IJMMS, Vol. 13, No. 1, (January-June 2017):243-254

ISSN: 0973-3329

AN EVALUATION OF SYSTEM NON-HOMOGENEOUS DIFFERENTIAL EQUATIONS USING LINEAR LEGENDRE MULTIWAVELETS

Meenu Devi* and S.R. Verma**

Abstract

In this paper, a method for the solution of the system of non homogeneous linear differential equations with initial conditions by using Linear Legendre Multi-wavelets is proposed. The orthonormality and high vanishing moment properties of Linear Legendre Multi-wavelets are used to find out an efficient, accurate and bounded solution for the system. Finally numerical results and exact solutions are compared by tables and graphs for two examples.

Keywords: *System of Differential Equations, Legendre Multi-wavelets, Operational Matrix of Integration, Approximation Methods.*

MSC2010: *33C45; 34F05; 34K28; 42C10; 42C40; 65L80; 65T60.*

1. INTRODUCTION

Wavelets [1-4] have been very successful in scientific and engineering fields. Due to, its wide applicability generate the interest of scientist and engineers in its study.

After having study of wavelet analysis, researchers applied several wavelets for analyzing problems of major complexity and proved wavelets to be strong tools to develop new ways in system analysis.

Chen and Hasio [1] have used Haar Wavelet for solving lumped and distributed-parameter systems [5]. Balachandren and Murugeshan have solved non linear system via STWS methods [6]. Some other researchers also have discussed on system of integro-differential equations [7, 8], time varying system [9], state space system [10] and optimal control time delayed system[11-12] by using different wavelets and multiwavelets [13-18]. Because of, wavelet and

* Department of Mathematics and Statistics, Faculty of Science, Gurukula Kangri University, Haridwar-249404, India

** *Corresponding author*, Department of Mathematics and Statistics, Faculty of Science, Gurukula Kangri University, Haridwar-249404, India. *Email: srvgkv@gmail.com*

multiwavelet have the potential to convert the system into system of algebraic equations which too much easy to solve [19].

In this paper, a clear method established for solving system of non homogeneous linear differential equation via linear Lengendre multiwavelet. For solving this system, the operational matrix will be use to eliminate the integral problem [13].

In section 2, introduce Linear Legendre Multi-wavelet (LLMW). And in section 3, a method for solving the system of non homogeneous linear differential equations is developed and the theorem on the bound of approximate solution of the system of differential equations is presented. Finally two examples are solved by the present method and obtained the approximate solution in section 3.

2. LINEAR LEGENDRE MULTIWAVELETS

We can define a wavelet [3] on a family of functions constructed from translation and dilation of a single function ψ as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right), a, b \in \mathbb{R}\,;\; a \neq 0, \tag{2.1}$$

where, a and b are dilation and translation parameters respectively.

If we restrict the parameters a and b to discrete values i.e. $a = 2^{-k}$ and $b = n2^{-k}$ [3], we have

$$\psi_{a,b}(t) = 2^{k/2}\psi\left(2^k t - n\right), k, n \in \mathbb{Z} \tag{2.2}$$

Let $\phi(t)$ be a function of $L_2(\mathbb{R})$ space. It said to be scaling function for V_0 if it satisfy the following condition

$$\int_{-\infty}^{\infty} \phi(t)dt = 1 \text{ and } V_0 = span_k\{\phi(t-k)\}. \tag{2.3}$$

2.1. Definition

The nested sequence $\{V_j\}_{j=-\infty}^{\infty}$ of the subspaces of $L_2(\mathbb{R})$ with scaling function $\phi(t)$ is called multi-resolution analysis (MRA) if it satisfies the following conditions:

$$\lim_{j\to+\infty} V_j = L_2\,(\mathbb{R})\; and\; \lim_{j\to-\infty} V_j = \{0\}$$

$$\phi(t) \in V_0 \Leftrightarrow \phi\left(2^j t\right) \in V_j$$

$\{\phi(t-n)\}_{n=-\infty}^{\infty}$ is a Riesz basis of V_0.

