimization problem can be stated as follows [3]:

$$\max \lambda \tag{1}$$

$$\text{Subject to } \mu_Z(X) \geq \lambda, \tag{2}$$

$$g_h(X) \leq 0, \quad \forall \text{ h} \tag{3}$$

$$0 \leq \lambda \leq 1 \tag{4}$$

Sasikumar and Mujumdar [2] proposed the MAX-MIN formulation which is the based on the crisp equivalent of the fuzzy multiple objective optimization problem,

(1) through (4), The model is expressed as

$$\max \lambda \tag{5}$$

subject to

$$\mu_{E_{il}}(C_{il}) \geq \lambda \;\; \forall \;\; i,l \tag{6}$$

$$\mu_{E_{jl}}(C_{jl}) \geq \lambda \;\; \forall \;\; j,l \tag{7}$$

$$\mu_{F_{imn}}(x_{imn}) \geq \lambda \;\; \forall \;\; i,m,n \tag{8}$$

$$\mu_{F_{jmn}}(x_{jmnl}) \geq \lambda \;\; \forall \; j,m,n \tag{9}$$

$$C_{il}^{L} \leq C_{il} \leq C_{il}^{D} \;\; \forall \;\; i,l \tag{10}$$

$$C_{jl}^{D} \leq C_{jl} \leq C_{jl}^{H} \;\; \forall \; j,l$$

$$x_{imn}^{L} \leq x_{imn} \leq x_{imn}^{M} \;\; i,\; m,\; n$$

$$x_{jmn}^{L} \leq x_{jimn} \leq x_{jimn}^{M}$$

$$x_{imn}^{MIN} \leq x_{imn} \leq x_{imn}^{MAX} \;\; \forall$$

$$x_{jmn}^{MIN} \leq x_{jmn} \leq x_{jmn}^{MAX} \;\; \forall \tag{11}$$

In the following example we have shown that, if the fuzzy program has alternative optimal solutions, then the Sasikumar and Mujumdar method [2] may not always present the “best” solution.

Consider the following problem:

(P) maximize $x_1 + x_2$

subject to

$$x_1 + 2x_2 \leq 10, \tag{17}$$

$$-2x_1 + x_2 \leq 3, \tag{18}$$

$$x_1 + x_2 \leq 12, \tag{19}$$

$$x_1 \geq 0, x_2 \geq 0 \tag{20}$$

Based on the Eqs. (5)-(16), we can obtain the following crisp problem for different aspiration levels.

(CP) maximize λ

subject to

$$\lambda \leq -2 + x_1 + x_2, \tag{21}$$

$$2\lambda \le 12 - x_1 - 2x_2, \quad (22)$$

$$3\lambda \le 6 + 2x_1 - x_2, \quad (23)$$

$$4\lambda \le 16 - 2x_1 - x_2, \quad (24)$$

$$x_1 \ge 0, x_2 \ge 0 \quad (25)$$

$$0 \le \lambda \le 1 \quad (26)$$

Define parameter λ which is the minimum satisfaction level in the system defined by using the constraints (21) through (24). Corresponding to maximum value of λ^* of the parameter λ, our objective is to find λ^*. The optimum value λ^* corresponds to the maximized minimum (max-min) satisfaction level in the system. The desired level and maximum acceptable level of pollutant treatment efficiencies set by the dischargers are expressed as $N_0 = 1$, $N_1 = 2$, $N_2 = 3$, $N_3 = 4$ and $N^{max} = 3$.

According to Sasikumar and Mujumdar method [2], the optimal solution of above problem (CP) is $X_1{*} = (x_1 = 3,\ x_2 = 0)$ and $\lambda^* = 1$. But, we observed that the problem (CP) has four alternative optimal solutions, $X_2{*} = (x_1 = 0,\ x_2 = 3)$, $X_3{*} = (x_1 = 0.8,\ x_2 = 4.6)$, $X_4{*} = (x_1 = 4.66,\ x_2 = 2.66)$ and the optimal values of the objective function corresponding to the different alternative optimal solutions is, $z_1^* = 3$, $z_2^* = 3$, $z_3^* = 5.4$, $z_4^* = 7.33$.

From above example, we examine that the Sasikumar and Mujumdar method [2] maximize the value of λ, but it do not tell which solution is prefer to the other. The optimal solution $X_1^* = (x_1 = 3,\ x_2 = 0)$ is provided, by Sasikumar andMujumdar method [2], to decision maker, whereas the best optimal solution is $X_4^* = (x_1 = 4.66,\ x_2 = 2.66)$. Hence the results proposed by Sasikumar and Mujumdar [2] are not reasonable.

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