For any orthogonal MRA with a multi-scaling function ϕ. There exists multi-wavelet function ψ orthogonal ϕ to each other, given by [3]:

$$\psi(t) = 2^{1/2} \sum_{n=-\infty}^{\infty} b_n \phi(2t-n) \tag{2.4}$$

And $\{\psi_{j,n}\}_{n=-\infty}^{\infty}$ form an orthonormal basis for $L_2(\mathbb{R})$ certain condition. For construction the linear legendre multiwavelet, firstly the scaling functions is defined $\phi_0\ (t)\ and\ \phi_1\ (t)$ as following:

$$\phi_0\ (t) = 1, \phi_1\ (t) = \sqrt{3}(2t-1), 0 \leq t < 1. \tag{2.5}$$

By the definition of MRA,

$$\psi^0(t) = \begin{cases} -\sqrt{3}(4t-1), 0 \leq t < \frac{1}{2} \\ \sqrt{3}(4t-3), \frac{1}{2} \leq t < 1 \end{cases};\ \psi^1(t) = \begin{cases} (6t-1), 0 \leq t < \frac{1}{2} \\ (6t-5), \frac{1}{2} \leq t < 1 \end{cases}. \tag{2.6}$$

We construct the Linear Legendre multi-wavelet by translating and dilating the mother wavelet and ψ are given by

$$\psi_{k,n}^j(t) = 2^{k/2}\psi^j\left(2^k t - n\right), k, n, j \in \mathbb{Z}. \tag{2.7}$$

The family $\{\psi_{k,n}^j(t)\}$ form an orthonormal basis for $L_{\mathbf{2}}(\mathbb{R})$ and subfamily $\psi_{k,n}^j$ where $n = 0, 1, 2, \ldots, 2^k - 1, k = 0, 1, 2, \ldots$ and $j = 0,1$ is an orthonormal for $L_2[0, 1]$.

$$\psi_{0,0}^0(t) = \begin{cases} -\sqrt{3}(4t-1), 0 \leq t < \frac{1}{2} \\ \sqrt{3}(4t-3), \frac{1}{2} \leq t < 1 \end{cases};\ \psi_{0,0}^1(t) = \begin{cases} (6t-1), 0 \leq t < \frac{1}{2} \\ (6t-5), \frac{1}{2} \leq t < 1 \end{cases}$$

$$\psi_{1,0}^0(t) = \begin{cases} -\sqrt{6}(8t-1), 0 \leq t < \frac{1}{4} \\ \sqrt{6}(8t-3), \frac{1}{4} \leq t < \frac{1}{2} \\ 0, \frac{1}{2} \leq t < 1 \end{cases};\ \psi_{1,1}^0(t) = \begin{cases} 0, 0 \leq t < \frac{1}{2} \\ -\sqrt{6}(8t-5), \frac{1}{2} \leq t < \frac{3}{4} \\ \sqrt{6}(8t-7), \frac{3}{4} \leq t < 1 \end{cases}$$

$$\psi_{1,0}^1(t) = \begin{cases} \sqrt{2}(12t-1), 0 \leq t < \frac{1}{4} \\ \sqrt{2}(12t-5), \frac{1}{4} \leq t < \frac{1}{2} \\ 0, \frac{1}{2} \leq t < 1 \end{cases};\ \psi_{1,1}^1(t) = \begin{cases} 0, 0 \leq t < \frac{1}{2} \\ -\sqrt{2}(12t-7), \frac{1}{2} \leq t < \frac{3}{4} \\ \sqrt{2}(12t-11), \frac{3}{4} \leq t < 1 \end{cases}. \tag{2.8}$$

3. METHOD OF THE SOLUTION OF SYSTEM OF NON HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS:

Consider the system [20]:

$$x'(t) + a_1 x(t) + a_2 y(t) = f_1(t) \tag{3.1}$$

$$y'(t) + b_1 x(t) + b_2 y(t) = f_2(t) \tag{3.2}$$

with initial conditions $x(0) = a, y(0) = b$,

where, a_1, a_2, b_1, b_2, a and b are constants and $f_1(t)$ and $f_2(t)$ are the known function of t.

After approximation $a_1, a_2, b_1, b_2, a, b, x'(t), y'(t), x(t), y(t)$, $f_1(t)$ and $f_2(t)$ with the help of function approximation[13,19] and Operational matrix of Integration [11, 13, 17, 18, 21], it can be get

$$\left.\begin{array}{c} a = A^T\Psi(t);\ b = B^T\Psi(t);\ a_i = A_i^T\Psi(t) \\ b_i = B_i^T\Psi(t); f_i(t) = F_i^T\Psi(t) \end{array}\right\}, \tag{3.3}$$

where, $i = 1, 2$. Now

and
$$\left.\begin{array}{c} x'(t) \approx\ \tilde{x}'^{(t)} = X^T\Psi(t) \\ x(t) = \int_0^x x'(t)dt + x(0) \approx \tilde{x}(t) = X^T P\Psi(t) + A^T\Psi(t) \\ y'(t) \approx\ \tilde{y}'^{(t)} = Y^T\Psi(t), \\ y(t) = \int_0^x y'(t)dt + y(0) \approx \tilde{y}(t) = Y^T P\Psi(t) + B^T\Psi(t) \end{array}\right\} \tag{3.4}$$

Now, by using equations (3.3) and (3.4), it can be reduce the equation (3.1) as below respectively

$$\Psi^T(t)X + A_1^T\Psi(t)[\Psi^T(t)P^TX + \Psi^T(t)A] + A_2^T\Psi(t)[\Psi^T(t)P^TY + \Psi^T(t)B]$$
$$= \Psi^T(t)F_1$$

i.e.,

$$\Psi^TX + \Psi^T\tilde{A}_1P^TX + \Psi^T\tilde{A}_1A + \Psi^T\tilde{A}_2P^TY + \Psi^T\tilde{A}_2B = \Psi^T(t)F_2$$

i.e.
$$\left(I + \tilde{A}_1P^T\right)X + \tilde{A}_2P^TY = F_1 - \tilde{A}_1A - \tilde{A}_2B$$

$$\alpha_1X + \alpha_2Y = \alpha_3 \tag{3.5}$$

where, $\alpha_1 = \left(I + \tilde{A}_1P^T\right), \alpha_2 = \tilde{A}_2P^T$ and $\alpha_3 = F_1 - \tilde{A}_1A - \tilde{A}_2B$.

similarly, equation (3.2) reduce as

$$\left(I + \tilde{B}_2P^T\right)Y + \tilde{B}_1P^TX = F_2 - \tilde{B}_1A - \tilde{B}_2B$$

i.e.,

$$\beta_1X + \beta_2Y = \beta_3, \tag{3.6}$$

where, $\beta_2 = \left(I + \tilde{B}_2P^T\right)$, $\beta_1 = \tilde{B}_1P^T$ and $\beta_3 = F_2 - \tilde{B}_1A - \tilde{B}_2B$.

One can get the values of X and Y from the equations (3.5) and (3.6) with the help of ref. [22] and putting these values in equation (3.4) we obtain $\tilde{x}(t)$ and $\tilde{y}(t)$.

Theorem 3.1: Let $y(t) = [y_1(t), y_2(t), \dots . y_m(t)]^T$ is the exact solution of the non homogeneous system $y'(t) = Ay(t) + b(t)$ [20]. If $\tilde{y}(t) \epsilon C^m[0,1]$ is the approximate solution of this system, then

$$\|\tilde{y}(t)\|^2 \le \left[\sum_{i=0}^{1} \mathrm{I}_i + \sum_{k=0}^{M} \sum_{j=0}^{1} \sum_{n=0}^{2^M-1} I_{k,n}^j\right] . [\|y_1(t)\|^2 + \|y_2(t)\|^2 + \cdots + \|y_m(t)\|^2] ,$$

where, $\mathrm{I}_i = 1$ and $I_{k,n}^j = 1$, for every i, j, k and n.

Proof: Let $\tilde{y}(t) = C^T \Psi(t)$, then

$$C^T \Psi(t) = [{C_1}^T \Psi(t), {C_2}^T \Psi(t), \dots\ {C_m}^T \Psi(t)]^T.$$

Taking the norm of both sides

$$\|C^T \Psi(t)\|^2 = \left[\left|{C_1}^T \Psi(t)\right|^2 + \left|{C_2}^T \Psi(t)\right|^2 + \cdots + \left|{C_m}^T \Psi(t)\right|^2\right] \tag{3.7}$$

$$= [|\langle C_1, \Psi(t)\rangle|^2 + |\langle C_2, \Psi(t)\rangle|^2 + \cdots + |\langle C_m, \Psi(t)\rangle|^2],$$

(Using inner product)

$$\le [\|C_1\|^2 \|\Psi(t)\|^2 + \|C_2\|^2 \|\Psi(t)\|^2 + \cdots + \|C_m\|^2 \|\Psi(t)\|^2],$$

(Using Schwarz inequality)

$$\le \|C_1\|^2 + \|C_2\|^2 + \cdots\ \|C_m\|^2, \tag{3.8}$$

(By orthonormality).

From [10, 11, 18, 21], we have

$$\left.\begin{array}{l} y_1(t) \approx \sum_{i=0}^{1} (c_i)_1 \phi_i(t) + \sum_{k=0}^{M} \sum_{j=0}^{1} \sum_{n=0}^{2^k-1} \left(c_{k,n}^j\right)_1 \psi_{k,n}^j(t) = {C_1}^T \Psi(t) \\ y_2(t) \approx \sum_{i=0}^{1} (c_i)_2 \phi_i(t) + \sum_{k=0}^{M} \sum_{j=0}^{1} \sum_{n=0}^{2^k-1} \left(c_{k,n}^j\right)_2 \psi_{k,n}^j(t) = {C_2}^T \Psi(t) \\ \quad\quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad . \\ \quad\quad\quad\quad\quad . \\ y_n(t) \approx \sum_{i=0}^{1} (c_i)_m \phi_i(t) + \sum_{k=0}^{M} \sum_{j=0}^{1} \sum_{n=0}^{2^k-1} \left(c_{k,n}^j\right)_m \psi_{k,n}^j(t) = {C_m}^T \Psi(t) \end{array}\right\} \tag{3.9}$$

where,

$$\left.\begin{array}{c} {C_1}^T = \left[(c_0)_1, (c_1)_1, \dots, \left(c_{M,2^M-1}^1\right)_1\right], \\ \dots, \\ {C_m}^T = \left[(c_0)_m, (c_1)_m, \dots .. \left(c_{M,2^M-1}^1\right)_m\right] \end{array}\right\} \tag{3.10}$$

and

$$(c_i)_l = \langle y_l(t),\ \phi_i(t)\rangle, \left(c_{k,n}^j\right)_l = \langle y_l(t),\ \psi_{k,n}^j(t)\rangle, \tag{3.11}$$

where, $i, j = 0, 1, l = 1, 2, 3, \dots, n, k = 0,1, \dots, M, n = 0,1, \dots, 2^M - 1$.

The equation (3.8) reduces in to the following form using (3.10) and (3.11),

$$\|C^T\Psi(t)\|^2 \leq \left[|\langle y_1(\mathrm{t}), \phi_0(t)\rangle|^2 + |\langle y_1(\mathrm{t}), \phi_1(t)\rangle|^2 + \cdots + \left|\langle y_1(\mathrm{t}), \psi^1_{M,2^M-1}\rangle\right|^2\right]$$
$$+ \left[|\langle y_2(\mathrm{t}), \phi_0(t)\rangle|^2 + |\langle y_2(\mathrm{t}), \phi_1(t)\rangle|^2 + \cdots + \left|\langle y_2(\mathrm{t}), \psi^1_{M,2^M-1}\rangle\right|^2\right]$$
$$+ \cdots + \left[|\langle y_m(\mathrm{t}), \phi_0(t)\rangle|^2 + |\langle y_m(\mathrm{t}), \phi_1(t)\rangle|^2 + \cdots + \left|\langle y_m(\mathrm{t}), \psi^1_{M,2^M-1}\rangle\right|^2\right]. \tag{3.12}$$

Again using Schwarz inequality, equation (3.12) gets the following form,

$$\|C^T\Psi(t)\|^2 \leq \|y_1(t)\|^2 + \|y_2(t)\|^2 + \cdots + \|y_m(t)\|^2.$$
$$\times \left[\|\phi_0(t)\|^2 + \|\phi_1(t)\|^2 + \cdots + \left\|\psi^1_{M,2^M-1}\right\|^2\right], \tag{3.13}$$

If $\|\phi_i(t)\|^2 = I_i = 1$,

where, $i = 0,1$ and

$$\left\|\psi^1_{M,2^M-1}\right\|^2 = I^1_{M,2^M-1} = 1,$$

then one has

$$\|\tilde{y}(t)\|^2 \leq \left[\sum_{i=0}^{1} I_i + \sum_{k=0}^{M} \sum_{j=0}^{1} \sum_{n=0}^{2^M-1} I^j_{k,n}\right]. [\|y_1(t)\|^2 + \|y_2(t)\|^2 + \cdots + \|y_m(t)\|^2]. \tag{3.14}$$

Corollary 3.2: If $M = 1$, any approximate solution $\tilde{y}(t)$ of the system

$$y'(t) = [A]_{2\times2} y(t) + [b(t)]_{2\times1},$$

where, $y(t) \epsilon\, C^2[0,1]$ will satisfy

$$\|\tilde{y}(t)\|^2 \leq 8\, [\|y_1(t)\|^2 + \|y_2(t)\|^2].$$

Proof: Taking $M = 1$ and $m = 2$ in equation (2.14), we get

$$\|\tilde{y}(t)\|^2 \leq 8\, [\|y_1(t)\|^2 + \|y_2(t)\|^2].$$

4. ILLUSTRATIVE EXAMPLES

Example 4.1: Consider the system of non homogeneous linear differential equations

$$x'(t) + x(t) + y(t) = 1 \tag{4.1}$$

$$y'(t) + y(t) + x(t) = 0 \tag{4.2}$$

with initial conditions $x(0) = 0, y(0) = 0$.

Approximating the unknown functions $x(t), y(t)$, $x'(t)$ and $y'(t)$ and constant 1, we have

and
$$\left.\begin{array}{c} x'(t) \approx \tilde{x}'^{(t)} = X^T\Psi(t), \\ x(t) = \int_0^x x'(t)dt + x(0) \approx \tilde{x}(t) = X^T P\Psi(t) \\ y'(t) \approx \tilde{y}'^{(t)} = Y^T\Psi(t), \\ y(t) = \int_0^x y'(t)dt + y(0) \approx \tilde{y}(t) = Y^T P\Psi(t) \end{array}\right\} \tag{4.3}$$

$1 \approx D^T\Psi(t)$,

where, $X = [x_0, x_1, \dots . x_7]^T$, $Y = [y_0, y_1, \dots . y_7]^T$, P is operational matrix of integration and LLMW bases $\Psi(t) = [\phi_0, \phi_1, \psi_{00}^0, \psi_{00}^1, \psi_{10}^0, \psi_{10}^1, \psi_{11}^0, \psi_{11}^1]^T$ by ref. [11, 13].

After using equation (4.3), the equations (4.1) and (4.2) resulted into the following forms respectively

$$\Psi^T(t)X + \Psi^T(t)P^TX - \Psi^T(t)P^TY = \Psi^T(t)E \tag{4.4}$$

$$\Psi^T(t)Y + \Psi^T(t)P^TY - \Psi^T(t)P^TX = 0 \tag{4.5}$$

From equation (4.5), we get

$$P^TX = [I + P^T]Y \tag{4.6}$$

With this value of X, equation (4.4), gives

$$[I + 2P^T]Y = P^TE \tag{4.7}$$

the simplification of the equations (4.6) and (4.7) yields:

$$X = \begin{bmatrix} \frac{10852921}{27691682}, \frac{4867081}{13845841\sqrt{3}}, -\frac{113484\sqrt{3}}{13845841}, \frac{90579}{55383364}, \\ -\frac{6\sqrt{6}}{3721}, -\frac{8214\sqrt{6}}{13845841}, \frac{3}{7442\sqrt{2}}, \frac{4107}{27691682\sqrt{2}} \end{bmatrix}^{\mathrm{T}}$$

$$\mathrm{Y} = \begin{bmatrix} \frac{1496460}{13845841}, \frac{4111679}{27691682\sqrt{3}}, \frac{113484\sqrt{3}}{13845841}, -\frac{90579}{55383364}, \\ \frac{6\sqrt{6}}{3721}, \frac{8214\sqrt{6}}{13845841}, -\frac{3}{7442\sqrt{2}}, -\frac{4107}{27691682\sqrt{2}} \end{bmatrix}^{\mathrm{T}}.$$

With these values of X and Y, one can get from equation (4.3)

$$\tilde{x}(t) = \begin{cases} \frac{1}{244}(1 + 218x), 0 \le x < \frac{1}{4} \\ \frac{613+10994x}{14884}, \frac{1}{4} \le x < \frac{1}{2} \\ \frac{79129+585386x}{907924}, \frac{1}{2} \le x < \frac{3}{4} \\ \frac{5(1431929+6510874x)}{55383364}, \frac{3}{4} \le x < 1 \end{cases};$$

$$\tilde{y}(t) = \begin{cases} \frac{1}{244}(-1+26x), & 0 \le x < \frac{1}{4} \\ \frac{-613+3890x}{14884}, & \frac{1}{4} \le x < \frac{1}{2} \\ \frac{-79129+322538x}{907924}, & \frac{1}{2} \le x < \frac{3}{4} \\ \frac{-7159645+22828994x}{55383364}, & \frac{3}{4} \le x < 1 \end{cases}$$

The exact $[x(t), y(t)]$ and approximate $[\tilde{x}(t), \tilde{y}(t)]$ solutions are depicted in Figure 1 and Figure 2:

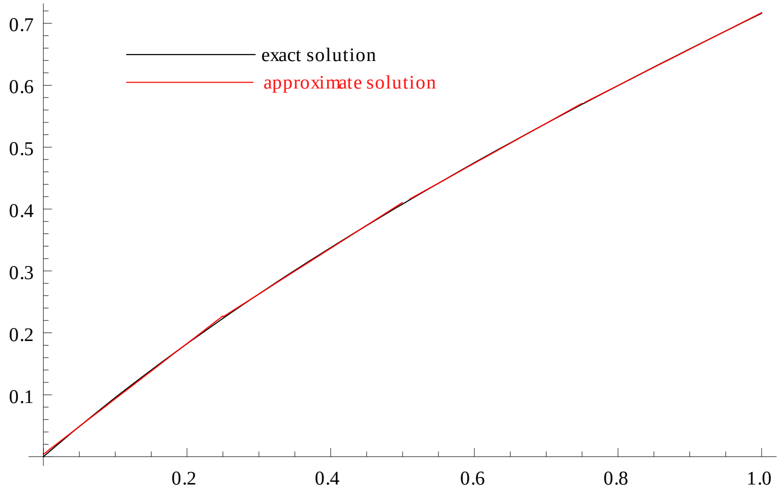

Figure 1: $\boldsymbol{x(t), \tilde{x}(t)}$

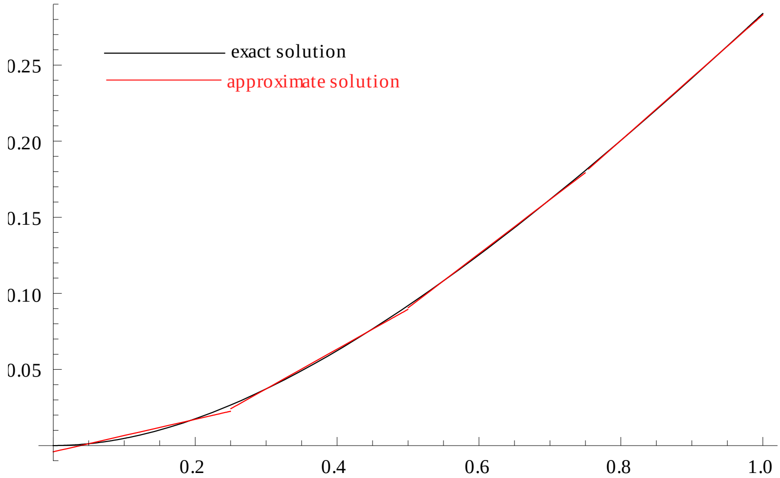

Figure 2: $\boldsymbol{y(t), \tilde{y}(t)}$

Absolute error of exact $[x(t), y(t)]$ and approximate $[\tilde{x}(t), \tilde{y}(t)]$ solution:

Table 1

t	*Exact solution* $x(t)$	*Approximate solution* $\tilde{x}(t)$	*Error* $\|x(t) - \tilde{x}(t)\|$	*Exact solution* $y(t)$	*Approximate solution* $\tilde{y}(t)$	*Error* $\|y(t) - \tilde{y}(t)\|$
0.0	0	0.00409836	4.09×10^{-3}	0	-0.00409836	4.09×10^{-3}
0.1	0.0953173	0.0934426	1.8×10^{-3}	0.00468269	0.00655738	1.8×10^{-3}
0.2	0.18242	0.182787	3.6×10^{-4}	0.01758	0.0172131	3.6×10^{-4}
0.3	0.262797	0.262779	1.8×10^{-5}	0.0372029	0.0372212	1.8×10^{-5}
0.4	0.337668	0.336643	1.02×10^{-3}	0.0623322	0.0633566	1.02×10^{-3}
0.5	0.40803	0.40953	1.4×10^{-3}	0.0919699	0.0904701	1.4×10^{-3}
0.6	0.474701	0.474005	6.9×10^{-4}	0.125299	0.125995	6.9×10^{-4}
0.7	0.538351	0.53848	1.2×10^{-4}	0.161649	0.16152	1.2×10^{-4}
0.8	0.599526	0.599515	1.1×10^{-5}	0.200474	0.200485	1.1×10^{-5}
0.9	0.658675	0.658295	3.8×10^{-4}	0.241325	0.241705	3.8×10^{-4}
1.0	0.716166	0.743439	2.7×10^{-2}	0.283834	0.256561	2.7×10^{-2}

Example.4.2: Consider the system of non homogeneous linear differential equations

$$x'(t) + y(t) = 1 \tag{4.8}$$

and

$$y'(t) - x(t) = 0 \tag{4.9}$$

with initial condition $x(0) = 0, y(0) = 0$.

Likewise the example 4.1, one can get

$$X = \begin{bmatrix} \frac{866860189305292800}{1885736598705120001} \\ \frac{268961243154645118\sqrt{3}}{1885736598705120001} \\ -\frac{18517804350289920\sqrt{3}}{1885736598705120001} \\ -\frac{345824896658803 8}{1885736598705120001} \\ -\frac{880128\sqrt{6}}{1373221249} \\ -\frac{3329924551100928\sqrt{6}}{1885736598705120001} \\ -\frac{215430\sqrt{2}}{1373221249} \\ -\frac{223403571206790\sqrt{2}}{1885736598705120001} \end{bmatrix} \text{ and } Y = \begin{bmatrix} \frac{298949473075323841}{1885736598705120001} \\ \frac{146933252980789440\sqrt{3}}{1885736598705120001} \\ \frac{33896831231250624\sqrt{3}}{1885736598705120001} \\ -\frac{1889237885423040}{1885736598705120001} \\ \frac{3446880\sqrt{6}}{1373221249} \\ \frac{3574457139308640\sqrt{6}}{1885736598705120001} \\ -\frac{55008\sqrt{2}}{1373221249} \\ -\frac{208120284443808\sqrt{2}}{1885736598705120001} \end{bmatrix}.$$

and
$$\tilde{x}(t) = \begin{cases} \frac{24(1+1528x)}{37057}, & 0 \le x < \frac{1}{4} \\ \frac{24(988995+53102584x)}{1373221249}, & \frac{1}{4} \le x < \frac{1}{2} \\ \frac{24(165463404677+1715014488568x)}{50887459824193}, & \frac{1}{2} \le x < \frac{3}{4} \\ \frac{24(16195818897487303+50233593698460664x)}{1885736598705120001}, & \frac{3}{4} \le x < 1 \end{cases},$$

$$\tilde{y}(t) = \begin{cases} \frac{-191+4608x}{37057} & 0 \le x < \frac{1}{4} \\ \frac{-89362943+501659136x}{1373221249} & \frac{1}{4} \le x < \frac{1}{2} \\ \frac{-8833099801919+29696339040768x}{50887459824193} & \frac{1}{2} \le x < \frac{3}{4} \\ \frac{-583024131114383999+1443605121207369216x}{1885736598705120001} & \frac{3}{4} \le x < 1 \end{cases}.$$

The exact solutions $[x(t), y(t)]$ and approximate solutions $[\tilde{x}(t), \tilde{y}(t)]$ are traced in Figure 3 and Figure 4:

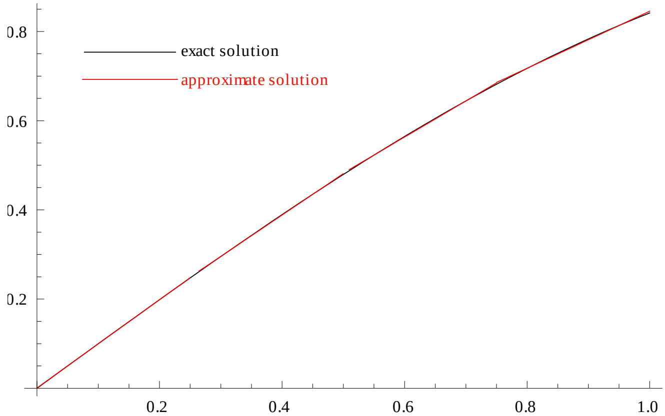

Figure 3: $x(t), \tilde{x}(t)$

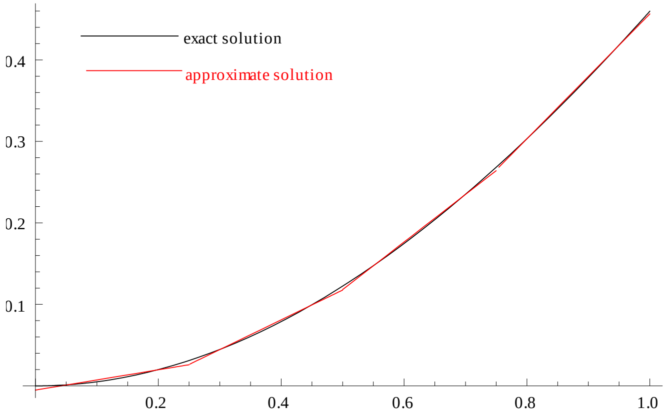

Figure 4: $\boldsymbol{y(t)}, \boldsymbol{\tilde{y}(t)}$

Absolute error of exact $[x(t), y(t)]$ and approximate $[\tilde{x}(t), \tilde{y}(t)]$ solution:

Table 2

t	*Exact solution* $x(t)$	*Approximate solution* $\tilde{x}(t)$	*Error* $\|x(t)-\tilde{x}(t)\|$	*Exact solution* $y(t)$	*Approximate solution* $\tilde{y}(t)$	*Error* $\|y(t)-\tilde{y}(t)\|$
0.0	0.000000	0.000647651	6.4×10^{-4}	0.00000	-0.00515422	5.15×10^{-3}
0.1	0.0998334	0.0996087	$2.2\times 10-^{4}$	0.00499583	0.00728068	2.2×10^{-3}
0.2	0.198669	0.19857	9.9×10^{-5}	0.0199334	0.0197156	2.1×10^{-4}
0.3	0.29552	0.295709	1.8×10^{-4}	0.0446635	0.0445193	1.4×10^{-4}
0.4	0.389418	0.388518	9×10^{-4}	0.078939	0.0810508	2.1×10^{-3}
0.5	0.479426	0.482463	3.03×10^{-3}	0.122417	0.118203	4.2×10^{-3}
0.6	0.564642	0.563348	1.2×10^{-3}	0.174664	0.17656	1.8×10^{-3}
0.7	0.644218	0.644233	1.5×10^{-5}	0.235158	0.234917	2.4×10^{-4}
0.8	0.717356	0.717589	2.3×10^{-4}	0.303293	0.303255	3.7×10^{-5}
0.9	0.783327	0.781522	1.8×10^{-3}	0.37839	0.379809	1.4×10^{-3}
1.0	0.841471	0.887581	4.6×10^{-2}	0.459698	0.392287	6.7×10^{-2}

4. CONCLUSION

In this paper, a well- organized method for solving system of non homogeneous linear differential equations is derived. Two examples are solved by this method and got more accurate solutions which are depicted by graphs because the exact and approximate solutions are all most over lapping and bounded too.

References

[1] B. Han, Q. T. Jiang, Multi-wavelets on the interval, Appl. Comput. Harm. Anal. Vol. 12(2002), 100-127.

[2] C. K. Chui, An Introduction to Wavelets, Boston Academic Press, 1992.

[3] I. Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, PA, 1992.

[4] H. L. Resnikoff, et. al, Wavelet Analysis, Springer-Verlag New York, Inc. 2004.

[5] C. F. Chen, C. H. Hsiao, Haar wavelet method for solving lumped and distributed-parameter systems, IEE Proc-Control Theory Appl. Vol. 144 No. 1(1997), 87-94.

[6] K. Balachandran, K. Murugeshan, Analysis of non linear singular system via STWS Methods, Int. J. Com. Math. Vol. 36(1990), 9-12.

[7] E. Yusufoglu, An efficient algorithm for solving integro-differential equations system, Appl. Math. Comput. Vol. 192(2007), 51–55.

[8] K. Maleknejad, M. Tavassoli Kajani, Solving linear integro-differential equation system by Galerkin methods with hybrid functions, Appl. Math. and Comp. Vol. 159(2004), 603–612.

[9] B. Sepehrain, M. Razzaghi, Solution of time-varying singular non linear systems by single term Walsh series, Mathem. Prob. in Eng., Vol. 3(2003), 129-136.

[10] H-Y Chung, Solution of state-space equation via Fourier series, Int. J. of Syst. Sci., Vol. 18 No.2(1987), 221-228.

[11] F. Khallat, Optimal Control of Linear Time Delayed System by Linear Legendre Multi-wavelet, J. Optim. Theory Appl., Vol. 143(2009)107-121.

[12] X. T. Wang, A Numerical Approach of Optimal Control for Generalized Delay System by Genaral Legendre Wavelets, Int. J. of Com. Math., Vol. 86 No. 4(2009), 743-752.

[13] F. Khellat, S. A. Yousefi, The linear Legendre mother wavelets operational matrix of integration and its application, J. Frankl. Inst. Vol. 343(2006), 181-190.

[14] J. S. Gu, W. S. Jiang, the Haar wavelet operational matrix of integration, International Journal of systems science, Vol. 27(1996), 623-628.

[15] H. Danfu, S. Xufeng, Numerical solution of integro-differential equations by using CAS wavelet operational matrix of integration, Appl. Math. Comput., Vol. 194(2007), 460-466.

[16] E. Babolian, F. Fattahzdeh, Numerical solution of differential equations by using Chebyshev wavelet operational matrix of integration, Appl. Math. Comput., Vol. 188(2007), 417-426.

[17] M. Devi, et al, Solving differential equations of second order using quadratic Legendre multi-wavelets (QLMW) with operational matrix of integration, Int. J. Comput. App. Vol. 75 No. 15(2013), 43-49.

[18] M. Razzaghi, S. Yousefi, The Legendre wavelets operational matrix of integration, Int. J. Syst. Sci, Vol. 32 No. 4(2001), 495-502.

[19] G. Beylkin, et al, Fast wavelet transforms and numerical algorithms-I, Commun. Pure Appl. Math. Vol. 44(1991), 141-183..

[20] L. Ya. Adrianova, Introduction to Linear System of Differential Equation (Translations of Mathematical Monographs) Vol. 146, AMS, Rhode Isaland (1995).

[21] M. I. Bhatti, P. Brachen, Solution of differential equation in Bernstein polynomial bases, J. Comput. Appl. Math., Vol. 205, (2007), 272-280.

[22] A. Cohan, Numerical Analysis of wavelet Methods, Imprint Jai Press Elsevier (April 2003).